

Mathematical Modeling Approaches to Solve the Line Balancing Problem

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Keywords: Framework, Review, Survey, Assembly Line, Line Balancing, Mathematical Model.

Abstract: The assembly line balancing problem belongs to the class of NP-hard combinatorial optimisation problem. For several decades' line balancing took attention of researchers who are trying to find the solutions for real world applications. Although tremendous works have been done, the gap still exists between the research and the real problems. This paper provides analysis of about 50 papers that used mathematical modeling in solving line balancing problems. Thereafter, a framework is proposed for future work.

1 INTRODUCTION

Assembly lines consist of a number of workstations that are arranged through a material handling equipment where tasks are assigned, and the workpieces are moving from one station to another until the final product is produced. The workstations are equipped with all required machines and skilled operators to perform specific tasks without violating cycle time, which represents the time between two consecutive units produced from the assembly line based on specific production plan. From the first day that Henry Ford introduced assembly line for a mass production in this company, the researchers are seeking for the optimal way to assign all tasks to workstations that is called Assembly Line Balancing Problem (ALBP) without violating assignment constraints (such as precedence constraints). (Dolgui & Battai 2013). For instance, when we increase the balance of workload among workstations that will lead to increase productivity by removing bottlenecks and reducing idle time. Each task has a particular time to perform called processing time and the workstation time is the sum of all processing times for all assigned tasks. Figure (1): An illustrative example for assembly line balancing problem.

Salveson made the first mathematical model formulation for assembly line balancing problem (Salveson 1955), and from this day the ALBP has become an attractive topic for more research.

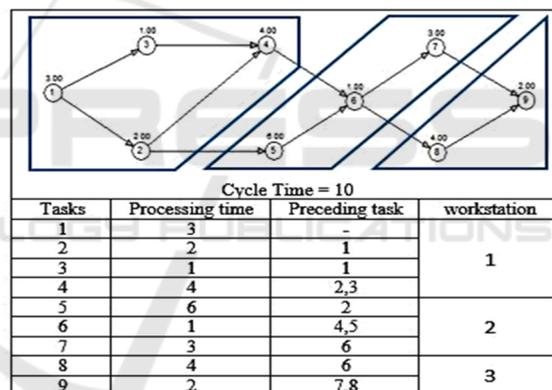


Figure 1: Assembly line balancing problem.

The ALBP is an NP-hard combinatorial optimisation problem (Gutjahr & Nemhauser 1964) and the widely used objective functions are to minimise the number of workstations with fixed cycle time (SALBP-1), minimise the cycle time with fixed number of workstations (SALBP-2) and maximize line efficiency (SALBP-E). The ALBP can be classified based on Industrial environment (Machining, Assembly, Disassembly). Another classification considers the number of product models in the line (Single model, Mixed model, Multi-model). The line layout is also a different theme of classification (Basic straight line, Straight lines with multiple workplaces, U-shaped lines, Lines with the circular transfer, Asymmetric lines). Last but not least the nature of task times (Deterministic – Stochastic)

(Dolgui & Battai 2013) (Sivasankaran & Shahabudeen 2014a).

In the past, the single-model lines were commonly used for producing large and homogeneous products, so it was a daunting task to provide any customized products. Nowadays, due to the increasing demand and competition for creating customised products, a large number of traditional lines are replaced by mixed-model lines to keep up with current market trends (Vilarinho & Simaria 2002) (Dong et al. 2014). On the other hand, the multi-model lines produce batches of different products that requires setup time to Initialize the machines between different batches. The difference between single model, mixed-model and multi-model are illustrated in figure (2).

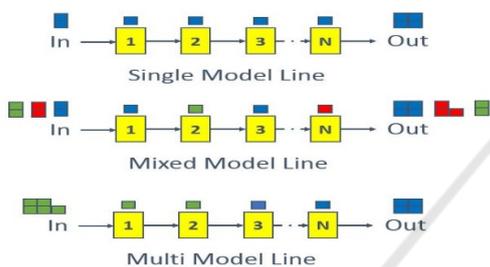


Figure 2: Different line configurations based on a number of models.

Additionally, there are many assumptions used to reduce the level of complexity of the ALBP such as deterministic processing time, fixed cycle time, etc. Consequently, the challenges facing researchers is to reduce these assumptions as possible to simulate the real-life problems. The U-shaped lines have been introduced for the first time by the Japanese Factories where high experience workers were hired to increase variability and quality of products. However, the balancing for U-lines is more complicated compared to traditional lines. Nevertheless, it can provide many advantages such as; less work-in-process, less worker movement, increase line efficiency and increase flexibility in production rate. The straight line and U-shaped line are illustrated in figure (3).

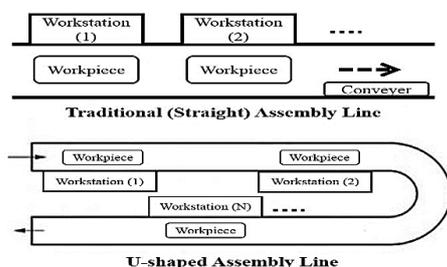


Figure 3: Different line layouts.

The ALBP has been intensively discussed in the literature. As a result, many recent reviews have been published (Boysen et al. 2007), (Battaia & Dolgui 2013) and (Sivasankaran & Shahabudeen 2014b). In this paper, we focus on analysing published articles that formulated mathematical models to solve different configurations of assembly line balancing problems. Furthermore, a framework with improvements in the model formulations is proposed to tackle ALBP.

The remainder of this paper is organised as follows: The second section presents the classification of assembly line balancing problems as well as reviewing and analysing the articles published in each category. The third section is dedicated to providing further research areas and concludes remarks of this study. The last section explains the proposed framework.

2 REVIEW ON MATHEMATICAL MODELS

This section represents a taxonomy of the mathematical models used in describing a wide range of different assembly lines configurations:

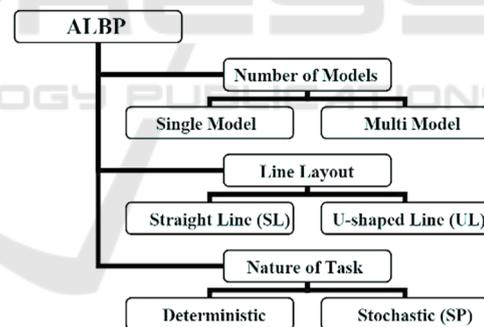


Figure 4: Assembly Lines Configurations.

2.1 Single-Model and Straight Type Assembly Lines with Deterministic Processing Times

The first formulation for SALBP by (Bowman 1960) used linear programming by using two different linear program forms. Also, Some modifications were introduced by (White 1961). Additionally, (Thangavelu & Shetty 1971) developed an improved 0-1 integer programming version of Bowman-White model by simplifying certain steps in (Geoffrion 1967) 0-1 integer programming algorithm. Moreover,

(Patterson & Albracht 1975) formulated an improved 0-1 integer-programming model draws heavily on the work done by (Bowman 1960) taking into consideration to determine feasibility and reduce computational time by reducing the required number of variables. Furthermore, (Talbot & Patterson 1984) presented an integer programming formulation for defining all feasible assignments for each task to a workstation with upper and lower bounds. Thereafter, they solved it by using a modified Balas algorithm (Balas et al. 1965). In (Vitria 2004) the authors analysed different ways for modelling precedence and incompatibility constraints in ALBP to obtain the best modelling formulation and solving procedure. (Pastor & Ferrer 2009) used the two efficient models of (Vitria 2004) for both SALBP-1 and SALBP-2 besides introducing additional constraints based on the upper bound of the number of workstations or the cycle time that belongs to the branch and bound technique. The research carried by (Özcan & Toklu 2009) presented mathematical model used a goal programming and a fuzzy goal programming for a two-sided assembly line to minimise the number of mated workstations at first and then minimise the number of the workstations as a secondary goal. (Esmailbeigi et al. 2015) presented the mixed integer programming for maximizing the line efficiency (SALBP-E) as well as providing secondary objectives (SALPB-1, SALBP-2, minimizing smoothness index) for the problem. Their proposed model is considered as the first MILP model for getting an exact solution directly in SALBP-E.

2.2 Single-Model and U-type Assembly Lines with Deterministic Processing Times

(Miltenburg & Wijngaard 1994) proposed the first model for the simple U-line balancing and used a dynamic programming procedure for obtaining the optimal solution. In (Urban 1998) the authors formulated an integer programming model for optimally solving UALBP-1. (Gökçen & Ağpak 2006) introduced the first multi-criteria for decision-making technique for U-shaped lines. They formulated a mathematical model using a goal programming for a UALBP based on the IP model proposed by (Urban 1998). Furthermore, their model was used by (Toklu & özcan 2008) as a base for formulating the first fuzzy goal programming model with multi-objectives aiming at optimising the conflicting goals as well as helping the decision maker to determine goals in the fuzzy environment. (Kara et al. 2009) proposed binary fuzzy goal

programming models for each of the traditional and U-shaped assembly lines. They extended the linear programming model of (Urban 1998) in developing their BFGP for balancing U-lines. The improved version of the previous model in (Urban 1998) addressed in the work of (Fattahi et al. 2014), They formulated an integer programming model for UALBP-1 that was able to reduce the binary variables to half by increasing the efficiency of LP relaxation.

2.3 Single-Model and Straight Type Assembly Lines with Stochastic Processing Times

The processing times in deterministic assembly line (AL) are assumed to take constant values. Nonetheless in real life, it takes values based on probability distribution resulting from machine breakdowns, the difference in skills between operators, complex tasks, environment, and so forth. (Moodie 1964) The first research work that addressed the stochastic nature to the ALBP. (Carraway 1989) proposed two dynamic programming approaches for minimising the number of workstations. The task times assumed to be independent and normally distributed. (Ağpak & Gökçen 2007) formulated a chance-constrained 0-1 integer programming model for balancing stochastic traditional assembly line. Additionally, a goal programming has been proposed for increasing the reliability of the assembly line. (Özcan 2010) presented the first study of two-sided assembly lines with variation in task time and formulated a chance-constrained, piecewise-linear and mixed integer programming for solving this problem. In two-sided assembly lines, the workers are assigned in both sides of the production line (left and right) and used in parallel. (Hamta et al. 2013) They formulated a mixed integer non-linear programming model. Their model considered multi-objectives to simultaneously minimise the cycle time, equipment cost and the smoothness index. Finally, they developed a solution method based on the combination of particle swarm optimisation and variable neighbourhood search to solve the problem in reasonable time. (Hazır & Dolgui 2013) proposed robust optimisation models for SALBP-2 considering uncertainty through operations time and they developed an exact decomposition algorithm. They developed two mathematical models in addition to decomposition based algorithm to find the optimal solution for large problems. (Ritt et al. 2016) did not consider the variability in task times rather, they considered it indirectly by representing the variability of the workforce due to absenteeism. They proposed

a two-stage mixed integer models to minimise the cycle time. Furthermore, they presented a local search heuristic procedure based on simulated annealing for solving large instances.

2.4 Single-Model and U-type Assembly Lines with Stochastic Processing Times

The research done by (Nakade et al. 1997) is considered the first work in balancing U-lines taking stochastic nature results from manual work into consideration. They proposed approximate formulation for the upper and lower bound of the expected cycle time. (Guerrero & Miltenburg 2003) They used dynamic programming in balancing U-lines and the recursive algorithm for determining the optimal solution. (Urban & Chiang 2006) formulated a chance constraint programming model for U-line balancing problem, then they used piecewise linear, integer programming for solving the model optimally. The further investigations are to develop an efficient heuristic for solving large problems. (Ağpak & Gökçen 2007) formulated 0-1 integer programming model by using a chance-constrained procedure for balancing stochastic traditional and U-shaped lines. They used the model of (Urban 1998) as the base for their work; also they presented two linear transformations (pure and approximate) to enable the model to solve large problems. Lastly, they introduced goal programming for smoothing the workload among workstations. Most of the researches done in the U-type assembly line problems focused on deterministic processing times comparing to the stochastic time.

2.5 Multi or Mixed Model and Straight Type Assembly Lines with Deterministic Processing Times

(Gökçen & Erel 1998) introduced a binary integer programming model for the Mixed-Model Assembly Line (MMAL). Flexibility ratio has also been presented that is used to compute the computational and storage requirements for solving the problem by measuring the number of possible sequences for the precedence diagram. (Vilarinho & Simaria 2002) developed a mathematical model for balancing mixed-model assembly lines that gives the decision maker the ability to define the limit number of parallel workstations and zoning constraints. (Simaria & Vilarinho 2009) formulated a mathematical model for balancing two-sided mixed-model assembly lines. Moreover, they proposed an ant colony optimisation

algorithm for optimally solving the model. (Fattahi & Salehi 2009) developed a mixed-integer linear programming model to minimise the total utility and idle costs. They tried to solve the problem using branch and bound method, but it was very time-consuming so, they used simulated annealing to resolve this issue. (Mosadegh et al. 2012) formulated a mixed-integer linear programming model to provide the exact solution of both balancing and sequencing problems simultaneously for mixed model assembly lines. For solving the problem, they developed a simulated annealing algorithm as well as Taguchi method for calibrating the algorithm parameters. (Kucukkoc & Zhang 2014) proposed a mixed integer programming model to investigate both sequencing and balancing problems simultaneously in mixed model parallel two-sided assembly lines. The objectives of their model were to minimise the number of workstations, reduce the length of production lines and maximise workload smoothness. Furthermore, they presented an agent based ant colony optimisation algorithm for solving the problem. (Zhao et al. 2016) formulated a mathematical model for MMAL focused on the effect of mental workload and the complexity of the operations on balancing the line. They concluded that, the mental workload considered as an essential rule when minimising cycle time also, the mental workload was influenced by the level of experience of the operator.

2.6 Multi or Mixed Model and U-type Assembly Lines with Deterministic Processing Times

(Sparling & Miltenburg 1998) are considered the pioneers in studying MMUL. They presented a model for U-line balancing problem for assigning a set of tasks in a minimum number of workstations. Furthermore, they presented an approximation algorithm to solve large size problems. (Miltenburg 2002) formulated a mixed, zero-one integer, non-linear programming model then used a genetic algorithm for searching for a good solution in a reasonable computational time. (Kara 2008) formulated a non-linear mathematical model to solve balancing and sequencing problem simultaneously for MMUL. The objective of their model was to minimise deviation of workloads among workstations. Due to the complexity so, they proposed simulated annealing algorithm to solve large-scale problems. (Kara & Tekin 2009) proposed a mixed integer linear programming model for optimally balancing mixed-model U-lines. The goal

of their model was to minimise the number of workstations for a given cycle time. (Kazemi et al. 2011) introduced an integer linear programming model. The objective of their presented model was to minimise the number of stations. Furthermore, they developed a two-stage genetic algorithm approach for the large-scale problem. (Rabbani et al. 2012) formulated a multi-objective mixed integer linear programming model for two-sided Als. Finally, they introduced a heuristic based on the genetic algorithm for solving this problem. (Rabbani et al. 2016) formulated a mixed-integer linear programming model for robotic mixed-model assembly lines. The model aimed to minimise the cycle time, robot purchasing and setup costs.

2.7 Multi or Mixed Model and Straight Type Assembly Lines with Stochastic Processing Times

(Paternina-Arboleda & Montoya-Torres 2006) proposed a mathematical model for balancing and sequencing MMAL. The model included multi-objective function aimed to minimise the number of workstations, increase throughput and find the appropriate sequence of models to remove bottlenecks through an assembly line. (Al-e-hashem 2009) formulated a mixed integer robust optimization model to minimize the total costs that include the cost of workstations and duplicated tasks.

2.8 Multi or Mixed Model and U-type Assembly Lines with Stochastic Processing Times

(Agrawal & Tiwari 2008) proposed a model for balancing and sequencing mixed-model U-disassembly lines where the processing times are different depending on the structure of the products and the human factor. The objective function was to minimise the variation of workload and maximise the line efficiency. They solved this problem by using collaborative Ant Colony Optimization, and they tested the results on benchmarks using a design of experiment and analysis of variance to determine which factor is significant in the objective. (Dong et al. 2014) formulated a 0-1 stochastic programming model to solve balancing and sequencing problem simultaneously for MMUL with independently and normally distributed task times. They proposed a simulated annealing algorithm to resolve the issue into both situations (Deterministic and stochastic).

3 DISCUSSIONS AND FUTURE RESEARCH

Although researchers have contributed in various configurations and application of assembly line balancing, the gap still exists between research and real life problems. To the best of our knowledge, only two papers have been published using a mathematical model in each branch of MMAL with stochastic processing times, so the further research may be carried out in developing multi-objective mathematical models including more constraints such as zoning and distance constraints. It is clear that the majority of authors neglected the use of statistical methods in comparing the results to clarify the significant improvement between their proposed methods and previous research in the literature. Also, statistical studies are useful in determining the effect of each variable on the objective function and calibrating algorithm parameters. It is clear that more studies are applied in SLs comparing to U-lines. Thus, further work can be done in a different configuration of U-lines such as two-sided, multi lines, disassembly and rebalancing U-lines. Most of the articles neglect the human factors (skills, experience, learning effect) and working environment that directly affects the operator's performance and productivity. Consequently, it is very crucial to enhance existing mathematical models to consider these aspects in further work. Further work may be directed to consider other objective functions such as maximise the line efficiency, minimising smoothness index and minimise total costs (equipment-duplication-setup). Enhance current meta-heuristics such as simulated annealing algorithm, genetic algorithm, practical swarm optimisation, etc. That will help in solving large instances of ALBP in less computational time and provide better results especially in the case of mixed-models. In MMAL it is important to handle both ALB and ALS problems jointly. Formulate a mathematical model for multi-optimization problems such as the incorporation of line design and balancing problems.

4 THE PROPOSED SOLUTION FRAMEWORK

The objective of the ALBP-2 is to minimise the cycle time as a result of minimising the workload of the bottleneck workstation. Nevertheless, it is also important to consider the second heavily loaded workstation, the third one and so on, to improve the

reliability and the quality of the balance through the line. The proposed solution approach is to modify the model formulated by (Kucukkoc et al. 2015) to include the objective of increasing balance between and within workstations to ensure that all workstations through the line have an equal amount of work also all the workers within the workstations have the same workload. Moreover, the zoning constraints will be added to the model to increase the ability to solve real-life problems with fewer assumptions as possible. The proposed mathematical model will be coded using LINGO optimisation modelling software to solve small-sized problems. The solution from the solver will be utilised as an input to a DES model to test the robustness of solutions when introducing the real-world variability such as stochastic times, breakdowns, etc. Then a comparison will be made between the initial solution and the proposed solution from the model using the performance indicators of the simulation. Finally, statistical analysis will be implemented to evaluate the significant improvement in the assembly line.

5 CONCLUSIONS

The research on ALBP is crucial because it affects the productivity and the competitiveness of the company. This paper surveyed studies of ALBP within the area of mathematical modeling that were published in the eight branches of ALBP. The goal of this analysis was to discover the research gaps in line balancing problems. Furthermore, a proposed framework is introduced to enhance the solution of the MMAL by modifying the objective function and adding more constraints that represent realistic world problems.

ACKNOWLEDGEMENTS

This research project is sponsored by the Mitsubishi Corporation Graduate Scholarship to the Egypt-Japan University of Science and Technology (E-JUST) and support of the Japanese International Cooperation Agency (JICA).

REFERENCES

Ağpak, K. & Gökçen, H., 2007. A chance-constrained approach to stochastic line balancing problem. *European Journal of Operational Research*, 180(3), pp.1098–1115.

- Agrawal, S. & Tiwari, M.K., 2008. A collaborative ant colony algorithm to stochastic mixed-model U-shaped disassembly line balancing and sequencing problem. *International Journal of Production Research*, 46(6), pp.1405–1429.
- Al-e-hashem, S.M., 2009. Mixed model assembly line balancing problem under uncertainty. , 2009. *CIE 2009*. ... Available at: http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=5223925 [Accessed September 27, 2016].
- Balas, E., Glover, F. & Zions, S., 1965. An Additive Algorithm for Solving Linear Programs with Zero-One Variables Author(s): *Operations Research*, 13(4), pp.517–549.
- Battaia, O. & Dolgui, A., 2013. A taxonomy of line balancing problems and their solution approaches. *International Journal of Production Economics*, 142(2), pp.259–277.
- Bowman, E.H., 1960. Assembly-Line Balancing by Linear Programming. *Operations Research*, 8(3), pp.385–389.
- Boysen, N., Fliedner, M. & Scholl, A., 2007. A classification of assembly line balancing problems. *European Journal of Operational Research*, 183(2), pp.674–693.
- Carraway, R.L., 1989. A Dynamic Programming Approach to Stochastic Assembly Line Balancing. *Management Science*, 35(4), pp.459–471. Available at: <http://pubsonline.informs.org/doi/abs/10.1287/mnsc.35.4.459> [Accessed September 24, 2016].
- Dolgui, A. & Battai, O., 2013. Int . J . Production Economics A taxonomy of line balancing problems and their solution approaches. , 142, pp.259–277.
- Dong, J. et al., 2014. Balancing and sequencing of stochastic mixed-model assembly U-lines to minimise the expectation of work overload time. *International Journal of Production Research*, 52(24), pp.7529–7548. Available at: <http://www.tandfonline.com/doi/abs/10.1080/00207543.2014.944280> [Accessed September 28, 2016].
- Esmailbeigi, R., Naderi, B. & Charkhgard, P., 2015. The type E simple assembly line balancing problem: A mixed integer linear programming formulation. *Computers & Operations Research*, 64, pp.168–177. Available at: <http://linkinghub.elsevier.com/retrieve/pii/S0305054815001446>.
- Fattahi, A. et al., 2014. A novel integer programming formulation with logic cuts for the U-shaped assembly line balancing problem. *International Journal of Production Research*, 52(5), pp.1318–1333. Available at: <http://www.tandfonline.com/doi/abs/10.1080/00207543.2013.832489> [Accessed September 21, 2016].
- Fattahi, P. & Salehi, M., 2009. Sequencing the mixed-model assembly line to minimize the total utility and idle costs with variable launching interval. *The International Journal of Advanced Manufacturing*. Available at: <http://link.springer.com/article/10.1007/s00170-009-2020-0> [Accessed September 25, 2016].

- Geoffrion, A.M., 1967. Integer Programming by Implicit Enumeration and Balas' Method. *SIAM Review*, 9(2), pp.178–190. Available at: <http://epubs.siam.org/doi/abs/10.1137/1009031> [Accessed October 16, 2016].
- Gökçen, H. & Ağpak, K., 2006. A goal programming approach to simple U-line balancing problem. *European Journal of Operational Research*, 171(2), pp.577–585.
- Gökçen, H. & Erel, E., 1998. Binary Integer Formulation for Mixed-Model Assembly Line Balancing Problem. *Computers & Industrial Engineering*, 34(2), pp.451–461. Available at: <http://www.sciencedirect.com/science/article/pii/S0360835297001423>.
- Guerriero, F. & Miltenburg, J., 2003. The stochastic U-line balancing problem. *Naval Research Logistics*, 50(1), pp.31–57. Available at: <http://doi.wiley.com/10.1002/nav.10043> [Accessed September 22, 2016].
- Gutjahr, A.L. & Nemhauser, G.L., 1964. An Algorithm for the Line Balancing Problem. , (August 2015).
- Hamta, N. et al., 2013. A hybrid PSO algorithm for a multi-objective assembly line balancing problem with flexible operation times, sequence-dependent setup times and learning effect. *International Journal of Production Economics*, 141(1), pp.99–111.
- Hazır, Ö. & Dolgui, A., 2013. Assembly line balancing under uncertainty: Robust optimization models and exact solution method. *Computers & Industrial Engineering*, 65(2), pp.261–267. Available at: <http://www.sciencedirect.com/science/article/pii/S0360835213000934>.
- Kara, Y., 2008. Line balancing and model sequencing to reduce work overload in mixed-model U-line production environments. *Engineering Optimization*, 40(7), pp.669–684. Available at: <http://www.tandfonline.com/doi/abs/10.1080/03052150801982509> [Accessed September 26, 2016].
- Kara, Y., Paksoy, T. & Chang, C.-T., 2009. Binary fuzzy goal programming approach to single model straight and U-shaped assembly line balancing. *European Journal of Operational Research*, 195(2), pp.335–347.
- Kara, Y. & Tekin, M., 2009. A mixed integer linear programming formulation for optimal balancing of mixed-model U-lines. *International Journal of Production Research*, 47(15), pp.4201–4233. Available at: <http://www.tandfonline.com/doi/abs/10.1080/00207540801905486> [Accessed September 26, 2016].
- Kazemi, S.M. et al., 2011. A novel two-stage genetic algorithm for a mixed-model U-line balancing problem with duplicated tasks. *The International Journal of Advanced Manufacturing Technology*, 55(9–12), pp.1111–1122. Available at: <http://link.springer.com/10.1007/s00170-010-3120-6> [Accessed September 26, 2016].
- Kucukkoc, I. et al., 2015. A mathematical model and artificial bee colony algorithm for the lexicographic bottleneck mixed-model assembly line balancing problem. *Journal of Intelligent Manufacturing*, pp.1–13. Available at: <http://link.springer.com/10.1007/s10845-015-1150-5> [Accessed October 24, 2016].
- Kucukkoc, I. & Zhang, D.Z., 2014. Mathematical model and agent based solution approach for the simultaneous balancing and sequencing of mixed-model parallel two-sided assembly lines. *International Journal of Production Economics*, 158, pp.314–333.
- Miltenburg, G.J. & Wijngaard, J., 1994. The U-line Line Balancing Problem. *Management Science*, 40(10), pp.1378–1388. Available at: <http://pubsonline.informs.org/doi/abs/10.1287/mnsc.40.10.1378> [Accessed September 21, 2016].
- Miltenburg, J., 2002. Balancing and Scheduling Mixed-Model U-Shaped Production Lines. *International Journal of Flexible Manufacturing Systems*, 14(2), pp.119–151. Available at: <http://link.springer.com/10.1023/A:1014434117888> [Accessed September 26, 2016].
- Moodie, C., 1964. A Heuristic Method of Assembly Line Balancing for Assumptions of Constantor Variable Work Element Times. Available at: <http://docs.lib.purdue.edu/dissertations/AAI6408691/> [Accessed September 23, 2016].
- Mosadegh, H., Zandieh, M. & Ghomi, S.M.T.F., 2012. Simultaneous solving of balancing and sequencing problems with station-dependent assembly times for mixed-model assembly lines. *Applied Soft Computing*, 12(4), pp.1359–1370.
- Nakade, K., Ohno, K. & George Shanthikumar, J., 1997. Bounds and approximations for cycle times of a U-shaped production line. *Operations Research Letters*, 21(4), pp.191–200.
- Özcan, U., 2010. Balancing stochastic two-sided assembly lines: A chance-constrained, piecewise-linear, mixed integer program and a simulated annealing algorithm. *European Journal of Operational Research*, 205(1), pp.81–97.
- Özcan, U. & Toklu, B., 2009. Multiple-criteria decision-making in two-sided assembly line balancing: A goal programming and a fuzzy goal programming models. *Computers and Operations Research*, 36(6), pp.1955–1965.
- Pastor, R. & Ferrer, L., 2009. An improved mathematical program to solve the simple assembly line balancing problem. *International Journal of Production Research*, 47(11), pp.2943–2959.
- Paternina-Arboleda, C. & Montoya-Torres, J., Mathematical formulation for a mixed-model assembly line balancing problem with stochastic processing times. *laccet.org*. Available at: http://www.laccei.org/LACCEI2006-PuertoRico/CopyrightPending/IT240_PaterninaArboleda.pdf [Accessed September 27, 2016].
- Patterson, J.H. & Albracht, J.J., 1975. Technical Note--Assembly-Line Balancing: Zero-One Programming with Fibonacci Search. *Operations Research*, 23(1), pp.166–172.
- Rabbani, M., Moghaddam, M. & Manavizadeh, N., 2012. Balancing of mixed-Model two-Sided assembly lines with multiple u-Shaped layout. *International Journal of Advanced Manufacturing Technology*, 59(9–12), pp.1191–1210.

- Rabbani, M., Mousavi, Z. & Farrokhi-Asl, H., 2016. Multi-objective metaheuristics for solving a type II robotic mixed-model assembly line balancing problem. *Journal of Industrial and Production Engineering*, 33(7), pp.472–484. Available at: <https://www.tandfonline.com/doi/full/10.1080/21681015.2015.1126656> [Accessed September 27, 2016].
- Ritt, M., Costa, A.M. & Miralles, C., 2016. The assembly line worker assignment and balancing problem with stochastic worker availability. *International Journal of Production Research*, 54(3), pp.907–922. Available at: <http://www.tandfonline.com/doi/full/10.1080/00207543.2015.1108534> [Accessed September 25, 2016].
- Salveson, M.E., 1955. The assembly line balancing problem. *Journal of Industrial Engineering*, 6(3), pp.18–25.
- Simaria, A.S. & Vilarinho, P.M., 2009. 2-ANTBAL: An ant colony optimisation algorithm for balancing two-sided assembly lines. *Computers & Industrial Engineering*, 56(2), pp.489–506.
- Sivasankaran, P. & Shahabudeen, P., 2014a. Literature review of assembly line balancing problems. *The International Journal of Advanced Manufacturing Technology*, 73(9–12), pp.1665–1694. Available at: <http://link.springer.com/10.1007/s00170-014-5944-y> [Accessed August 20, 2016].
- Sivasankaran, P. & Shahabudeen, P., 2014b. Literature review of assembly line balancing problems. *The International Journal of Advanced Manufacturing Technology*, 73(9–12), pp.1665–1694. Available at: <http://link.springer.com/10.1007/s00170-014-5944-y> [Accessed September 29, 2016].
- Sparling, D. & Miltenburg, J., 1998. The mixed-model U-line balancing problem. *International Journal of Production Research*, 36(2), pp.485–501.
- Talbot, F.B. & Patterson, J.H., 1984. An Integer Programming Algorithm With Network Cuts for Solving the Assembly Line Balancing Problem. *Management Science*, 30(1), pp.85–99. Available at: <http://search.ebscohost.com/login.aspx?direct=true&db=bth&AN=7357463&site=ehost-live&scope=site>.
- Thangavelu, S.R. & Shetty, C.M., 1971. Assembly Line Balancing by Zero-One Integer Programming. *AIIE Transactions*, 3(1), pp.61–68.
- Toklu, B. & Özcan, U., 2008. A fuzzy goal programming model for the simple U-line balancing problem with multiple objectives. *Engineering Optimization*, 40(3), pp.191–204. Available at: <http://www.tandfonline.com/doi/abs/10.1080/03052150701651642> [Accessed September 21, 2016].
- Urban, T.L., 1998. Note. Optimal Balancing of U-Shaped Assembly Lines. *Management Science*, 44(5), pp.738–741. Available at: <http://pubsonline.informs.org/doi/abs/10.1287/mnsc.44.5.738> [Accessed September 21, 2016].
- Urban, T.L. & Chiang, W.C., 2006. An optimal piecewise-linear program for the U-line balancing problem with stochastic task times. *European Journal of Operational Research*, 168(3), pp.771–782.
- Vilarinho, P.M. & Simaria, A.S., 2002. A two-stage heuristic method for balancing mixed-model assembly lines with parallel workstations. *International Journal of Production Research*, 40(6), pp.1405–1420. Available at: <http://www.tandfonline.com/doi/abs/10.1080/00207540110116273> [Accessed September 25, 2016].
- Vitria, J., 2004. Different ways of modelling and solving precedence and incompatibility constraints in the Assembly Line Balancing Problem. *Recent Advances in Artificial Intelligence Research*. Available at: <https://www.google.com/books?hl=en&lr=&id=P7FYs0IvbpoC&oi=fnd&pg=PA359&dq=Pastor,+R.,+Corominas,+A.+and+Lusa,+A.,+2004.+Different+ways+of+modelling+and+solving+precedence+and+incompatibility+constraints+in+the+assembly+line+balancing+problem.+Frontiers+in>.
- White, W.W., 1961. Comments on a Paper by Bowman. *Operations Research*, 9(August 1961), pp.274–276.
- Zhao, X. et al., 2016. A genetic algorithm for the multi-objective optimization of mixed-model assembly line based on the mental workload. *Engineering Applications of Artificial Intelligence*, 47, pp.140–146.