Discrete Strategy Game-theoretic Topology Control in Wireless Sensor Networks

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Abstract: One of the most significant problems in Wireless Sensor Network (WSN) deployment is the generation of topologies that maximize transmission reliability and guarantee network connectivity while also maximising the network's lifetime. Transmission power settings have a large impact on the aforementioned factors. Increasing transmission power to provide coverage is the intuitive solution yet with it may come with lower packet reception and shorter network lifetime. However, decreasing the transmission power may result in the network being disconnected. To balance these trade-offs we propose a discrete strategy game-theoretic solution, which we call TopGame that aims to maximize the reliability between nodes while using the most appropriate level of transmission power that guarantees connectivity. In this paper, we provide the conditions for the convergence of our algorithm to a pure Nash equilibrium as well as experimental results. Here we show, using the Indriya WSN testbed, that TopGame is more energy-efficient and approaches a similar packet reception ratio with the current closest state of the art protocol ART.

1 INTRODUCTION

A significant problem in Wireless Sensor Network (WSN) topology management is to guarantee connected network topologies that have a high transmission reliability. The simple approach would be to increase the radio transmission power levels of unconnected nodes. However, this is too simple and does not account for the complexities of the wireless channel. An increase in transmission power might cause an increase in interference, decreasing the number of packets received (i.e. lowering Packet Reception Ratio, or PRR). On the other hand, as we see in (Spyrou and Mitrakos, 2015b), if the distance between the transmitter-receiver and interferer-receiver is difference by approximately a factor of 2, interference does not cause packet loss. This indicates that a node may select a high transmission power level, in order to strengthen its signal, without suffering from packet loss. There is a sweet spot in PRR related to transmission power levels that can keep PRR to a high level while not using a larger transmission power level than necessary. The transmission power also affects the energy consumption of the node, directly influencing the lifetime of the WSN (Antonopoulos et al., 2009). In order to handle this trade-off we present a discrete strategy distributed game-theoretic approach that maximizes each node's PRR while using the optimal transmission power from an optimisation problem; guaranteeing connectivity. We call our approach TopGame.

Specifically, we focus on the trade-offs between energy consumption, and PRR. We use game theory, since it can appropriately describe the behavior of selfish nodes and find an optimal solution in a distributed manner. Modeling systems with selfish algorithms have been shown to provide efficient solutions that improve network performance (Yeung and Kwok, 2006). We consider nodes to be individual players that play selfishly in order to find a best response for their objectives. In this paper we present our model and prove that this game is a potential game (Monderer and Shapley, 1996). Potential games are games where the incentive of players to change their strategy can be expressed in a single global function, the potential function. Potential games have been used in wireless networks in a plethora of problems, including power control (Heikkinen, 2006) (Spyrou and Mitrakos, 2015a), cognitive radio (Neel et al., 2004), gateway selection (Song et al., 2011) and channel allocation (Chen et al., 2011). In our game-theoretic formulation we prove that there is an equilibrium

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point. Next, we provide testbed results to show the convergence of our proposed algorithm and we compare it to the closest state of the art algorithm, Adaptive Robust Topology (ART)[1], with respect to connectivity, energy-efficiency and PRR. To our knowledge this is the first practical topology control game that has been evaluated on a real testbed system, and show the following:

- TopGame exhibits slightly lower network PRR than ART, since it exploits a Transmission Reliability metric to determine each node's final transmission power.
- Using TopGame, the network's relative energy consumption is less by 5% than ART's, due to the fact that TopGame can use a per node transmission power setting, which remains so after the optimisation process. Also, connectivity is preserved.
- TopGame's operation increases contention for accessing the wireless medium, since it keeps a steady transmission power level and it includes the bootstrapping period. This explains the slightly less PRR of our approach.
- TopGame includes mathematical proofs to support the convergence of each node's transmission power in the form of the Nash equilibrium of a potential game.
- We prove that the Price of Stability and Price of Anarchy of TopGame is 1. This shows that TopGame can find the optimal equilibrium of the game.

The paper is structured as follows: Section 2 provides the related work, Section 3 introduces topology control in WSN, Section 4 describes game theory basics and potential games, Section 5 formally describes TopGame, Section 6 shows the experimental results obtained and Section 7 presents the conclusions.

2 RELATED WORK

The characteristics and behaviors of wireless links are now more understood. There has been work measuring the effects of varying power levels and showing the irregularity of radio ranges and the lack of link symmetry (Son et al., 2005) (Zhao and Govindan, 2003). The relationship between PRR and RSSI for the Chipcon CC2420 radio was established in (Lin et al., 2006). Subsequent work then looked at the differences in behavior between indoor and outdoor networks, and fluctuations in link quality over longer durations of time (Hackmann et al., 2008).

Regarding of Topology Control (TC) specifically, (Hackmann et al., 2008) contributes a comprehensive review of this field which we summarise. Given the diversity of link behaviors influenced by their environment, experimentation for much of the early TC work was carried out using graph theory and simulation studies for tractability reasons. Yet, this work did not consider aspects like realistic radio ranges, node distributions or node capability/capacities into account, limiting their usefulness for real sensor networks (Li et al., 2005a; Li et al., 2005b) (Burkhart et al., 2004) (Blough et al., 2007) (Gao et al., 2008). For example, some have assumed that link costs are proportional to link length, but in reality a more complex relationship is evident (Son et al., 2005) (Ganesan et al., 2002) (Zhao and Govindan, 2003). The main competitors in the practical Topology control area are PCBL (Son et al., 2005) and ART (Hackmann et al., 2008), which we introduce next.

PCBL was derived from link quality observations showing that links with a very high PRR remain quite stable. They then categorise links as blacklisted, middling or highly reliable. The power in the latter is minimised to their lowest stable power setting while the blacklisted are not used at all. The middling links are those that lie between the two and are set to full power. Given the expense of probing the network to establish the link categories, this protocol cannot work with dynamic routing protocols such as CTP (Gnawali et al., 2009). CTP aims to find the least expensive routes through the network. To overcome such link probing, link quality metrics have been used to approximate PRR in ATPC (Lin et al., 2006). Specifically there is a link between RSSI and PRR, and LQI and PRR over a monotonically-increasing curve. Further, linear correlations between transmission power levels and RSSI/LQI are observed at the receiver but are different for each environment monitored. Therefore, ATPC estimates the slope and uses closed feedback to adjust the model to the current situation to achieve lower bound RSSI (PRR).

Hackmann et al., showed that RSSI and LQI cannot always realistically estimate PRR in indoor environments (Hackmann et al., 2008), nor can instantaneous probing represent the behaviors of a link over time. They propose ART, which does not rely on estimates of link quality nor does it involve long bootstrapping phases. Being more dynamic, ART adapts link power to changes in the environment as well as contention using a gradient. Also, where applications expect acknowledgment messages, ART can piggyback these to reduce communication overhead. ART

[1] Note that by ART we mean the optimised ART.

selects the appropriate transmission power based on the failures observed when the target PRR is 95% and a contention gradient.

In (Hao et al., 2015), the authors proposed a distributed topology control and channel allocation game-theoretic algorithm. The main objective of the work is the relief of interference and the energy consumption balancing. They examined the connection between topology control and channel allocation. They designed a game-theoretic model that takes into account transmission power, energy consumption and interference suffered by a node. They have proven the existence of Nash Equilibrium and they developed an algorithm that preserves connectivity by jointly setting the transmission power and channel. Lastly, their algorithm converges to Pareto optimality.

Tan et al. (Tan et al., 2015), suggested a topology control scheme where every node tunes its transmission power adaptively, in order to use its harvested energy in an efficient manner. The authors, proposed an ordinal potential game model where high harvesting nodes cooperate with the low harvesting nodes to ensure network connectivity. They proved the existence of a Nash Equilibrium and they designed an algorithm that achieves it.

Abbasi at al. (Abbasi and Fisal, 2015), investigated the issue of topology control in wireless sensor networks, in order to perform energy consumption minimisation and energy balancing. Their approach accomplished their objectives by adjusting transmission power on the nodes and preserving connectivity. The authors utilised a game-theoretic scheme to address energy welfare topology control. They showed that their proposed game-theoretic solution is a potential game and it achieves a unique Nash equilibrium, which is Pareto optimal as well.

Nahir et al. (Nahir et al., 2008), provided a game-theoretical solution to the topology control problem, by addressing three major issues: the price of establishing a link, path delay and path congestion proneness. They established that bad performance due to selfish play in the considered games is significant, while all but one are guaranteed to have a Nash equilibrium point. Furthermore, they showed that the price of stability is typically 1; hence, often optimal network performance can be accomplished by being able to impose an initial configuration on the nodes. Furthermore, the authors express their concern regarding the computational tractability of their solution.

Komali et al. (Komali et al., 2008), analysed the creation of energy efficient topologies with two proposed algorithms. Specifically, their game-theoretic model specified that nodes have the incentive to preserve connectivity with a sufficient number of neighbours and that the network will not partition. They proved that their game is an exact potential game and that a subset of the resulting topologies is energy efficient. They addressed the major issue of fair power allocation by providing the argument of efficient allocation vs fair allocation.

3 WSN AND TOPOLOGY CONTROL

Wireless sensor networks are networks of small computational devices fitted with radio transceivers for communication and sensors to capture data. Topology control can be defined by the construction of a graph that represents the nodes and links in the network that does not consist of any disjoint parts. Good topology control mechanisms can be characterized by providing an energy efficient network, offering high throughput and doing so with a low overhead. Energy-efficiency equates to the use of the minimum transmission power that guarantees connectivity, where throughput can be maximised by reducing interference and contention on the wireless medium. However, minimum transmission power does not guarantee a high reliability of transmission resulting in high throughput. This is due to a weak signal that may be significantly influenced by a small portion of interference.

For the most part, hitherto link asymmetry has been ignored, and the use of different transmission power levels when a node transmits to different neighbours may cause undesired packet loss. In addition, in a dense network, a node having a large number of neighbours may not be able to cope with transmission power changes when unicasting to different recipients in that neighbourhood while expecting to achieve a high PRR as well. As observed by Ahmed et al. (Ahmed et al., 2009), environmental effects and different node transmission powers are the major cause of link asymmetry in WSNs.

4 GAME THEORY AND POTENTIAL GAMES

Game theory studies mathematical models of conflict and cooperation (Von Neumann et al., 2007), between nodes in our work. Therefore, our meaning of the term game corresponds to any form of social interaction between two or more nodes. The rationality of a node is satisfied if it pursuits the satisfaction of its preferences through the selection of appropriate strategies. The preferences of a node need to satisfy general rationality axioms, then its behavior can be described by a utility function. Utility functions provide a quantitative description of the node's preferences and the main objective is therefore the maximization of its utility function.

In this work, we focus on strategic non-cooperative games, since we consider nodes to act as selfish players that want to preserve their interests. The intuition behind this is that the nodes will reach an optimal state, without having to pay a price to maximize their payoffs. The Nash equilibrium is the most important equilibrium in non-cooperative strategic form games. It is defined as the point where no node will increase its utility by unilaterally changing its strategy. It got its name from John F. Nash who proposed it (Nash Jr, 1950).

In 2008, (Daskalakis et al., 2008) Daskalakis proved that finding a Nash equilibrium is PPAD-complete . Polynomial Parity Arguments on Directed graphs (PPAD) is a class of total search problems (Papadimitriou, 1994) for which solutions have been proven to exist, however, finding a specific solution is difficult if not intractable. This development lead researchers to concentrate a specific class of games called 'Potential Games', due to the important properties that pure Nash equilibria will always exist and best response dynamics are guaranteed to converge.

This class of games consists of the exact and ordinal potential games. In this paper we utilise exact potential games and refer the reader to (Monderer and Shapley, 1996) for details on potential games. In order to use exact potential games, it is essential to have a potential function that has the same behavior as the individual utility function, when a player unilaterally deviates.

More formally:

A game $G\langle N, A, u\rangle$, with N players, A strategy profiles and u the payoff function, is an exact potential game if there exists a potential function

$$V : A \rightarrow \mathbb{R} \tag{1}$$

subject to

$$\forall i \in N, \forall \sigma_{-i} \in A_{-i}, \forall \sigma_i, \sigma_i' \in A_i \tag{2}$$

where σ_i is the strategy of player i, σ_i' is the deviation of player i, σ_{-i} is the set of strategies followed by all the players except player i and A_{-i} is the set of strategy profiles of all players except i such as

$$V(\sigma_i, \sigma_{-i}) - V(\sigma_i', \sigma_{-i}) = u_i(\sigma_i, \sigma_{-i}) - u_i(\sigma_i', \sigma_{-i}) \tag{3}$$

5 TopGame

We developed the TopGame algorithm that aims to guarantee connectivity, by locating the best response of PRR and transmission power. The intuition behind this research is that TopGame will force nodes to converge to the best transmission power.

A WSN consists of a set of nodes N and each node $i \in N$ can switch its transmission power $p_i^k \in P$, where $k \in 3, 7, 11, 15, 19, 23, 27, 31$ and P is the set of the available transmission power levels of our example CC2420 transceiver. In this paper, we employ 4 transmission power levels, namely $11, 15, 19, 23$, in order to identify the PRR when transmission powers that operate mostly on the gray area (Son et al., 2004) are used. Let a vector $\mathbf{P} = (\mathbf{p_1}, \mathbf{p_2}, \ldots, \mathbf{p_{|N|}})$ be an allocation of the transmission power level of each sensor node. The total number of possible power allocations is $4^{|N|}$. The aim of this paper is to determine a power allocation in a distributed way, which can achieve a best response rade-off between network connectivity, energy-efficiency and transmission reliability, using game theory.

5.1 Connectivity Definition and Measurement

In this paper we consider the small-world Model A from (Ganesh and Xue, 2007), where there are N nodes in the network and each one arbitrarily selects m nearest neighbours to connect to. Essentially, we utilise the variant of this small-world model, where node locations are being modeled by a stochastic point process. The number of neighbours consists of nearest neighbours and shortcuts. A shortcut is an edge between two nodes if either of the two nodes exist in the nearest neighbour set of the other. If a node is connected by a nearest neighbour and a shortcut, multiple edges are replaced by a single one. The presence of the shortcuts reduces the network diameter. Furthermore, we have to note that m is the number of neighbours a node has in terms of a spatial graph, and $(N-1)p$ is the number of neighbours it has via shortcuts. In order to ensure connectivity the quantities $m = (1+\delta)\sqrt{2\log(N)}$ and $Np = (1+\delta)\sqrt{2\log(N)}$, where $\delta > 0$, are sufficient. Hence connectivity is preserved with a smaller degree of (nearest neighbours plus shortcuts). We select a degree of 6 for each each node. It is well known that the node degree can be reached by adjusting the transmission power; hence, the transmission power level that satisfies connectivity satisfies the condition that more than 6 nodes exist in the neighbourhood of each node.

5.2 Transmission Reliability (TR)

For a wireless link (i, j), the Packet Reception Ratio $PRR_{i,j}$ is defined as the ratio of the number of packets received by node j over the number of packets sent by node i. It can be expressed by approximation as

$$PRR_{i,j} = (1 - \xi_{i,j})^l \tag{4}$$

where l is the packet length in bits.

The Bit Error Rate (BER), which we denote as $\xi_{i,j}$, is given by the following formula (Fu et al., 2012)

$$\xi_{i,j} = \frac{1}{2}\left(1 - \sqrt{\frac{\gamma_{i,j}}{1+\gamma_{i,j}}}\right) \tag{5}$$

where $\gamma_{i,j}$ is the Signal-to-Interference-plus-Noise Ratio (SINR) of the transmission from node i to node j. $\gamma_{i,j}$ is given by

$$\gamma_{i,j} = \frac{H_{i,j} p_i}{\sum\limits_{t \neq i, t \neq j} p_t H_{t,j} + N_0} \tag{6}$$

where N_0 is the white noise and $H_{i,j}$ is the channel gain of the wireless link (i, j) and $H_{t,j}$ is the channel gain between the receiver and an interferer. Due to the path loss, the larger the distance between nodes t and j the smaller the $H_{t,j}$. We focus on static WSNs, hence, we assume that the channel is slow fading in nature and the channel gain of every link remains constant before the convergence of the TopGame algorithm.

To measure the reliability of links around node i, we define a new metric called Transmission Reliability (TR_i) as

$$TR_i(p_i, p_{-i}) = \frac{\sum\limits_{j \in N_i(p_i, p_{-i}), k \in N_j(p_i, p_{-i}), k \neq i} PRR_{k,j}}{\left| \bigcup_{j \in N_i(p_i, p_{-i})} (N_j(p_i, p_{-i}) - \{i\}) \right|} \tag{7}$$

where p_i is the power level of node i, p_{-i} means the power levels of all nodes except i, $N_i(p_i, p_{-i})$ is the set of nodes such that $\forall j \in N_i(p_i, p_{-i})$, $PRR_{i,j} > 0$.

For instance, Figure 1 shows a sub-graph of a WSN for a given transmission power allocation. For each link (i, j) $PRR_{i,j} > 0$. In this sub-graph $N_6 = 2, 15, 7, 11$ and $\beta TR_6 = (PRR_{1,2} + PRR_{15,2} + PRR_{12,2} + PRR_{2,15} + PRR_{9,15} + PRR_{7,15} + PRR_{15,7} + PRR_{10,7} + PRR_{11,7} + PRR_{7,11} + PRR_{14,11})/11$.

In practice, every node i can obtain TR_i at run time by every node j in N_i calculating the $\overline{PRR}_{k,j}$ as the average $PRR_{k,j}$, $k \in N_j - \{i\}$ and periodically broadcasting $\overline{PRR}_{k,j}$. Thereafter node i calculates TR_i.

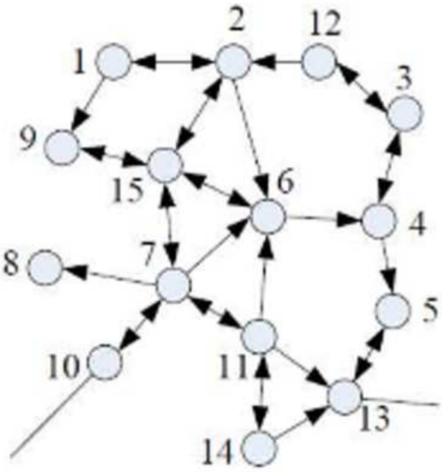


Figure 1: An example to explain Transmission Reliability metric.

5.3 Utility and Potential Function

We define the utility function of each node i as,

$$u_i(p_i) = TR_i - c_i p_i \tag{8}$$

where c_m is the price assigned to each strategy played by a node/player.

Our strategy domain consists of 4 strategies, which are $11, 15, 19, 23$, which correspond to the values in table 1. Notably, the 2 smallest and 2 largest transmission power levels of the CC2420 radio have been excluded. The main reason is to see TopGame operate under medium to large SINR regime. The second reason is to simplify TopGame.

Table 1: Transmission Power Levels and Values.

PA_LEVEL	dB	mA
11	-10	11.2
15	-7	12.5
19	-5	13.9
23	-3	15.2

It is straightforward to see that the above utility function has a minimum under the following condition of medium to high SINR values. We do the price assignment in a similar way with (Candogan et al., 2010). The prices assigned at every node has the value 1 except when it reaches its maximizer. Each node then assigns the price given below:

$$c_i = diff(TR_i) \tag{9}$$

Hence, if we take the first derivative to obtain the minimum, it follows that there is a local minimum. Since we wish to maximize the function we simply take the negative of (8).

$$u_i(p_i) = c_i p_i - TR_i \tag{10}$$

Thereafter we wish to define the potential function and prove that the game G is a potential game.

Proposition 1. *The game G is a potential game. The potential function is given by*

$$V(\boldsymbol{p}) = \sum_i c_i p_i - \sum_i TR_i, p_i \in A \quad (11)$$

Proof. This comes as a result by taking the characterisation of the potential games in (Monderer and Shapley, 1996) where $\frac{\partial V(\mathbf{p})}{\partial p_i} = \frac{\partial u_i(\mathbf{p})}{\partial p_i}, i \in N$.

$$V(p_i, p_{-i}) - V(p_i^{'}, p_{-i}) = u_i(p_i, p_{-i}) - u_i(p_i^{'}, p_{-i}) + \sum_{m \in N, m \neq i}^{N} (u_m(p_m, p_{-m}) - u_m(p_m^{'}, p_{-m}))$$

Since only one node can deviate $\sum_{m \in N, m \neq i}^{N} (u_m(p_m, p_{-m}) - u_m(p_m^{'}, p_{-m})) = 0$. Hence we conclude that Γ is an exact potential game. This proof comes as a result of the fact that given a strategy of a node/player m, $p_m \in N$ and an alternative strategy $p_m' \in N$ and taking the assumption that the strategies of all the other nodes remain the same, we have

$$u_i(p_i, p_{-i}) - u_i(p_i, p'_{-i}) = V_i(p_i, p_{-i}) - V_i(p_i, p'_{-i}) \quad (12)$$

where p_{-i} is the transmission power strategy of all the nodes excluding that of the node i. Hence, the game is a potential game.

□

Remark 3.1: The potential function is significant since its maximisation, when a specific policy is played, results in this policy being an equilibrium of the designed game. In this work, the strategy set is discrete; hence, in the case that the potential function satisfies particular types of concavity, such as the Larger Midpoint Property (LMP) (Ui, 2008), the converse is true as well. If a policy is an equilibrium, it maximises the potential function. Thus, we may consider the TopGame as the following optimisation problem.

$$\hat{p}_i = \arg\max V_i(i) \quad (13)$$

As presented in (Altman et al., 2009), we consider two n-dimensional vectors $\delta(1), \delta(2)$. *Definition 1:* (Marshall et al., 2010) A vector $\delta(2)$ majorises $\delta(1)$, which we denote as $\delta(1) \prec \delta(2)$, if $\delta(2)$ is more "unregular" in the following fashion:

$$\begin{cases} \sum_{i=1}^{k} \delta_{[i]}(1) \leq \sum_{i=1}^{k} \delta_{[i]}(2), k = 1, 2, ..., n-1 \\ \sum_{i=1}^{k} \delta_{[i]}(1) = \sum_{i=1}^{k} \delta_{[i]}(2) \end{cases} \quad (14)$$

where $\delta_{[i]}(m)$ is a permutation of $\delta_i(m)$ satisfying the condition $\delta_{[1]}(m) \geq \delta_{[2]}(m) \geq ... \geq \delta_{[n]}(m), m = 1, 2$

Equation (8) suggests that the largest element of $\delta(2)$ is larger than the largest element of $\delta(1)$. Consequently, the smallest element of $\delta(2)$ is smaller than the smallest element of $\delta(1)$. Thereafter we proceed in Schur convexity properties of majorisation.

Definition 2:: A function $f : \mathbb{R}^n \to \mathbb{R}$ is Schur concave if $\delta(1) \prec \delta(2)$ suggests $f(\delta(1)) \geq f(\delta(2))$. f is Schur convex if the inequality suggests that $f(\delta(1)) \leq f(\delta(2))$.

Definition 1 dictates that there is strong majorisation; however, at least one of the inequalities of (8) is strict. Furthermore, Proposition $C.2$ of (Marshall et al., 2010) dictates that a function $f : \mathbb{R}^n \to \mathbb{R}$ that is symmetric and convex (concave), is also Schur-convex (concave). Hence, we need to show that our potential function is Schur-concave, in order to proceed with the majorisation properties.

Lemma 1. *Function V is concave in N*

Proof. It is obvious that the function is concave, since if we take the second derivative test the first term will be set to 0 and the second term is a concave term (raised to power) for medium to large SINR values. Note that for very high SINR values the second derivative of (10) become positive and the function becomes convex as we can deduct from (Meshkati et al., 2006).

□

Proposition 2. *If the function $u(p)$ is concave then the function $V(p)$ is Schur concave.*

Proof. The proof is given by using the following corollary from (Marshall et al., 2010).

□

Corollary 5.0.1. *Let $\phi(x) = \sum_{i=1}^{n} g(x)$ where g is concave (convex). Then ϕ is Schur-concave (convex)*

Theorem 5.1. *The Game G reaches the global optimum via the potential function $V(p)$ maximisation.*

Proof. Recall that the potential $V(p)$ Schur concave and it satisfies the LMP. It follows that if p^* is a Nash equilibrium strategy, then it maximises the potential and is the global maximum. Assume that there is another strategy profile p'^* that maximises the potential and is the global maximum. This means by p^* majorises p'^*. Since $V(p)$ is Schur concave it follows by definition that $V(p'^*) \geq V(p^*)$. Since, $p*$ maximises the potential, this is only possible when $V(p'^*) = V(p^*)$. Hence, p^* is the global optimum.

This also comes as a result of the fact that we have shown that there is a critical point in the function

$V(p)$. It follows from (Jorswieck and Boche, 2006) - Theorem 2.22 - that the critical point p^* is the global optimum. □

Notably, Schur concavity of V not only allows us to capture the optimal policies, but it allows the comparison of the performance of two non-optimal strategies, whenever one of the policies majorises the other.

Theorem 5.2. *The price of stability of the game is* 1

Proof. It follows from the previous theorem that shows that the game reaches the global optimum. □

Thereafter, we will proceed with the derivation of the Price of Anarchy (PoA) (Nisan et al., 2007), in order to further check the optimality of the game. Firstly, though, we start with the following result.

Definition 5.1. (Pareto efficient) (Myerson, 1991) A strategy profile $(p_i^{OPT}, p_{-i}^{OPT})$, is considered to be strongly Pareto efficient if and only if there exists no other strategy profile (p_i, p_{-i}) such that $u_i(p_i, p_{-i}) \geq u_i(p_i^{OPT}, p_{-i}^{OPT}), \forall i \in N$ and $u_i(p_i, p_{-i}) > u_i(p_i^{OPT}, p_{-i}^{OPT})$ for at least one node m. On the other hand, a strategy profile $(p_i^{OPT}, p_{-i}^{OPT})$ is weakly Pareto efficient if and only if there exists no strategy profile (p_i, p_{-i}) such that $u_i(p_i, p_{-i}) > u_i(p_i^{OPT}, p_{-i}^{OPT}), \forall i \in N$. We use the term Pareto efficient for both weak and strong cases.

Definition 5.2. A pure strategy NE is a Pareto efficient pure strategy NE if it is Pareto efficient.

Theorem 5.3. *A maximizer of V, which coincides with the optimal solution of (11), is a Pareto efficient pure strategy NE.*

Proof. We have shown previously that the game G reaches the maximum which is a pure strategy NE. Hence $(p_i^{OPT}, p_{-i}^{OPT})$ constitutes an optimal solution of (11). There is no other strategy that maximises the potential. That is that there is no strategy profile $(p_1, ...p_i) \in P_{i i \in N}$, such that

$$u_i(p_1, ...p_i) = V(p_1, ...p_i) > u_i(p_m^{OPT}, p_{-i}^{OPT}) = V(p_i^{OPT}, p_{-i}^{OPT}), \forall i \in N \quad (15)$$

Thus, considering Definition 5.2, $(p_i^{OPT}, p_{-i}^{OPT})$ is Pareto efficient. Moreover, let us assume the $\forall i \in N$, p_i is an alternative strategy of node/player i, where $p_i \neq p_i^{OPT}$. Then, we obtain

$$u_i(p_i, p_{-i}^{OPT}) \geq u_i(p_i^{OPT}, p_{-i}^{OPT}) \quad (16)$$

We see that there is no node that can unilaterally change its transmission power/ strategy, in order to increase its utility. Furthermore, the strategy profile $(p_1^{OPT}, ..., p_i^{OPT})$ is also a pure strategy NE. To summarise, $(p_1^{OPT}, ..., p_i^{OPT})$ is a Pareto efficient pure strategy NE. □

Since, the game G may have more than one pure strategy NEs, we will check the optimality of the NE to show the relationship between the local optimal NE and the Pareto efficient NE. Even though we have shown that the Game G goes to the global optimum, we will strengthen this proof even further, by evaluating the ratio between the highest utility and the worst-case NE, namely the PoA.

Theorem 5.4. *$PoA = 1$, i.e. a pure strategy profile of G is Pareto efficient.*

Proof. We assume that p_i^{OPT} is a Pareto efficient NE. Also, assume that p_i^* is an arbitrary pure strategy NE $\mathbf{p}_i^* = (p_i^*, p_{-i}^*)$. Then for any arbitrary node/player i, we have

$$u_i(\mathbf{p}^*) = V(\mathbf{p}^*) = c_i p_i^* - TR_i^* \quad (17)$$

Note that $u_i(\mathbf{p}^*) \geq u_i(p_i^{OPT}, p_{-i}^*)$ according to the definition of a game. Therefore, we have

$$u_i(\mathbf{p}^*) = V(\mathbf{p}^*) = c_i p_i^* - TR_i^* \geq u_i(\mathbf{p}^*) = V(\mathbf{p}^{OPT}) = c_i p_i^{OPT} - TR_i^{OPT} \quad (18)$$

Furthermore , since we have assumed that $\mathbf{p}^{OPT}$ is a Pareto-optimal pure strategy NE, $\forall i \in N$

$$V(\mathbf{p}^{OPT}) \geq V(\mathbf{p}^*) \quad (19)$$

Combining (18) and (19) we have $V(\mathbf{p}^{OPT}) \geq V(\mathbf{p}^*), \forall i \in N$. Hence, $PoA = 1$. □

5.4 Algorithm Design

The TopGame algorithm is a cross-layer approach that encapsulates information taken from the routing, MAC and physical layers. In particular, the PRR and neighbour information are obtained from the routing layer, the transmission power used is acquired from the radio and the MAC is responsible for triggering the game to determine the topology in the case of nodes failing or newly added to the network.

Initially, all nodes start communicating at their maximum transmission power $p_{max} = 23$. Node i collects the neighbour information, such as current transmission power levels used by its neighbours and their respective PRR. This occurs simultaneously, since the number of neighbours is determined via periodic beacons being broadcast and the PRR obtained by unicasting to random neighbours using a gossip-based protocol (Dammer and Hinrichsen, 2003). The nodes are also synchronised using beacons with a firefly-based (Breza and McCann, 2008) algorithm.

Node i iterates through its 4 available transmission power levels, it computes TR for each power level and it finally maximises its utility function u_i. Note that for practical reasons the pricing of each node's utility function is set to 1. The global optimum is accomplished as we can see in Theorem 5.1. Pseudocode of TopGame is presented in Algorithm 1.

Algorithm 1: TopGame at node i.

Require: $A_i = \{p^i_1, p^i_2, ..., p^i_{max}\}$
Require: $degree = 6$, $p_i = p_{max}$
for $i = 1$ to $N_{i,p}$ **do**
  $get\ p_{-i}, PRR_{i,k} \in N_i$
  $N_i \leftarrow N_i^{p_i}$
  $compute\ TR_i$
  $u_i(p_i, p_{-i}) = c_i * p_i - TR_i$
end for
$\hat{p}_i = \arg\max u_i(m)$

In the case of the addition or a failure of node, nodes that detect a change in their neighbour table initiate TopGame from the start, since their TR will be affected by the topological change. This is due to the fact that TopGame is a repeated game only on topological alterations.

5.4.1 Message Overhead

The message overhead per transmission power consists of the sum of the broadcast messages for synchronisation and the unicast messages transmitted to each of the neighbours of every node. That is,

$$O_{TopGame} = f_{sync} * N + N * f_{link} * m * M \qquad (20)$$

where N is the number of nodes, m is the window of the unicast transmission to obtain PRR and M is the number of neighbours of every node for a given transmission power.

Since $f_{sync} = \Theta(1)$, $f_{link} = \Theta(N)$ and $M = \Theta(1)$, provided some constants c_1, c_2, c_3 we have: $f_{sync} \leq c_1$, $f_{link} \leq c_2 * N, M \leq c_3$. Therefore, 20 can be expressed as follows:

$$\begin{aligned} &O_{TopGame} \leq f_{sync} * N + N * f_{link} * m * M \\ &\Rightarrow O_{TopGame} \leq c_1 * N + c_2 * N * m * c_3 \\ &\Rightarrow O_{TopGame} \leq N * (c_1 + m * c_2 * c_3) \\ &\Rightarrow O_{TopGame} = O(N) \end{aligned}$$

6 EXPERIMENTAL EVALUATION AND RESULTS

In order to evaluate TopGame, in comparison to ART, we performed 120 minute experiments using 50 nodes selected at random on Indriya. The data rate of the nodes was 250 $kbps$ and each node transmits 4 packets per second. In addition each node calculates its PRR over a window of 8 packets. Each node finishes iterating through all the transmission power levels and the utility function of each power level has been obtained in order to proceed with the maximisation.

Our aim is to show that converging to lower transmission powers, may provide similar reception performance. In this section we will provide results that show that TopGame approaches ART (Hackmann et al., 2008) in terms of PRR and is slightly better in energy-efficiency while ensuring that the network is connected.

6.1 Performance and Energy Consumption

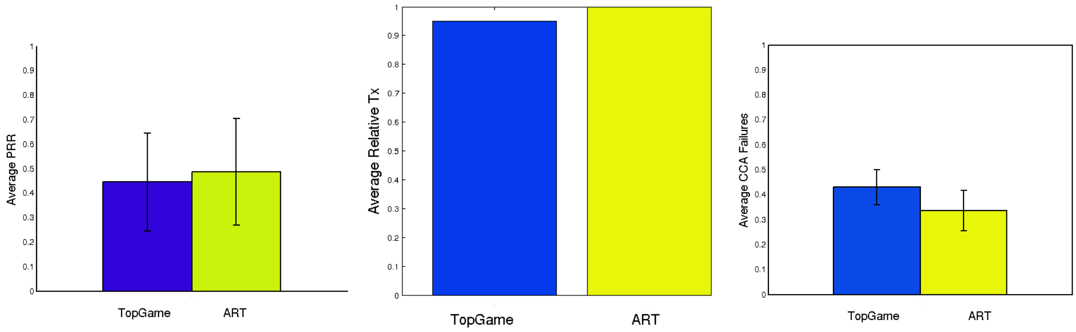
Initially, we obtained the average PRR and relative energy consumption, in order to evaluate both algorithms globally. The network average PRR is provided in Figure 2 (a). Specifically, we observe that TopGame exhibits an average network PRR of 44.7%, while ART 48.1%. Moreover, the standard deviation of ART is higher than TopGame's by 3%.

The difference in the PRR between the two schemes is not quite significant; However, we have shown that a game theoretic algorithm, with a more systematic approach, exhibits similar performance with a state-of-the-art practical algorithm such as ART. Further, the formation of less links does indicate that TopGame uses its utility function that finds the sweet spot per node. On the other hand, ART fluctuates between its two packet failure thresholds; thus, forming more links. As we have seen in a previous section, contention is related to the number of neighbours (K) of each node. Table 2 presents the average degree of the network and the number of links that are formed with TopGame and ART.

Table 2: Average K and formed links.

	Average K	Number of Links
ART	7	360
TopGame	7	338

In order to examine whether the difference in the PRR average of TopGame and ART is a result of channel collision we performed 2 hour experiments measuring the Clear Channel Assessment (CCA) failures. Figure 2 (c) presents the CCA failures ratio of TopGame and ART. Briefly, a CCA operation occurs when the MAC layer receives a packet to transmit, then it instructs the physical layer to check channel availability (CCA) in two consecutive slots. If the channel is found to be available in both slots, the node proceeds with its transmission. Otherwise, the node



(a) Average PRR

(b) Average Relative Energy Consumption

(c) Average CCA Failures Ratio

Figure 2: TopGame and ART average Relative Energy, Mean PRR and CCA failures Mean of 50 nodes.

attempts CCA again after a random back-off, which it repeats a certain number of times and it calls a failure of access to the upper layer. Hence, with TopGame exhibiting nearly 10% more CCA failures than ART, it is natural to assume that the difference in average PRR comes from a higher interference and collisions of TopGame of the bootstrapping period, since it initially forms a larger number of links that are included in the graph. Specifically, TopGame exhibits 43% failures, while ART's percentage is 33.6%.

In our experiments we were unable to directly monitor the energy consumed by the listening and transmitting periods of each node. Thus, we decided to use unicast communications as an indicator to calculate the relative average energy. Making the assumption that all nodes spend the same amount of energy in listening, to get a rough idea of relative energy consumption, we added the number of unicast messages transmitted by ART and TopGame with their respective transmission powers and multiplied them with the corresponding *mA* radio energy consumption. The relative energy consumption of the two algorithms can be seen in Figure 2 (b). TopGame consumes 5% less energy than ART, including the bootstrapping period.

This is due to the fact that the nodes do not fluctuate on a per packet basis and they are not targeting a very high PRR value as dictated in the ART thresholds; hence, TopGame is slightly more energy efficient. We present an example of two links from both TopGame and ART, in order to show the difference in the switching between transmission powers and the convergence of TopGame. From the figures 3 and 4, it is clear that ART switches its transmission power according to packet drops; hence, the Tx fluctuation in the figure. On the other hand, TopGame collects TR for each available transmission power and converges to the transmission power maximizing the utility function. Also, we are not aware of the energy cost

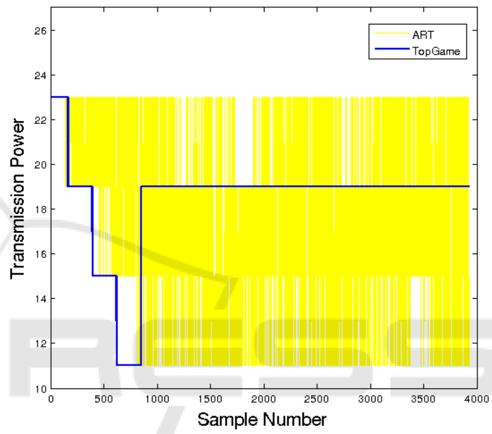


Figure 3: ART and TopGame Node 13 Tx levels.

of the continuous Tx switch. We assume it is negligible. Note that TopGame is repeated only when a neighbourhood change is detected.

Recall that ART's intention is to reach the target PRR of 95%, yet we observe that its reception quality is significantly lower. TopGame also does not attain this lofty figure. We believe that is the case for our scheme because of the bimodal distribution of 802.15.4 link qualities (Srinivasan et al., 2007).

By looking at the Cumulative Density Functions (CDF) of the two algorithms in figure 5, we observe that TopGame has a slightly higher probability of forming poorer quality links of PRR lower than 20%. ART has a lower probability of forming medium to high quality links. Furthermore, TopGame exhibits a sightly higher probability of establishing links with PRR over 80%. It would be strong to claim that TopGame is better than ART; however, approaching the numbers of ART is significant, since it relies in concrete theoretical basis.

Finally, we compared the RAM and ROM overhead of TopGame with ART. Table 3 shows that

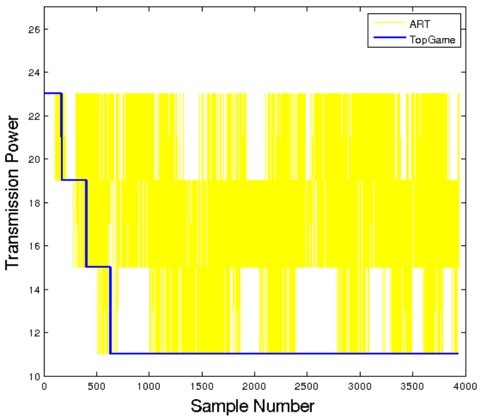


Figure 4: ART and TopGame Node 41 Tx levels.

TopGame consumes 2348 more ROM bytes than ART, while it produces an overhead in RAM of 342 bytes.

Table 3: RAM and ROM (bytes).

	RAM	ROM
TopGame	3426	25228
ART	3084	22880

6.2 Connectivity

After we obtained the results we evaluated connectivity offline using method that determines whether the resulting graph is connected. This was to evaluate the connectivity model we discussed in a previous section. We have shown that the average degree of each node is greater than 6 nodes. We derived the available links from TopGame and ART's data sets and we created their respective adjacency matrices. Thereafter, we used the matrices to find a zero eigenvalue. In the case that the corresponding eigenvector has 0s, then a sum of non-zero number of rows/columns of the adjacency is 0 (Horn and Johnson, 2005). Hence, the degrees of these nodes are 0 and the graph is disconnected. Both TopGame and ART resulted in fully connected graphs.

7 CONCLUSIONS

Compared with the state of the art protocol ART, we showed that TopGame is a general solution providing efficient robust Topology Control minimising the costs of communications while ensuring connectivity. First, we evaluated ART and TopGame on Indriya using 50 nodes to determine the average PRR and the average relative TX power. We evaluated connectivity based on a method that checks the eigenvector of

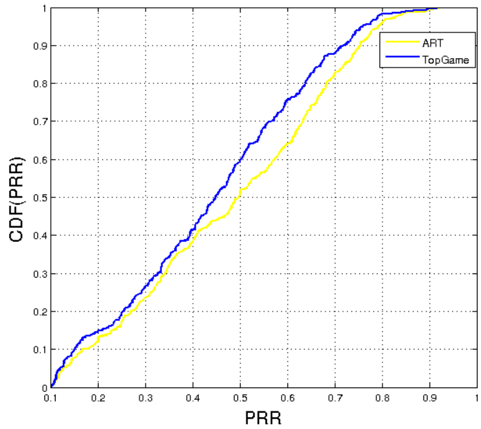


Figure 5: CDF for ART and TopGame.

each algorithms adjacency matrix (links) and determined that the resulting graph is connected. The experiments on Indriya showed that TopGame reduces power consumption compared with ART without significantly degrading link quality. Macro-benchmarks comparing TopGame to ART protocol indicated that TopGame provides guaranteed connectivity and exhibited slightly lower PRR than its competitor and 5% improvement on energy consumption. The corresponding Probability Density Functions showed that TopGame has a slightly higher probability of having links of low quality ($< 20\%$) than ART. Moreover, ART has a lower probability of creating links of $> 20-80\%$ PRR. Finally, TopGame has a better probability of creating high PRR links ($> 80\%$) This is a promising factor of the comparison between the two algorithms in terms of performance, since the average network PRRs were not significantly different. In terms of energy consumption, we presented results that show that TopGame converges to lower transmission power levels than ART making it more energy-efficient.

The main differences between ART and TopGame are that ART establishes per-link power levels while TopGame establishes power settings for a given neighbourhood of nodes, and thus can be seen as non link-based. A node running ART will have to switch between transmission powers to transmit packet to its neighbours. This has an impact on the transmission power selection in larger networks, since the target PRR (95%) is not reached and nodes select high transmission powers. TopGame's power is set to cover the neighbourhood and therefore has no such switching overhead. ART obtains data and makes decisions by indirectly considering link asymmetry in that; hence ART selected higher transmission power levels. In fact, their unoptimised version not using the contention gradient verify these phenomena and even the improved version also shows a decrease in PRR. However, link asymmetry is taken into account in TopGame; where bi-directional information helps to ensure both the connectivity of all nodes and that we will converge at a Nash Equilibrium. Finally, in terms of implementation, ART is closely coupled to CTP whereas, though TopGame is slightly more expensive in terms of speed and footprint, it is agnostic to WSN Operating System or stack implementations and is therefore more generally applicable. We aim to interface our approach with CTP or other state of the art routing protocol such as the Backpressure Collection Protocol (BCP) (Moeller et al., 2010).

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