Segmentation Technique based on Information Redundancy Minimization

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Abstract: In this paper, a problem of image segmentation quality is considered. The problem of segmentation quality is viewed as selecting the best segmentation from a set of images generated by segmentation algorithm at different parameter values. We use superpixel algorithm SLIC supplemented with the simple post-processing procedure for generating a set of partitioned images with different number of segments. A technique for selecting the best segmented image is proposed. We propose to use information redundancy measure as a criterion for optimizing segmentation quality. It is shown that proposed method for constructing the redundancy measure provides it with extremal properties. Computing experiment was conducted using the images from the Berkeley Segmentation Dataset. The experiment confirmed that the segmented image corresponding to a minimum of redundancy measure produces the suitable dissimilarity when compared with the original image. The segmented image that was selected using the proposed criterion, gives the highest similarity with the ground-truth segmentations, available in the database.

1 INTRODUCTION

The paper deals with the problem of image segmentation quality. According to Haralik and Shapiro (Haralik and Shapiro, 1985), segmentation is the process of partitioning image represented as a region Ω into n non-overlapping subregions Ω_1, Ω_2 ,..., Ω_n . The elements in subregions are grouped by some feature and differ from the elements of the adjacent areas. Formal definition of segmentation is given in (Gonsales and Woods, 2008). Any of segmentation algorithms has one or more parameters. A problem of setting parameters of the algorithm arises. Parameters should be set in order to provide the best quality of the segmentation result. The problem of finding parameter values is rather difficult. In this work, we formulate the problem of segmentation quality as follows. Suppose, for a given input image U we obtain a set of Q segmented images $\mathfrak{V} = \{V_1, V_2, ..., V_q, ..., V_Q\}$. It is necessary to choose image V_q providing minimum for a given performance criterion $M(U, V_q)$:

$$q_{\min} = \arg\min_q \left(M(U, V_q) \right),\ q = 1, 2, ..., Q .$$

When solving different tasks of image analysis, suitable quality criterion should be applied. This may be a visual evaluation of an expert or any quantitative measure. The results of segmentation are usually compared with an image partitioned manually and accepted as ground-truth (Arbelaez, 2011). If the segmentation operation is considered as clustering of pixels, then the set-theoretical, statistical, and information-theoretical measures (Wagner, 2007) proposed to compare data clustering results, are used. The most commonly used are: chi-square measure; Rand Index (Rand, 1971) and its variants; Fowlkes-Mallows measure (Fowlkes and Mallows, 1983); mutual information and normalized mutual information (Ana, 2003); variation of information (Meilă, 2003, 2005). These measures make it possible to compare different versions of partitioning image into non-overlapping regions. In paper (Arbelaez, 2011), the authors noted that the standard methodology for estimating efficiency of segmentation algorithms is not yet developed.

In paper (Frosio, 2015) another approach is proposed. Parameters of the superpixel segmentation algorithm (Felzenszwalb, 2004) were chosen depending on the result of estimating similarity of

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segmented and original images. As a measure of similarity the authors proposed to use weighted uncertainty index calculated using the values of the normalized mutual information (Witten, 2002; Ana and Jain, 2003) between the color channels of the input and segmented images. The authors proposed to choose parameter value that provides the best segmentation in terms of visual perception. The dependence of the uncertainty index on parameter value (and accordingly, the number of the subregions) is approximately monotonous (see (Frosio, 2015) and Fig.1). At the training step, the expert estimations of the results of segmenting series of images at different parameter values were obtained. Next, using SVM-like classifier the areas of under-segmentation, over-segmentation, and optimal segmentation were formed in the space "parameter - uncertainty index". At image processing step, parameter of graph-cut segmentation algorithm is selected using an iterative procedure. Procedure starts from the parameter boundary values. Parameter is adjusted till the uncertainty index reaches the region of optimal segmentation. The drawbacks of this approach are the subjectivity of expert assessments and the fact that the segmentation algorithm will produce acceptable results only for those types of images that were involved in the training process.

In this paper, we say that the segmentation V of image U is "good" if applied segmentation algorithm does not produce a significant loss of information. Information losses are estimated by theoretical-information dissimilarity measure between original image U and segmentation V. "Good"segmentation contains information only on the most important objects fixed in the original image, and as in (Frosio, 2015), is the best in terms of visual perception. In work (Atick, 1990), a theoretical-information model of the human visual system is proposed. The model is based on Barlow hypothesis (Barlow, 1961) about minimizing data redundancy at the early stages of signal processing in the human visual system.

In this work, basing on principle of minimizing data redundancy (Atick 1990), we propose to use a measure of information redundancy as a segmentation quality criterion. We show that a particular method of forming information-theoretical model of segmentation system provides the redundancy measure with extremum. In order to demonstrate that segmented image corresponding to minimum of the redundancy measure is the best, i.e., it yields an acceptable dissimilarity with the original image and ground-truth segmentations, we conduct an experiment on images taken from Berkeley Segmentation Dataset BSDS500 (Arbelaez, 2011).

2 SEGMENTATION ALGORITHM AND POSTPROCESSING PROCEDURE

A method for choosing the best variant of segmentation is applied to the superpixel algorithm SLIC (Simple Linear Iterative Clustering) (Achanta, 2012) supplemented with the post-processing procedure. The procedure is proposed below. In the next section a brief description of the SLIC algorithm is given.

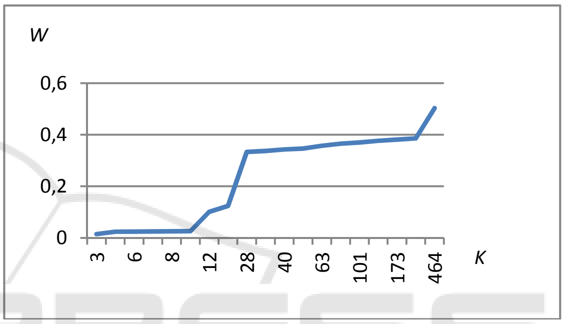


Figure 1: Uncertainty index W as a function of number of segments K computed for test image taken from BSDS500 dataset.

2.1 SLIC Segmentation Algorithm

The main idea of the segmentation algorithm SLIC (Achanta, 2012) consists in clustering pixels in restricted areas, into which the analyzed image is divided in a regular manner.

Each point of the image is characterized by five-dimensional vector $p = (c_1, c_2, c_3, x, y)^T$, where c_1, c_2, c_3 - are the point coordinates in the selected color space, x, y - are the spatial coordinates of an image pixel. The authors of the algorithm (Achanta, 2012) used CIE *Lab* color space.

The algorithm includes the following steps.

1. The image is divided into K fragments of size $a \times a$, which are taken as an initial approximation of superpixel clusters. Geometric centres C_k of the fragments are selected as the initial centres of superpixels.

2 Fragment centres are moved to the lowest color gradient position in a 3×3 neighborhood.

3. The local clusters are formed in a $2a \times 2a$

neighborhood of the centers C_k similarly to k-means algorithm. Distance D between the center and the fragment point is computed as a combination of Euclidean distances d_c and d_s of the color and spatial components describing point.

$$d_c = \sqrt{(c_{1j} - c_{1i})^2 + (c_{2j} - c_{2i})^2 + (c_{3j} - c_{3i})^2}, \tag{1}$$

$$d_s = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2}, \tag{2}$$

$$D = \sqrt{d_c^2 + \left(\frac{d_s}{a}\right)^2 m^2}, \tag{3}$$

where m is a parameter specifying the ratio of the contributions of the two components of the image description in the distance value D; i and j are the point numbers.

4. New cluster centers are determined and the displacements of cluster centers are computed.

5. Steps 3 and 4 are repeated as long as the displacements of centers between iterations will not exceed a predetermined value.

To allocate homogeneous regions corresponding to objects fixed in the image, it is necessary to merge superpixels. For this purpose, a post-processing procedure is proposed in the next section.

2.2 Post-Processing Procedure

In order to merge superpixels into homogeneous regions corresponding to objects in the original image, a two-step post-processing procedure is proposed.

At the first step neighboring superpixel areas are combined. For making a decision on merging, a threshold decision rule is used. This rule allows merging if the following inequality is taking place:

$$d_c(C_i, C_j) \le \Delta_1, \tag{4}$$

$$d_c(C_i, C_j) = \\ = \sqrt{(c_{1j} - c_{1i})^2 + (c_{2j} - c_{2i})^2 + (c_{3j} - c_{3i})^2}, \tag{5}$$

where $d_c(C_i, C_j)$ is the distance between centers of adjacent superpixels with numbers i and j in the selected color space; c_{1k}, c_{2k}, c_{3k} are the coordinates of centre C_k; Δ_1 is a threshold value.

The second step is intended to merge superpixel clusters throughout the entire image. As at the first step, the decision rule allows merging if the following inequality holds:

$$d_c(C_i, C_j) \le \Delta_2, \tag{6}$$

where Δ_2 is a threshold value.

Procedure includes the following operations: (a) scanning array of centers of superpixel image clusters and forming a logical matrix for combining neighboring superpixels by the rule (4, 5); (b) merging neighboring superpixels; (c) determining new cluster centers; (d) scanning array of centers of superpixel image clusters and forming a logical matrix for combining superpixels by the rule (6); (e) merging superpixels.

Results of segmenting an image, taken from dataset BSDS500, is shown in Figure 2.

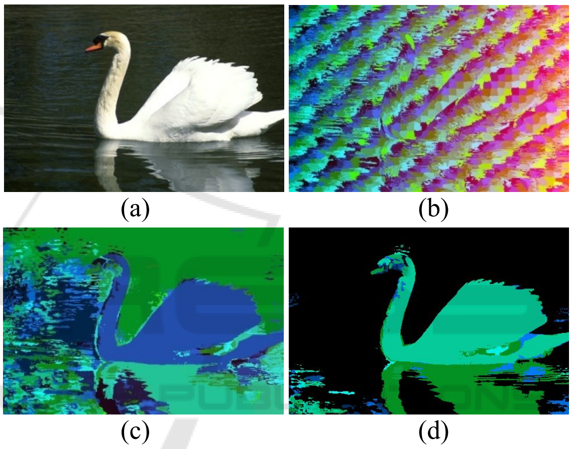


Figure 2: Results of segmenting image taken from dataset BSDS500: (a) input image; (b) superpixels produced by SLIC algorithm; output of the first (c) and second (d) steps of the post-processing procedure.

The segmentation technique based on SLIC algorithm with post-processing is controlled by four parameters: initial superpixel size a; color and spatial component ratio m; threshold values Δ_1 and Δ_2. Segmentation result depends on the choice of these parameters. In the next section, an information-theoretical technique for obtaining the best segmentation result is proposed.

3 CHOOSING THE BEST SEGMENTATION

Parameters of the algorithm are chosen as follows. The initial superpixel size a fit the size of the smallest objects that should be outlined in image. Parameter m the authors of works (Achanta, 2010; Achanta, 2012) set equal to 2. The result of segmentation also depends on parameters Δ_1 and Δ_2 of the conditions (4-6). These parameters will be chosen using information-theoretical measures. To apply information-theoretical approach, a probabilistic model of relationship between the input and the segmented images is needed. Segmentation quality will be estimated using one of the color channel (for example *L*) of images in the CIE *Lab* color space.

Let the initial and segmented images be the input and the output of a stochastic information system. Levels of lightness in images are the continuous random variables U and V with probability mass functions of $p(u)$ and $p(v)$, where u and v are the values of U and V, respectively. Operation of segmentation can be represented by an information channel model:

$$V = F(U+\eta), \tag{7}$$

where U is an input signal, V is a channel output, F is a transformation function, and η is a channel noise. We assume that noise η is Gaussian random variable with zero mean value and variance σ_η^2; variables V and η are independent.

We propose to use a redundancy measure as a criterion of segmentation quality. The redundancy measure is defined as follows (Atick, 1990):

$$R = 1 - \frac{I(U,V)}{C(V)}, \tag{8}$$

where $I(U;V)$ is a mutual information between the system input and output, $C(V)$ is a channel capacity. We take $C(V)=H(V)$, where $H(V)$ is an entropy of the output. Then, taking into account that $I(U;V)=H(V)-H(V\,|\,U)$, the expression (8) takes the form:

$$R = \frac{H(V\,|\,U)}{H(V)}, \tag{9}$$

where $H(V\,|\,U)$ is a conditional entropy of the output V under condition that the input is equal to U.

We will show that the redundancy measure of the segmentation system described by the model (7-9) depends on number of segments and can have a minimum.

Probability mass function of the output may be represented by a sum

$$p(v) = \sum_{k=1}^{K} P(v_k)\delta(v-v_k), \tag{10}$$

where $P(v_k)$ is a probability of lightness value v_k assigned to pixels of a segment having number k, $\delta(v-v_k)$ is a delta-function, K is a number of segments in the output image. To find analytic dependence $R(K)$, we will use a continuous version of model (7). Taking into account expression (10), differential entropy of the output can be written as follows:

$$H(V) = -\int_{-\infty}^{+\infty} p(v)\log p(v)dv = \\ = -\sum_{i=1}^{K}\left[P(v_i)\log P(v_i)\right]. \tag{11}$$

Let all values v_i be equiprobable: $P(v_i)=1/K$. Then it follows from (11) that

$$H(V) = \log K. \tag{12}$$

Next, we shall find an expression for differential conditional entropy $H(V\,|\,U)$. Conditional entropy $H(V\,|\,U)$ is a measure of information about signal noise η measured at the system output. In this case, we may take (Haykin, 1999):

$$H(V\,|\,U) = H(\eta). \tag{13}$$

Differential entropy of the Gaussian noise is equal to (Haykin, 1999)

$$H(\eta) = \frac{1}{2}\left[\log e + \log(2\pi\sigma_\eta^2)\right], \tag{14}$$

where σ_η^2 is a variance of the system noise.

We assume that the probability mass function of the input image lightness is represented as a Gaussian mixture model of K components, which may overlap partially. The components of the mixture correspond to the segments of the output image V. Areas of component overlappings generate noise η. The overlapping areas are formed by pixels of U having the same lightness values, but related to different segments in image V. Substituting (12)-(14) into (9), we get the following expression for redundancy measure:

$$R(K) = \frac{\log e + \log(2\pi\sigma_\eta^2)}{2\log K}. \tag{15}$$

It follows from (15) that the redundancy measure depends linearly on logarithm of system noise variance and inversely on logarithm of number of produced segments K. Function (15) have minimum at a point $K_{\min}$ if the noise variance σ_η^2 is close to zero at small K and rapidly grows when K increases. Computing experiments confirmed that such behavior of the noise variance is taking place.

Taking into account dependency of the redundancy measure R on number of segments K, the best segmented image should be selected in the following way. The input image U is segmented using algorithm SLIC with post-processing procedure at different values of parameter Δ_1. As a result, a set of Q segmented images $\mathfrak{V} = \{V_1, V_2, ..., V_Q\}$ is obtained. Next, for input image U and each of the segmented images $V_q, q = 1, 2, ..., Q$, the redundancy measure R is computed. We choose image V_q providing minimum to R: $R(V_q) = R_{\min}$. Image V_q divided into $K_{\min}$ segments fits parameter value $\Delta_1 = \Delta_{1\min}$. If it is necessary to apply the second step of the post-processing procedure, the output of the first step (which is the input image at the second step) should be redundant. It means that Δ_1 should be chosen as $\Delta_1 < \Delta_{1\min}$. Then the proposed above technique should be applied for finding the best value of Δ_2.

4 COMPUTING EXPERIMENT

In this work, in the experiments we used 25 images from the Berkeley Segmentation Dataset BSDS500 (Arbelaez, 2011) transformed to CIE Lab color space. The experiment includes three stages. At the first stage, each of the test images is segmented using algorithm SLIC and post-processing procedure at different values of parameter Δ_1. Each of the images generates a set of Q segmented images $\mathfrak{V} = \{V_1, V_2, ..., V_Q\}$. For input image U and each of the segmented images $V_q, q = 1, 2, ..., Q$ the redundancy measure R is computed. To involve all color channels, we use the weighted version of the redundancy R:

$$R_W(U, V_q) = \frac{R_L H_L(U) + R_a H_a(U) + R_b H_b(U)}{H_L(U) + H_a(U) + H_b(U)}, \quad (16)$$

where R_i is the redundancy measure determined in color channel $i \in \{L, a, b\}$ of images U and V_q; H_i is the entropy of the color channel i of the input image.

At the second stage, segmentation quality is estimated. We estimate the amount of information about the input image, which was lost in segmentation process. For this purpose we compare the set of Q segmented images with the input image U using normalized version of variation of information proposed in (Meilă, 2003, 2005) for comparing clusterings. This metric was also used in (Arbelaez, 2011) for comparing segmented images. Here we use the weighted index based on this metric:

$$VI_W(U, V_q) = \frac{VI_L H_L(U) + VI_a H_a(U) + VI_b H_b(U)}{H_L(U) + H_a(U) + H_b(U)}, \quad (17)$$

$$VI_i(U, V_q) = \frac{H_i(U) + H_i(V_q) - 2I_i(U, V_q)}{H(U, V_q)}, \quad (18)$$

where $VI_W(U, V_q)$ is the weighted variation of information; VI_i is the distance between color channels i of images U and V_q; I_i is their mutual information; $H(U, V_q)$ is the joint entropy.

At the third stage, using the weighted index (17) based on metric (18), we compare a set of Q segmented images with the ground-truth segmentations V_t^{GT}, $t = 1, 2, ..., T$, (T is a number of ground-truth segmentations for a test image U) available in BSDS500 dataset.

The results of the experiments are demonstrated on the images shown in Figure 3.

At the first stage of the experiment we apply SLIC algorithm and the first step of post-processing procedure to all test images. For each of the test images a set of segmented images is generated at initial superpixel size $a = 16$ pixels, $m = 2$ (see Section 3), and threshold values Δ_1 changing in the range $0 \le \Delta_1 \le 3.6$ with increment equal to 0.2.

(a) (b)

Figure 3: Test images taken from BSDS500 dataset.

Relationship between threshold Δ_1 and number of segments K in images V_q generated by one of the test images is shown in Figure 4.

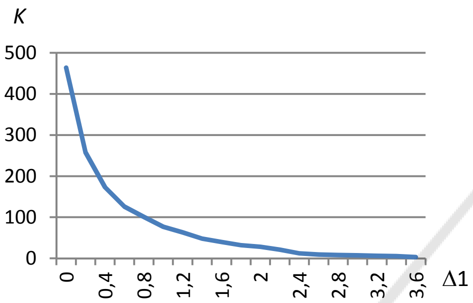
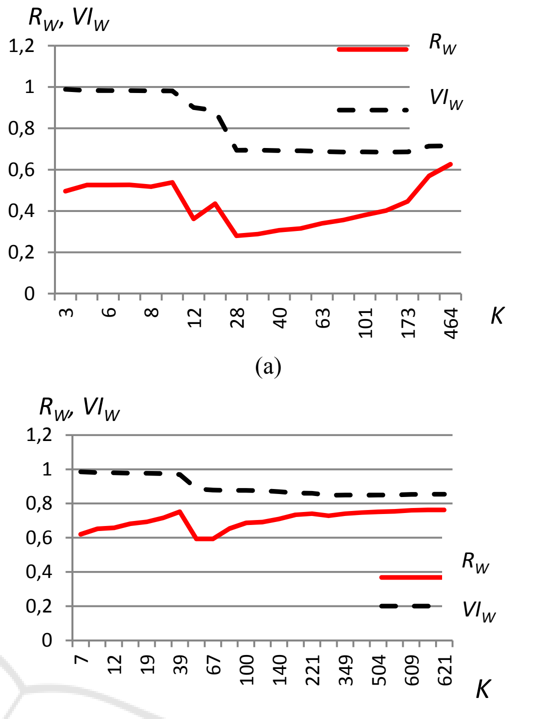


Figure 4: Relationship between threshold value Δ_1 and number of segments K.

For each test image and related set of segmented images we computed the weighted redundancy measure R_w. Dependencies of measure R_w on number of segments K for the test images shown in Figure 3(a,b) are depicted in Figure 5 (a) and (b). Minima of R_w are reached at $K = 28$, and $K = 55$ that correspond to threshold values $\Delta_1 = 2$ and $\Delta_1 = 2.6$, respectively.

In order to estimate the distance between the input and the segmented images, we compute weighted normalized variation of information (17). The curves representing $VI_w(U,V_q)$ as the functions of number of segments are shown in Figure 5(a, b) by dashed lines. One can see that distance between the input and segmented image decreases when K grows and become nearly stable at $K_{\min}$ corresponding to minimal redundancy value.

Normalized variation of information and its components computed in the lightness image channel are represented in Figure 6 as the functions of number of segments.



(b)

Figure 5: Dependency of redundancy R_w and normalized variation of information VI_w on number of segments K for images shown in Figure 3.

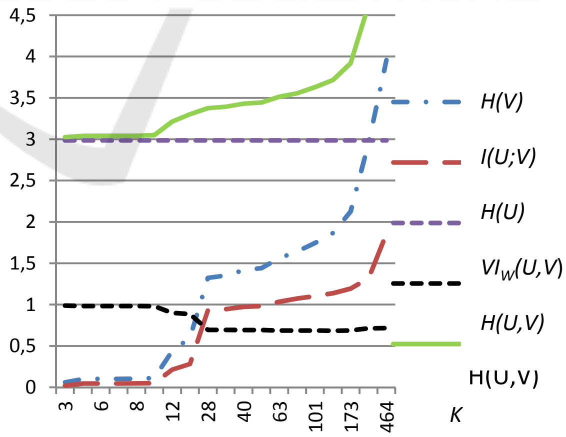


Figure 6: Normalized variation of information $VI(U,V)$ in one of the color channels and its components: marginal entropies $H(U)$ and $H(V)$, mutual information $I(U;V)$, and joint entropy $H(U,V)$ as the functions of K.

At the last stage we compared sets of segmented images with ground-truth segmentations. The result of comparing obtained for image shown in Figure

3(b) is represented in Figure 7 as the curves reflecting relationship between normalized variation of information $VI_w(V_t^{GT}, V_q)$, $q = 1, 2, ..., Q$, $t = 1, 2, ..., T$, and number of segments K in images V_q. It can be seen from Figure 7 that for the majority of the ground-truth segmentations, the distance $VI_w(V_t^{GT}, V_q)$ is minimal when image V_q is partitioned into 55 segments. This V_q gives minimum to redundancy measure R_w. Taking into account the fact that ground-truth segmentations were produced manually, we can conclude that the proposed technique allows one to obtain the best segmentation in terms of visual perception.

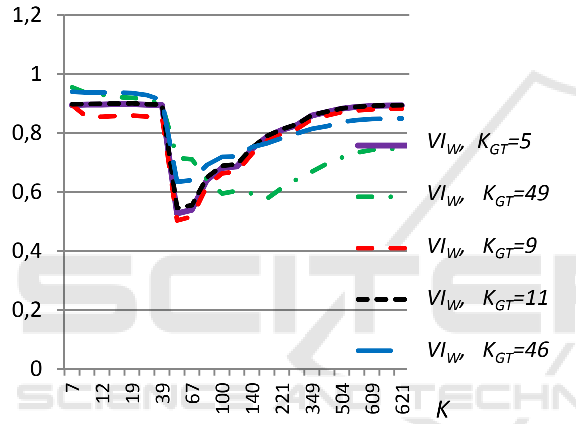


Figure 7: Normalized variation of information $VI_w(V_t^{GT}, V_q)$ computed for segmented images V_q and ground-truth segmentations with different number of segments K_{GT}.

Ground-truth segmentations of images shown in Figure 3 and segmented images fitting condition of minimum of the redundancy measure, are depicted in Figure 8. It can be seen from Figure 8 that the main details of the original images are captured in the segmented images as well as in the ground-truth segmentations.

To show the efficiency of the proposed technique, we introduce the following relative difference:

$$\Delta K = \frac{K_{\min} - K_{\min}^{GT}}{K_{\max}}, \tag{19}$$

where $K_{\min}$ is a number of segments corresponding to $R_{\min}$; $K_{\min}^{GT}$ is a number of segments in image V_q, which corresponds to the minimum of distance $VI_w(V_t^{GT}, V_q)$; $K_{\max}$ is the highest possible number of segments in images V_q obtained from input image U.

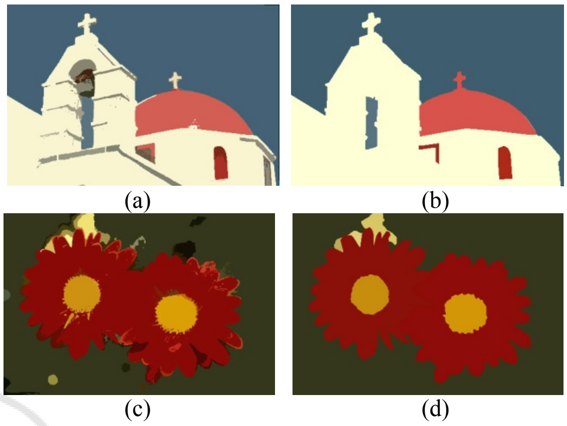


Figure 8: Segmented and ground-truth images: (a) segmented image from Figure 3(a), $K = 28$; (b) ground-truth segmentation, $K = 12$; (c) segmented image from Figure 3(b), $K = 55$; (d) ground-truth segmentation, $K = 9$.

For example, for image shown in Figure 3 (b) $K_{\min} = 55$, $K_{\max} = 621$, and $K_{\min}^{GT} = 181$ for the ground-truth segmentation with number of segments $K_{GT} = 49$; $K_{\min}^{GT} = 55$ for other ground-truth segmentations (see Figure 7). Histogram of ΔK values computed for 25 test images and 125 ground-truth segmentations (5 ground-truth segmentations per each of the test images) is depicted in Figure 9. Figure 9 shows that there exists a sufficiently large group of test images such that magnitude of ΔK is rather small. The ground-truth segmentations of these images are close enough in the sense of measure (17-18) to segmentations, which minimize redundancy of information R_w.

5 CONCLUSIONS

In this work, the problem of image segmentation quality was considered. The problem of segmentation quality was studied as a task of selecting the best segmentation from a set of images generated by segmentation algorithm at different parameter values.

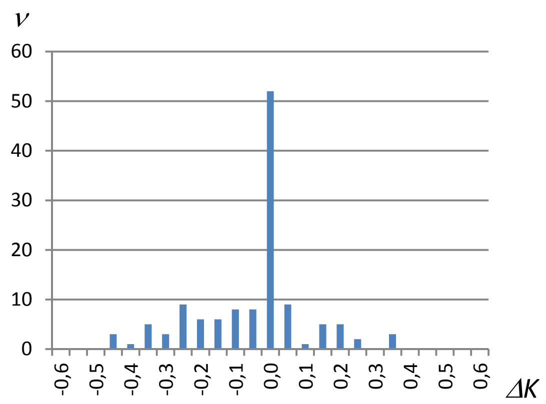


Figure 9: Histogram of ΔK values computed for 25 test images and 125 ground-truth segmentations; ν is a frequency of occurrence of particular ΔK value.

A technique based on theoretical-information criterion was proposed for selecting the best segmented image. We proposed to use information redundancy measure as a performance criterion. It was shown that the proposed way of constructing the redundancy measure provides the performance criterion with extremum. Computing experiment was conducted using 25 images from the Berkeley Segmentation Dataset. The experiment confirmed that the segmented image corresponding to a minimum of redundancy measure, produced the suitable information dissimilarity when compared with the original image. The segmented image, which was selected using the proposed criteria, gives the minimal distance from the majority of ground-truth segmentations available in BSDS500 database.

We used SLIC segmentation algorithm supplemented with the post-processing procedure for generating sets of partitioned images with different number of segments. The proposed technique of optimizing segmentation quality can be combined with other segmentation algorithms.

The future research will be aimed at the improving segmentation noise model and estimating the boundaries of application domain.

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REFERENCES

Haralick, R., Shapiro, L., 1985. Image Segmentation Techniques. *Computer Vision, Graphics, and Image Processing,* 29(1), 100-132.

Gonzalez, R., Woods, R., 2008. *Digital Image Processing. Third Edition*. Pearson Prentice Hall.

Arbelaez, P., Maire, M., Fowlkes, C., and Malik, J., 2011. Contour Detection and Hierarchical Image Segmentation. *IEEE TPAMI*, 33(5), 898-916.

Wagner, S., Wagner, D. 2007. *Comparing Clusterings - An Overview. Technical Report No 2006-04*, Universität Karlsruhe (TH).

Rand, W., 1971. Objective Criteria for the Evaluation of Clustering Methods. *Journal of the American Statistical Association,* 66(336), 846-850.

Fowlkes, E., Mallows, C., 1983. A Method for Comparing Two Hierarchical Clusterings. *Journal of the American Statistical Association,* 78(383), 553. doi:10.2307/2288117.

Witten, I., Eibe, F. 2002. *Data Mining: Practical Machine Learning Tools and Techniques*, Elsevier.

Ana, L., Jain, A., 2003. Robust data clustering. In: *Proc. CVPR 2003*. IEEE, 2, 128 – 133.

Meilă, M., 2003. Comparing Clusterings by the Variation of Information. *Learning Theory and Kernel Machines*. *LNCS*, Springer, 2777, 173-187.

Meilă, M., 2005. Comparing Clusterings: An axiomatic view. In: *Proceedings of the 22nd International Conference on Machine Learning (ICML 2005)*.

Frosio, I., Ratner, E., 2015. Adaptive Segmentation Based on a Learned Quality Metric. In: *Proc. VISAPP 2015,* SCITEPRESS, 1, 283-291.

Felzenszwalb, P., Huttenlocher, D., 2004. Efficient Graph-Based Image Segmentation. *International Journal of Computer Vision,* 59(2), 167–181.

Atick, J., Norman, A., 1990. Towards a theory of early visual processing. *Neural Computation archive*, 2(3), 308–320.

Barlow, H., 1961. Possible Principles Underlying the Transformations of Sensory Messages. In: *Rosenblith, W.A. (ed) Sensory Communication*. Cambridge: M.I.T. Press.

Haykin S. 1999. *Neural Networks: A Comprehensive Foundation*. 2nd ed. Upper Saddle River, NJ, USA: Prentice Hall Inc.

Achanta, R., Shaji, A., Smith, K., Lucchi, A., Fua, P., Susstrunk, S., 2010. SLIC Superpixels. *Technical report*, EPFL, Lausanne.

Achanta, R., Shaji, A., Smith, K., Lucchi, A., Fua, P., Susstrunk, S., 2012. SLIC Superpixels Compared to State-of-the-Art Superpixel Methods. *IEEE Transactions on Pattern Analysis and Machine Intelligence,* 34(11), 2274-2282.