Optimization Techniques for Routing Design Problems over Wireless Sensor Networks: A Short Tutorial

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Abstract: This paper is intended to serve as an overview of, and mostly a short tutorial to illustrate, the optimization techniques used in several different key design problems that have been considered in the literature of routing over wireless sensor networks. For each routing design problem, a key paper that implements optimization techniques is selected, and for each we present the formulation techniques and the solution methods implemented. We observed that good formulation is the key to fully exploiting the features of the techniques. Hence we focus on presenting the formulation techniques, to facilitate the use of "on the shelf" efficient algorithms in the operations research literature. This we believe will help researchers in better understanding the issues and how to improve further on solution techniques.

1 INTRODUCTION

As the technology evolves, the wireless sensors manufactured become technically more powerful and economically viable. In wireless sensor networks (WSNs) each node consists basically of units for sensing, processing, radio transmission, position finding and sometimes mobilizers (Al-Karaki and Kamal, 2004), (Papadopoulos et al., 2016). These sensors measure desired phenomenal conditions in their surroundings and digitize them to process the received signals to reveal some characteristics of the conditions in the surrounding area. A large number of these sensors can be networked in many applications that require unattended operations, hence producing a WSN. In general, the sensor nodes in a wireless sensor network WSN sense and gather data from surrounding environment and route it to one or more sinks, to perform more intensive processing. The number of applications for WSNs is large, many of these are in the fields of weather monitoring, surveillance, health care, etc. More fields are deploying WSNs as their reliability, performance and capabilities keep getting even better and wider.

In many applications, replacement of damaged or energy depleted nodes is not possible. Moreover planned nodal placement may not be a possible thing to do. Therefore, two of the main requirements for WSNs to operate reliably are to consume the minimum amount of energy to prolong the network's life time, and to be able to self organize themselves when the network topology changes. Other requirements (e.g limited delay, good signal to noise ratio, etc.) are usually application specific. Moreover, there are differences in the nature of WSNs. For example, there could be WSNs with either rechargeable or non-rechargeable sensor batteries, either single sink WSNs or multiple sink WSNs, which could either be immobile or mobile. Depending on these different variants of WSNs, different types of applications and the traffic types they handle, different design considerations will need to be taken into account. Optimization techniques that have been in the operations research (OR) literature for almost a century provide a rich reservoir of different types and classes of optimization problems that have been studied extensively. For these, different solution techniques are available that have experienced development over the years until they have reached to a mature level in which their computational and storage performance have been extensively tested and assessed. Among these are the different variants of Lagrangian relaxation (Fisher, 2004), dual decomposition methods (Sontag et al., 2011) column generation (Desaulniers et al., 2006) and many others. One of the benefits of dual decomposition techniques is that an optimization prob-

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lem can be decomposed and each node given a local part of it hence enabling a distributed solution scheme rather than a centralized one.

We believe that in order to exploit the full strength of the extensive tools in the optimization literature, good formulation is necessary to reduce the design problem to one of the classical optimization problems for which those well studied solution techniques could be used. In this paper, such techniques are illustrated for the different routing design problems in WSNs, for which we have selected seven papers to discuss. We summarize the various routing design problems and the corresponding optimization techniques used in Table 1 whose columns show:

1. the design problem,
2. the initial optimization problem formulations,
3. the design objective,
4. the centralized/distributed possible algorithmic implementation to solve the initial formulations,
5. any reformulations performed,
6. solution algorithms that were proposed,
7. whether the proposed algorithms are distributed or centralized,
8. the nature of the solution that could be obtained, whether it is suboptimal or global optimal,
9. the convergence speed or computational complexity of the solution algorithms.

A section is dedicated for each routing problem in which we give the system model and the design objectives, the problem formulation and solution methods considered in those papers. The notation in each section is restricted to that section only. We use the same notation used by the original papers to make it easier for the reader to connect the paper with the originals we consider here.

2 ROUTING FOR MULTI-HOP WSNS WITH A SINGLE IMMOBILE SINK

A multi-hop wireless sensor network was considered in (Madan and Lall, 2006) that focuses on computing multi-hop routes from each node to a single immobile sink such that the network lifetime is maximized. The lifetime in (Madan and Lall, 2006) was defined as the time at which the first node runs out of energy. Each node can generate information due to its sensing capabilities and relay packets from other nodes to the sink node. The nodes' battery energies are limited and adjustments of transmission powers for each node is possible depending on the distance between nodes.

2.1 Optimization Problem Formulation

The initial formulation given in (Madan and Lall, 2006) is a max-min non-linear program (NLP) where the **objective function** maximizes the minimum of all lifetimes across the nodes in the network. The life time of each node is defined as the quotient of the initial battery energy of the node to the sum of expended energies on each of the node's outgoing flows. The **decision variables** are continuous and represent the transmission rates r_{ij} for a node i to node j. The **constraint set**, is a linear equality set of conservation of flow constraints for all the nodes in the network. They simply state that the difference between the outgoing flows from each node and its incoming flows should strictly be equal to the data generated by the node itself.

To make the problem easier, the minimum term in the objective function is replaced by an auxiliary continuous variable which is upper bound constrained by the lifetime of all sensor nodes. This adds more structure to the problem making it a quadratically constrained program. Further reformulation techniques were used to reduce it to an equivalent linear program which introduced an auxiliary continuous variable q.

2.2 Solution Methods

Two solution methods were provided in (Madan and Lall, 2006), one is a *partially distributed* scheme and the other is *fully distributed*. For both, the Lagrangian for the objective function is obtained. The dual function is the minimum Lagrangian function where the minimizers are the primal variables of the problem which are r_{ij} and q. The primal decision variables appear in separate additive terms in the Lagrangian and hence the dual function can be evaluated separately at each node.

The resulting linear primal objective function gets modified, by squaring it, to an equivalent strictly convex function plus a strictly convex regularization term. Also a loose upper bound is imposed on the auxiliary variable so that all decision variables have bound constraints that form a bounded polyhedron. These two modifications ensure that the dual function is differentiable and hence enables the use of the subgradient algorithm with guarantee that it converges to the solution of the strictly convexified primal problem. The dual function for the strictly convexified problem is still separable in the primal variables. In each iteration of the subgradient algorithm, a box con-

Table 1: Summary of the covered WSN routing problems and the corresponding optimization techniques.

Routing Design Aspect	Classification of Initial Problem Formulation	Objective	Initial problem formulation: Centralized / Distributed?	Classification of Reformulated Problem	Solution Method	Implemented solution scheme for the problem : Centralized /Distributed?	Nature of Solution	Convergence Speed / Complexity
Routing for a WSN Multi-hop with Single Immobile Sink (Madan and Lall, 2006)	bilinear quadratically constrained program	Maximizes the shortest lifetime across all nodes	Centralized	1. reformulated to linear program 2. reformulated to strictly convex quadratic program	Dual Decomposition + subgradient algorithm	1. Partially distributed for linear program reformulation 2. Fully distributed for the strictly convex quadratic program reformulation	Global Optimal	1. Partially distributed algorithm : around 600 subradient iterations to converge 2. Fully distributed algorithm : around 3500 subgradient iterations to converge
Routing in a delay-tolerant WSN to a Single Mobile Sink (Yun et al., 2013)	Quadratically Constrained Program	Maximize the number of cycles made by mobile sink	Centralized	Linear program	Lagrangian relaxation +subgradient algorithm, greedy algorithm for fractional knapsack problems+ trivial heuristic for box constrained linear program	Decentralized algorithm	Good Suboptimal . Global Solution is also attainable in a long time	Good suboptimal in few iterations (100-200) and global after too many iterations.
Joint routing, power and bandwidth allocation in FDMA WSNs (Leinonen et al., 2013)	Non-strictly convex non-linear constrained program	Minimize the aggregate power for all network links	Centralized	Convex non-linear global consensus problem	Augmented Lagrangian +Alternating direction method of multipliers	Distributed among nodes	Global optimal	Fast, converged in 21 iterations of the ADMM algorithm
				Strictly convex non-linear program (using proximal regularization)	Dual decomposition and projected subgradient algorithm	Distributed among nodes and decomposed in protocol layers	Global optimal	Slower than ADMM, converges in 133 iterations of th projected subgradient algorithm
Energy and route allocation from multiple source to multiple destination in rechargeable WSNs (Chen et al., 2012)	Complicated Convex NLP	Maximize Aggregate node utilities	Centralized	Decomposed to two easier strictly convex subproblems by decoupling the time component	Dual decomposition +subgradient algorithm used in a heuristic scheme	Distributed	suboptimal in finite time but optimal in infinite time	Too slow, took 5000 minutes to reach the best suboptimal solution in the simulations done in (Chen et al., 2012)
Account for uncertainties in the distance between sensor nodes (Ye and Ordonez, 2008)	Linear Program	Aggregate transmission and reception normalized energies	N/A	• Polyhedral uncertainty set: yields Linear Program • Ellipsoidal uncertainty set: yields a second order cone program (SOCP)	Interior point methods, CPLEX solver was used to solve the problem	N/A	Global optimum	Can be solved in polynomial time
	Linear Program	Maximize the transmitted data to the sink						
	Bilinear Constrained Program	Maximize the network lifetime						
Joint Routing and Scheduling in WSNs with Multiple Sinks and Different Sink Location Possibilities (Gu et al., 2011)	Non -Linear and non-deterministic (due to the integration function)	Maximize the network lifetime	Centralized	Non-Linear Quadratically constrained Program	Column Generation Method	Centralized	Global Optimal	Too quick for small problems (less than one second and 6 iterations) but unknown for large problems
Joint Routing and TDMA scheduling for delay sensitive underwater acoustic WSNs (Ponnavaikko et al., 2013)	NLP with general undetermined nonlinear term in the objective function	Minimize the transmission energy	Centralized	Mixed Integer Linear Program	Not stated, an MILP solver is assumed to have been used	Centralized	Global optimal to the MILP but is suboptimal to the original NLP problem	Depends on the desired approximation error. Expected to be fast for large approximation errors but slow for small approximation errors, hence there is a trade-off.

strained single variable quadratic and convex problem gets solved. The obtained values of the primal variables are used to calculate the values of the dual variables in the next iteration by the subgradient formulas.

At a given iteration k, the values of the dual variables needed to solve for the flow transmission rate variables r_{ij}^k of a node are locally available to the sensor node. The transmission rates of a node's links could hence be computed at that node only without the need of any communication with other nodes. The calculation of the problem's auxiliary variable q^k however, needs all the values dual variable values from all the nodes in the network to be transmitted to the node responsible its computation. The auxiliary variable values obtained in every iteration hence need to be broadcasted to all the nodes as they are needed for the subgradient calculation in the following iteration.

For the i^{th} element of the subgradient to be computed at node i for iteration k, it needs the rate variables values at iteration r_{ji}^k for all its neighbors in the set N_i to be transmitted to it. Therefore, each node has to broadcast its r_{ij}^k values to its neighbors. Then, using those, the value of q^k received from a broadcast and the locally calculated r_{ij}^k, the i^{th} element of the subgradient gets calculated at node i. The values of the new dual variables are calculated locally at each node.

Since every node contributes in the computation of the primal variables, dual variables and the subgradient at each iteration, the algorithm is therefore like a distributed one. However, the algorithm is not fully distributed since at iteration k, node i still needs other calculated variables from other nodes.

Another algorithm was proposed in (Madan and Lall, 2006) that is *fully distributed*. The linear program is transformed into a strictly convex quadratic optimization problem by introducing a separate auxiliary variable q for each node i. Then, instead of maximizing the primal objective function in a single variable q^2 as in the problem, the sum of q_i^2 is maximized. By enforcing an equality constraint $q_i = q_j, \forall i \in V, \forall j \in N_i$ (where V is the set of nodes and N_i is the set of neighbors of i) which guarantees that for any feasible solution the objective function is just $|V|q^2$ which yields the same set of feasible solutions and the same optimal solution. This change enables the dual problem to be decomposed to separate node local problems which each node can solve independently with only the exchange of dual variables with its neighbors.

3 ROUTING IN SINGLE MOBILE SINK

A WSN with a mobile sink was considered in (Yun et al., 2013). Each node can postpone data transmission until the sink is at the most favorable position to extend the lifetime of the network. The problem is to find how long the sink should stay at potential stops and how buffered data could be routed to the sink when it stops taking into account a maximum delay toleration. A distributed algorithm was used in which the problem is decomposed to smaller decision problems and each can be solved by a sensor node. Only local information from the neighbors is needed by each node.

The network was modeled as a directed graph where the cost of each arc is proportional to the distance between the nodes of that edge. The sink must complete each of its tours through all available sink positions and back to the initial position within the maximum tolerable delay. The problem was found to be equivalent to maximizing the number of sink tours T within the tolerable delay for which the life time is the total number sink cycle durations. The sink does not have to visit all the locations in an optimal solution. Also, when it is at a particular position , only a set of nodes $R_l \subseteq \mathcal{N}$ (where $\mathcal{N}$ is the set of nodes in the network) can transmit data via multi-hops to the sink. The other nodes not in R_l do not transmit to the sink when it is at l and buffer their data instead. The set R_l is chosen depending on experimental trials. For each location l, there is a graph $\mathcal{G}^l = \left(\mathcal{N} \cup \{l\}, \mathcal{A}^l\right)$ where $\mathcal{A}^l = \{(i,j) \in \mathcal{A} | i \in R_l \cup \{l\}\}$ and $\mathcal{A}$ is the set of arcs in the main directed graph that represents the network.

An expanded graph $\mathcal{G}_{exp}$ that comprises all the graphs $\mathcal{G}^l$ and a vertex node s, that represents the sink , is constructed. The details of the construction are given in (Yun et al., 2013).

3.1 Optimization Problem Formulation

The optimization problem was initially formulated as a quadratically constrained program *QCP*. The **decision variables** $x_{ij}^{(l)}$ represent the data flow between two nodes i and j with the sink at position l, $y_i^{(l)}$ representthe amount of buffered data at node i just as the sink leaves location l and T represents the number of cycles the mobile sink makes. All the decision variables are continuous.The **objective function** maximizes the number of cycles the mobile sink makes i.e. $\max_{T, x_{ij}^{(l)}, y_{ij}^{(l)}} T$. This is a linear objective function in one continuous variable.

There are two **constraint sets**. A linear equality constraint set that combine the transmission flows and buffered data for all possible sink locations to enforce conservation of flow constraints which guarantee that the total incoming flows for node i plus the buffered data is equal to the outgoing flows. A quadratic constraint set that guarantees that all the energy expended due to data transmission on the links for all possible sink positions is within the available node's battery remaining energy. It was given by $\left(\sum_{l=1}^{L}\sum_{j\in N_l(i)} e_{ij}^{(l)} x_{ij}^{(l)}\right) T \leq E_i, \ \ \forall i \in \mathcal{N}$ where $e_{ij}^{(l)}$ is the energy spent per unit data on the link (i, j) when the sink is at position l and E_i is the available energy for node i and E_i is the available battery energy for node i.

The problem was reformulated to a linear program by minimizing the reciprocal of the number of sink cycles and substituting that reciprocal with another continuous variable $z = 1/T$. The quadratic constraint now becomes $\left(\sum_{l=1}^{L}\sum_{j\in N_l(i)} e_{ij}^{(l)} x_{ij}^{(l)}\right) \leq zE_i, \ \ \forall i \in \mathcal{N}$ in the new formulation.

3.2 Solution Method

Lagrangian relaxation was used to dualize the set of flow equality constraints. The Lagrangian dual function is then minimized with respect to the primal variables $z, \mathbf{x}$ and $\mathbf{y}$ where the vectors $\mathbf{x}$ and $\mathbf{y}$ comprise of the variables $x_{ij}^{(l)}$ and $y_i^{(l)}$ respectively. The Lagrangian dual optimization problem minimizes the Lagrangian dual function subject to the energy constraint for each node.

The Lagrangian dual optimization problem is decomposed into two subproblems $S1$ and $S2$. The subproblem $S1$ consists of a linear objective function in the primal vector $\mathbf{y}$ and decision variable bound constraints on the variables $y_i^{(l)}$. Subproblem $S2$ consists of a linear objective function in z and the vector $\mathbf{x}$. The constraints for $S2$ are the energy constraints $\left(\sum_{l=1}^{L}\sum_{j\in N_l(i)} e_{ij}^{(l)} x_{ij}^{(l)}\right) \leq zE_i, \ \ \forall i \in \mathcal{N}$ and variable bound constraints. One subproblem was reduced to a linear box constrained problem whose minimum value was obtained in a distributed manner by each node by setting its corresponding buffering variable $y_i^{(l)}$ to their upper bound if their respective coefficient in the objective function of the subproblem is negative and zero otherwise. The second subproblem was reduced to multiple fractional knapsack problems that could be solved separately by each node (hence decentralized approach) in polynomial time. The subgradient algorithm was used to evaluate the values of the dual variables on an iteration by iteration basis.

4 JOINT ROUTING, POWER AND BANDWIDTH ALLOCATION

In (Leinonen et al., 2013) a frequency division multiple access (FDMA) based single-immobile-sink WSN was considered. The objective is to jointly allocate data flows and bandwidth for the network links in order to minimize the total transmission power in the WSN. Flat fading was assumed which makes the link rates dependent only on the power levels and the bandwidth of the link irrespective of frequency dependent gains. Each sensor i has a limited total power P and a total preallocated bandwidth W_i. The power and bandwidth allocated to the sensor's links $l \in O(i)$ should satisfy $\sum_{l\in O(i)} p_l \leq P$ and $\sum_{l\in O(i)} w_l \leq W \ \forall i$ where p_l and w_l are the power and bandwidth on link l respectively.

Each node allocates disjoint and continuous frequency bands to its outgoing links. All nodes are assumed to have a maximum communication range, therefore a link between two nodes exists only if both are within the communication range of each other. The flow on a link cannot exceed Shannon 's capacity of the link.

4.1 Optimization Problem Formulation

All the **decision variables** are continuous non-negative variables. There are three sets, one set of variables is for the power values on the links p_l, the second is for the flow capacities on the links f_l and the third is for the amounts bandwidth spectrum allocated to the links in the network w_l. The **objective function** is a linear function in the aggregate link powers i.e. $\sum_{l\in \mathcal{L}} p_l$ (where $\mathcal{L}$ is the set of links). There are four **constraint sets**,

1. a linear equality constraint set in the flow variables f_l for the conservation of data rate flows. These guarantee that for every node the difference between the out-going flows and the sum of the in-going flows is strictly equal to the rate of data generated by each node t_i,
2. a convex non-linear constraint set that guarantees that the flows on each link are upper bounded by the Shannon capacity of the link. These constraints are function in both the powers on the links p_l and the bandwidths allocated to the links w_l and are given as, $f_l \leq c_l(p_l, w_l) \ \ \forall l \in \mathcal{L}$,
3. a linear constraint set in the power variables of the links that guarantees that for each node the aggregate transmission power on all its links does not exceed sensor's battery power, i.e. $\sum_{l\in O(i)} p_l \leq$

$P, \forall i \in S$ where S is the set of nodes and $O(i)$ is the set of links for node i,

4. a linear constraint set in the bandwidth variables that guarantee that the sum of bandwidths allocated on all the links of every node does not exceed the nodes' pre-allocated bandwidth W, i.e. $\sum_{l \in O(i)} w_l \leq W, \forall i \in S$.

The objective function and all constraints but the flow conservation constraint set can be considered an independent local problem for each node to solve. Consensus reformulation is used by introducing local copies $\hat{f}_{il}$ of the associated global flow variables for each node.The local variables were interpreted in (Leinonen et al., 2013) as the node's opinion about the corresponding global flow variables. By carrying out the following modifications to the formulation, the problem becomes a global consensus problem where except for the consensus constraints, the rest of the modified constraints are local to each node.

1. $\hat{f}_{il} \leq c_l(p_l, w_l) \quad \forall l \in \mathcal{L}, \ \forall i \in \mathcal{S}, \ l \in O(i)$
2. $\sum_{l \in \mathcal{L}(i)} a_{il} \hat{f}_{il} = r_i, \ \forall i \in \mathcal{S}$
3. $\hat{f}_{il} = f_l \forall i \in \mathcal{S}, l \in \mathcal{L}(i)$, which represent the consensus constraints.

where $a_{il} \in \{-1, 0, 1\}$ is a constant whose possible values describe the incidence of a graph edge on a node, and $\mathcal{L}(i)$ denotes the set of links connected to sensor node i and r_i is the source rate for node i.

4.2 Solution Method

The augmented Lagrangian for the problem's global consensus reformulation is obtained with respect to the consensus constraints. An L_2 norm penalty term is added to regularize the non-differentiable optimization function so that convergence is possible due to the non-differentiable nature in the objective function. The *alternating direction multiplier method* (ADMM) method is used to solvetheglobal consensus formulation. It consists of a sequence of optimization phases over the primal variables followed by a gradient method that updates the dual variables.

In each node, phase 1 minimizes the augmented Lagrangian over the node local variables power, bandwidth and flow variables p_i, w_i and $\hat{f}$). The second phase minimizes over the global flow variable (f_i) for each node i. Then, the dual variable corresponding to each link for the node is updated with the constant step size ρ. Phase 1 problem is a quadratic convex optimization problem in the local resource variables and the local flow variables. Interior-point methods were used by the authors to solve it. The phase 2 problem was manipulated algebraically to give the simple form:

$$f_l^k = \begin{cases} \frac{1}{2}\left(\hat{f}_{tran(l)}^k + \hat{f}_{rec(l)}^k\right), for\ all\ but\ the\ sink\ node \\ \hat{f}_{tran(l)}^k, for\ the\ sink\ node \end{cases} \tag{1}$$

where k is the iteration index, $tran(l)$ and $rec(l)$ are the transmitting and receiving nodes on the link l respectively. Thus, the global flow variables are obtained at each iteration k by averaging out the corresponding updated local variables.

The only information that needs to be shared among the nodes are the local flow variables. These have to be broadcasted by each node to its neighbors. The communications overhead therefore depends on the network density, rather than the number of nodes.

5 RECHARGEABLE WSNS WITH MULTIPLE SOURCES AND DESTINATIONS

In (Chen et al., 2012), a rechargeable WSN whose batteries' replenishment profile is unknown a priori is considered. Routing and energy allocation are performed such that the aggregate utility functions for the sensors is to be maximized with low complexity. It was shown that the problem can be formulated as a standard convex optimization problem with energy and routing constraints. However, the solution requires centralized control and full knowledge of replenishment profiles in the future, which may not be available in practice. Therefore, a low-complexity heuristic solution was developed that is asymptotically optimal and can be approximated by a distributed algorithm.

The following are the main elements of the system model in (Chen et al., 2012) are as follows. The system is time slotted system with finite number of slots and the battery of each sensor is assumed to have an infinite rechargeable capacity. Multiple sensing sources and multiple destination nodes are considered. A utility function that reflects the "satisfaction" of the node is associated with each source node when it transmits at an average data rate $\hat{x}^s(t)$ that is equal to the aggregate amount of data from that source to a particular destination over all time slots averaged over the duration of the frame. It is defined generally to be concave monotonically increasing in the average data rate of the source node.

5.1 Problem Formulation

A formulation that maximizes the sum of general utilities of all sensor nodes was formulated as a convex

NLP. All **decision variables** are continuous variables. There are three sets of those, $w_{ij}(t)$ is the amount of data on the outgoing link (i, j) for time slot t, $x^s(t)$ is the amount of data delivered from source f_s to the destination d_s, $e_s(t)$ represents the amount of energy expended by a node. The **objective function** is the sum of individual node utilities where each of these, $U_s\left(\frac{1}{T}\sum_{t=1}^{T} x^s(t)\right)$, is a function of the amount of data delivered from source node f_s to destination node d_s in all T time slots over possibly multiple hops and multiple paths. Each utility function is assumed to be a continuous non-linear concave function. There are two **constraint sets**, the first is conservation of flow constraint sets, which are linear constraints in $w_{ij}(t)$ and $x^s(t)$. The second ensures that the sum of flows emanating from a node i belongs to the set Λ_i of the different amounts of data in different time slots under a given replenishment profile vector $\overrightarrow{r}_i$. For any data vector in Λ_i, there exists an energy vector e_n that achieves that amount of data for a given modulation and coding scheme. The set Λ_i was proved to be convex in works earlier to (Chen et al., 2012).

5.2 Solution Method

A heuristic method named DualNet was proposed that obtains an infeasible upper bound and a feasible lower bound and iteratively solves the problem until it converges to the optimal solution infinity. First an upper bound was obtained on the value of the objective function at the optimal solution of the problem after a long period of time (theoretically infinity). The solution that gives the upper bound is obtained by an infeasible energy allocation i.e. energy allocation that is higher than the average replenishment rate. The energy allocation (and hence the routing solution) are the same over all time slots and is more than the average replenishment rate, yielding infeasiblity. Using the energy allocation obtained, a routing sub-problem that is strictly convex and computationally easier than the original problem is obtained because of the decoupling of the time component. This requires solving the problem every time slot.

The lower bound solution is obtained by assigning a feasible energy value in each time slot for each node. The energy assignment for a node is the minimum of either the average harvested energy or the available battery energy (including the instantaneous replenishment for a given time slot). This assignment is done by each node on its own and hence is a distributed energy assignment. Using the energy assignment values the routing subproblem that obtains the lower bound, is again a similar routing subproblem to that of the upper bound subproblem.

Dual decomposition was used to solve the problem which enabled a distributed implementation of the scheme. Each source node solves two problems, one to determine the amount of data to inject in the network at a given time slot t, $x_s(t)$, the other subproblem to determine the routes and their flows, $w_{ij}(t)$. All the nodes that are not sources of data, and only responsible for relaying data over multiple hops, solve the routing problem only. The dual variables are computed using the subgradient algorithm.

6 ROUTING WITH DISTANCE UNCERTAINTIES

Optimization models were considered in (Ye and Ordonez, 2008) for WSNs subject to distance uncertainty for three different objectives, 1) minimizing the energy consumed, 2)maximizing the data extracted and 3) maximizing the network lifetime. Robust optimization was used to take into account the uncertainty present. In a robust optimization model the uncertainty is represented by considering that the uncertain parameters belong to a bounded, convex uncertainty set. A robust solution is the one with best worst case objective over this set. It was shown in (Ye and Ordonez, 2008) that solving for the robust solution in these problems is just as difficult as solving for the problem without uncertainty. The computational experiments in (Ye and Ordonez, 2008) showed that, as the uncertainty increases, a robust solution provides a significant improvement in worst case performance at the expense of a small loss in optimality when compared to the optimal solution of a fixed scenario.

6.1 Problem Statement and Design Objectives

For the three different types of problems, energy consumption was considered. The transmission and reception energy for each node is accounted for after normalizing with respect to the radio energy dissipation of the transmitter and receiver circuits. The expression for the total normalized energy has two components. One for the normalized received energy which is equivalent to the number of received bytes, i.e. $\sum_{j|(j,i)\in A} f_{ji}$, and one for the the normalized transmitted energy, which is equivalent to the number of transmitted bytes times a linear function in the transmission distance, i.e. $\sum_{j|(j,i)\in A} f_{ji}\left(1+\beta d_{ij}^2\right)$, where

- A is the set of nodes in the network,
- f_{ij} is the number of transmitted bytes from node j to node i.

- d_{ij} is the distance from node i to node j and β is a constant depending on transceiver parameters.

6.2 Formulations for the Three Problems

A brief description of each of the three different optimization problems that were given in (Ye and Ordonez, 2008) is as follows:

1. *The Minimum Energy Problem:* The **decision variables** are the continuous variables f_{ij}. The **objective function** is a continuous linear objective function in f_{ij} which is the sum of the transmission and reception normalized energies of all nodes in the network. The objective is to minimize that aggregate energy function. There are two **constraint sets**, the first constraint set enforces a minimum data transmission requirement constraint that requires the aggregate data transmitted from all nodes to the sink node, to be greater than a minimum number of bytes. The second and third constraint sets are conservation of flow constraints that require the difference between the amount of data bytes transmitted and received by a node to be less than the available data bytes at the node and greater than zero.
2. *The Maximum Data Extraction Problem:* The **decision variables** are also f_{ij}. The **objective function** maximizes the data transmitted to the sink node. It was given as the sum of data bytes f_{ij} transmitted from each node i to the sink node $n+1$ on the arcs $(i, n+1)$. It is a continuous linear objective function. As for the **constraint sets**, besides the conservation of flow in the *Minimum Energy Problem*, there is a set of energy limitation constraints for each node, that guarantees that the the sum of transmitted and received energy for each node does not exceed the available energy of the node, i.e.
$$\sum_{j|(i,j)\in A} f_{ij}\left(1+\beta d_{ij}^2\right) + \sum_{j|(i,j)\in A} f_{ij} \leq E^i \;\forall i \in N. \tag{2}$$
Note that in (Ye and Ordonez, 2008), the energy is normalized such that E^i is the number of bytes that could be transmitted with the available energy and the left hand side of the constraint is the amount of bytes transmitted for an expended amount of energy. All constraints are linear and hence the problem is a linear program ignoring the uncertainties.
3. *Maximum Lifetime Problem:* The **objective function**: maximize the lifetime T of the network which is defined as the lifetime of the first sensor whose battery gets depleted, i.e. $T = min\{T_1, T_2, \ldots, T_n\}$. The **constraints** are Conservation of flow typical to those in *Minimum Energy Problem*, in addition a quadratic constraint with bilinear terms that guarantees that the energy expended by transmission of a node i does not exceed its available energy, this is given by:
$$T\left(\sum_{j|(j,i)\in A} f_{ji} + \sum_{j|(j,i)\in A} f_{ji}\left(1+\beta d_{ij}^2\right)\right) \leq E^i \tag{3}$$
This gives a quadratically constrained program, which is transformed to a linear program by substituting the variable T in the problem with $q = 1/T$ and minimizing the objective function instead of maximizing it.

6.3 Accounting for Uncertainties in the Formulation

According to (Ye and Ordonez, 2008) a robust solution for an optimization problem under uncertainty is defined as the solution that has the best objective value in its worst case uncertainty scenario. For an optimization problem under uncertainty with decision vector x and uncertainty vector parameter u, the robust solution is defined as:

$$\min_x \left\{\max_u f(x,u) : g(x,u) \leq 0 \;\forall u \in \mathcal{U}\right\} \tag{4}$$

which is equivalent to :

$$\min_{x,\gamma} \gamma : g(x,u) \leq 0, f(x,u) \leq \gamma \;\forall u \in \mathcal{U} \tag{5}$$

where $\mathcal{U}$ is a closed convex uncertainty set. According to (Ye and Ordonez, 2008), the complexity of solving the robust counterpart of an optimization problem is equivalent to solving the deterministic problem for many problems. Moreover the increase in size of the problem is polynomial in the deterministic problem dimensions.

For the three optimization problems, the distance parameter was considered as the uncertainty parameter and hence in (Ye and Ordonez, 2008) was made to belong to the uncertainty set $\mathcal{U}$. The set $\mathcal{U}$ defines distance vectors that are within a certain distance from a given estimate of the distance vector between nodes. The paper presented two general convex sets for the uncertainty region, these are polyhedral sets and ellipsoidal sets. For both types of uncertainty sets, the three deterministic optimization problems that were considered were formulated to their robust counterparts. The type of optimization problems obtained for the three cases are as follows:

a. Polyhedral uncertainty sets: Using LP duality, it was shown that optimization problem remains as a linear program with additional variables and constraints.

b. Ellipsoidal sets: Using a known closed form solution for an embedded ellipsoidal optimization subproblem with respect to the uncertainty variables, the robust optimization problem becomes a conic convex problem that can be solved by interior point methods in polynomial time. With a simple reformulation trick of replacing the conic component in the objective function with an upper bound linear component and bringing in the conic component in the constraint set, the problem can be rewritten as a second order cone program (SOCP).

Furthermore, simulation results showing the obtained objective function values for the robust and deterministic solutions were also provided in (Ye and Ordonez, 2008). The robust solutions were obtained for different degrees of uncertainty, while the deterministic solutions assumed complete certainty. The results showed the average and standard deviations of the objective functions over 100 random experiments. The deterministic solution gave a higher average objective function value compared to the robust solution while the standard deviation of the uncertainty level of the robust objective value was closer to its mean values as compared to the deterministic objective values.

7 WSNS WITH MULTIPLE SINKS HAVING DIFFERENT LOCATION POSSIBILITIES

In (Gu et al., 2011) scheduling and routing of data to multiple sinks having multiple position possibilities are considered. The objective is to maximize the network lifetime which is defined in (Gu et al., 2011) as the time elapsed since the launch of the network till the instant a living node cannot find a route to send its data to the sinks due to many dead nodes. The authors propose two formulations for the same problem which they state that they are equivalent but do not provide a proof for that.

7.1 Initial Problem Formulation: Time based Formulation

The first formulation considered the time to be continuous and an independent variable based upon which all decision variables are dependent. It was named as *time-based formulation* and was considered very hard to tackle. A brief description of this formulation is as follows:

- Decision variables: T is a continuous variable that represents the network's lifetime, $g_{ij}(t)$ is a continuous variable that represents the data rate on the link from node i to node j at a given time t, $g_{io}(t)$ is a continuous variable that represents the data rate from node i to one of the possible sinks' positions o at a given time t, $x_{s,o}(t)$ is a binary variable that is set only when sink s resides in position o.
- Objective function: is to maximize the lifetime of the network which is given as $\max\limits_{T, g_{ij}(t), g_{io}(t), x_{so}(t)} T$,
- Constraint Sets: The constraints sets of the initial formulation are explained below:
  1. Constraint set 1: is a linear constraint set in the binary variables $x_{s,o}(t)$ that guarantees that a possible position o at t can get occupied by no more than one sink node, this is given as $\sum_{s \in V_s} x_{s,o}(t) \leq 1, \;\; \forall o \in V_o$, where V_s and V_o are the sets of sink nodes and possible sink positions respectively.
  2. Constraint set 2: is a linear constraint set in the binary variables $x_{s,o}(t)$ that guarantees that sink s at time t can only reside in one location, this is given as $\sum_{o \in V_o} x_{s,o}(t) \leq 1, \;\; \forall s \in V_s$.
  3. Constraint set 3: linear conservation of flow equality constraints in the flow variables $g_{i,j}(t)$ and $g_{i,o}(t)$.
  4. Constraint set 4: variable upper bounds on the flow variables $g_{i,j}(t)$ from node to node links that represent the link capacity.
  5. Constraint set 5: a mixed integer linear constraint in the variables $g_{i,o}(t)$ and $x_{s,o}(t)$ that impose link capacity on the flows from node i to the possible sink location o if any sink is assigned to that location, otherwise the flow is enforced to be zero. The constraint is $g_{i,o}(t) \leq C_{i,o}.\sum_{s \in V_s} x_{s,o}(t) \forall l_{i,o}$ where $C_{i,o}$ is the capacity of the link $l_{i,o}$.
  6. Constraint set 6: An energy constraint for each sensor $i \in S$ which is an integration of linear terms with respect to the time parameter t with the decision variable T in the upper limit of the integral.
  7. Constraint set 7: non-negativity constraints on all the decision variables.

7.2 Reformulation: Pattern based Formulation

As was mentioned for the time-based formulation in (Gu et al., 2011), the life-time variable T is connected to the rest of the variables through the energy constraint by integrating over time. This constraint complicates the problem and makes it difficult to solve according to (Gu et al., 2011). Therefore a reformulation was performed to obtain an easier problem which discretized the time parameter into different durations to give an easier problem which has no integration in any of the constraints. For each duration, a placement pattern can be assigned such that the amount of energy expended over all time durations is within the initial available energy level of each node. The life time of the network is hence equivalent to the aggregate discretized time durations. In each time duration, an assigned pattern should satisfy all the constraint sets that were explained for the time-based formulation. The energy constraint in a given time duration t_p for placement pattern p becomes a linear constraint in the flow variables $g^p_{i,j}$ and $g^p_{i,o}$. The elements of the reformulated problem are (Gu et al., 2011):

- Decision variables:
  1. the continuous variables t_p which represent the assigned time durations for the possible patterns p,
  2. continuous variables e^p_i for the energy consumption rate for node i in pattern p for all nodes and patterns,
  3. binary decision variables $x^p_{s,o}$ tell whether sink node s is assigned to location o in the pattern placement p for all nodes, sinks and patterns
  4. continuous variables $g^p_{i,o}$ for the data rate flow from node i to the sink position o in pattern p for all nodes, sinks and patterns,
  5. continuous variables $g^p_{i,j}$ for the data flow rate on the link between the nodes i and j in pattern p for all nodes and patterns,
- Objective function: Maximizes the aggregate durations assigned to the patterns which in (Gu et al., 2011), was stated to be equivalent to the lifetime of the network, i.e. $max\ \ T = \sum_{p\in P} t_p$.
- Constraint sets: The same constraint sets for the time-based formulation should be satisfied for each pattern, the energy constraints however are replaced by two constraint sets for each pattern, one is linear and the other is bilinear quadratic. The linear one is an equality constraint that links the energy expended by the node in a given placement pattern with the flow variables $g^p_{i,j}$ and $g^p_{i,o}$.

  The bilinear constraint set guarantees that all the energies expended by every node in all pattern durations do not exceed their initial battery energies.

7.3 Solution Method: Column Generation Method

Since the possible patterns are too large to enumerate, the column generation method was used in (Gu et al., 2011) for solving the following master problem obtained from the pattern-based formulation in (Gu et al., 2011):

$$\max_{t_p, e^p_i} T = \sum_{p\in P} t_p : \sum_{p\in P} e^p_i t_p \le E_i \ \forall i \in V,\ e^p_i, t_p \ge 0 \,\forall\, i, p \tag{6}$$

where V_s is the set of sink nodes , V_o is the set of sink possible locations and V is the set of non-sink sensor nodes.

Different patterns correspond to columns in (Gu et al., 2011). The master problem is given by the pattern-based formulation except for e^p_i which is treated as a constant. It is hence a linear programming problem. The subproblem that determines which column to enter solves for the pattern that would give the maximum increase in the objective function of the master problem. If the objective function of the master problem cannot be improved any further then the optimal solution has been reached.

An initial set of patterns P_0 can be obtained by random assignment of sinks to locations and using shortest path Dijkstra's algorithm for routing to the nearest sink. The master problem is then solved and the corresponding optimal dual variables are obtained to substitute in the objective function of the sub-problem, which is given by a linear equation representing the reduced cost of the master problem. The reduced cost is function in e^p_i, which is the only variable in the objective function of the subproblem. The constraint sets for the subproblems are linear conservation of flow constraints and flow capacity constraints on all links including the links to possible sink locations.

8 DELAY-SENSITIVE ROUTING IN UNDERWATER WSNS

Underwater acoustic WSNs (UWA-SNs) were considered in (Ponnavaikko et al., 2013). The propagation delay in UWA-SNs is five times larger than in RF networks which has a non-negligible impact on the performance of UWA-SNs especially since they cover much larger areas (square kilometers) unlike the RF WSNs. In (Ponnavaikko et al., 2013) the propagation delay sensitivity was accounted for in optimizing energy for routing purposes. According to the authors of (Ponnavaikko et al., 2013), this was not considered before their work due to the added complexity it brings in.

The sensor nodes are immobile and have relaying capabilities that enables the WSN to use multi-hop communication to convey a packet from any sensor to a sink node. Slotted synchronous time division multiple access (TDMA) was the type of medium access control (MAC) scheme considered. The objective was to design an offline routing scheme to pre-calculate the number of slots per link and the amount of data to be transmitted on each link while factoring in the large propagation delays of water acoustic signals. The routing scheme proposed in (Ponnavaikko et al., 2013) was named Delay-constrained Energy-constrained Routing (DER).

8.1 Initial Problem Formulation

The initial problem formulation in (Ponnavaikko et al., 2013) is a *nonlinear program* (NLP) with a non-deterministic generic objective function in the decision variables and linear sets of constraints. A brief explanation of the formulation is given as follows:

- Decision variables:
  1. the number of bits transmitted on each link (W_{ij}),
  2. the transmission time allocated for each link (Δ_{ij}).
- Objective function: is the sum of all energy consumed on all links in the network. It contains $P\left(\frac{W_{ij}}{\Delta_{ij}}\right)$ which is a non-deterministic general term that is function in the number of bits W_{ij} to be transmitted on each link and the allocated transmit time Δ_{ij}. This term represents power consumption which is dependent on the modulation/channel coding schemes used for transmission.
- Constraint sets:
  1. A linear delay constraint , that takes into account the propagation delay of each link and the transmission delay that is inherent in the allocated transmission time. The total delays on all links from the source sensor nodes to the sink should not exceed a maximum allowable delay T, that was given by $\sum_{i=1}^{N-1}\sum_{j\in\psi_i}(\Delta_{ij}+\tau_{ij})\leq T$ where τ_{ij} is the propagation delay of link (i,j) and ψ_i is the set of neighbors for node i.
  2. A linear conservation of flow set of constraints for every node that guarantees that difference between incoming and outgoing data is equal to the amount of data generated by the sensor node, that is $\sum_{j\in\psi_i}W_{ij}-\sum_{j\in\psi_i'}W_{ij}=r_iT$ where r_i is the data generation rate of node i.

8.2 Reformulation

The objective function is linearized by substituting $P\left(\frac{W_{ij}}{\Delta_{ij}}\right)\Delta_{ij}$, the non-linear term, with an auxiliary variable ε_{ij} and adding to the constraint set the following equality constraint:

$$log\left(\varepsilon_{ij}\right)=P_L log\left(W_{ij}-log\left(\Delta_{ij}\right)\right)+log\left(\Delta_{ij}\right)\quad(7)$$

as well as the following equality constraint that connects the logarithm of the transmission rate with the number of bits transmitted and the transmission time allocated:

$$log\left(R_{ij}\right)=log\left(W_{ij}\right)-log\left(\Delta_{ij}\right)\quad(8)$$

where

- ε_{ij} is a new variable that represents the energy expended for transmission of data on link (i,j).
- P_L is a constant that represents the relation between the logarithm of the power on a particular link with respect to the logarithm of the transmission rate on that link.

The non-linear inequality constraints (7) and (8) are piecewise linearized by Special Ordered Set type 2 (SOS2) variables and break points giving a *Mixed Integer Linear Program* (MILP). To approximate the logarithms of the energy , power and the allocated time on each link, five vectors of k SOS2 variables were introduced where k is such that the approximation error is within 1%. Not more than two adjacent SOS2 variables can be non-zero otherwise the rest of the variables are enforced to zeros.

8.3 The Solution Method

There was no mention for the solution algorithm to be deployed by the network. A generic MILP solver is believed to be used for the numerical experiments, however it was not stated which procedures of those should be implemented in the network entity responsible for the routing implementation.

9 CONCLUSIONS

In this paper we explored and illustrated how different optimization techniques were used in solving different routing problems over WSNs. We explained the formulations that were done for these problems, classified them according to their types and explained the solution techniques used. A common set of constraints in most routing problems turns out to be the conservation of flow constraint sets. Mostly the objective is to minimize the energy consumption or maximize the network lifetime. We explained how distributed schemes are highly desirable in WSNs to reduce the communication overhead among nodes and balance the computational energy consumption across the nodes. Distributed computation is hence an important element that we included in our observations of the optimization techniques used.

As Table 1 shows, almost all the problems considered in this paper had initial formulations that could only be solved in centralized fashion. However using some good reformulation and solution techniques like the dual decomposition or problem specific heuristics, the problems could be solved in a distributed fashion. Therefore, as demonstrated throughout the paper, formulation techniques are always the key to the algorithms to implement. The speed of convergence of these algorithms and whether they can be implemented in distributed schemes, are consequences of the type of formulation the problem gets reduced to.

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