A Polynomial Algorithm for Merging Lightweight Ontologies in Possibility Theory Under Incommensurability Assumption

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Abstract: The context of this paper is the one of merging lightweight ontologies with prioritized or uncertain assertional bases issued from different sources. This is especially required when the assertions are provided by multiple and often conflicting sources having different reliability levels. We focus on the so-called egalitarian merging problem which aims to minimize the dissatisfaction degree of each individual source. The question addressed in this paper is how to merge prioritized assertional bases, in a possibility theory framework, when the uncertainty scales are not commensurable, namely when the sources do not share the same meaning of uncertainty scales. Using the notion of compatible scale, we provide a safe way to perform merging. The main result of the paper is that the egalitarian merging of prioritized assertional bases can be achieved in a polynomial time even if the uncertainty scales are not commensurable.

1 INTRODUCTION

In many applications, information pieces are provided by several and potentially conflicting sources where gathering them leads to inconsistent information. Information merging aims to define combination operations that take as input information provided by different sources and produce a consistent and unified point of view that syntheses the best of the sources. Knowledge bases merging or belief merging (e.g. (Bloch et al., 2001; Konieczny and Pino Pérez, 2002)), is a problem largely studied within the propositional logic setting. Several merging approaches have been proposed which depend on the nature and the representation of knowledge such as merging propositional knowledge bases and prioritized or weighted logical knowledge bases.

In this paper we place ourselves in the context of *Ontology-Based Data Integration* (Wache et al., 2001) and we investigate merging of uncertain assertional bases provided by different sources. In such a setting the ontology is assumed to be coherent and fully reliable. However the data, although refer to the same coherent ontology, are often provided by conflicting sources generally affected with uncertainty due for instance to the reliability of sources. Uncertainty here is represented in the framework of possibility theory. This theory is particularly appropriate when one uses a finite ordinal scale $\{0, \alpha_1, ..., \alpha_n, 1\}$ to asses the certainty degrees associated with each assertion. As ontology language, we use the well-known lightweight description logics *DL-Lite*. *DL-Lite* is recognized as powerful logical-based frameworks for *Ontology-Based Data Access*. In such a setting, we use a knowledge base formed of a terminological base, called TBox, and an assertional base, called ABox. The TBox contains ontological (or generic) knowledge of the application domain whereas the ABox stores data (or individuals or constants) that instantiate generic knowledge.

When priorities or uncertainty degrees are attached with assertional facts, there exist two main approaches (Konieczny and Pino Pérez, 2002) to merge or aggregate uncertain information: utilitariant (majority) approaches and egalitarian (or egalitarest) approaches. Within possibilistic *DL-Lite*, an egalitarian merging operator was proposed in (Benferhat et al., 2013) to merge DLs knowledge bases when uncertain pieces of information are represented in the possibility theory framework. However, the presented work in (Benferhat et al., 2013) is based on the assumption that the scales used to represent uncertainty in the merged knowledge bases are commensurable, namely the sources share the same meaning of uncertainty scales. In some applications and especially in the Web applications ones, the commensurability

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assumption may appear to be strong and needs to be dropped. In the propositional setting (Benferhat et al., 2007) a merging under incommensurability assumption has been proposed. However, this approach is very computationally hard (at least in Δ_p^2).

In this paper, we propose an efficient egalitarian-based merging of uncertain assertional bases under the incommensurability assumption. We assume that generic knowledge (the TBox) is fully coherent and fully certain and seen as a constraint to be satisfied during the merging process. Namely, the TBox should be present in the merging result (i.e. the resulting DL knowledge base), while assertional facts can be either accepted, ignored or weakened during the merging process. The assertional facts (ABox), issued from different sources, may be affected with uncertainty. Sources are not assumed to share the same meaning of uncertainty scales. To tackle the incommensurability problem, we use the concept of compatible scales. A compatible scale is a re-assignment of certainty degrees to assertional facts such that the initial plausibility ordering inside each ABox is preserved. Using the notion of compatible scale, we provide a safe way to perform merging. The nice result of this paper is that merging uncertain assertional bases, without the commensurability assumption, can be achieved in a polynomial time. The last part of the paper proposes a way to deal with incommensurability assumptions by normalizing uncertainty scales.

Before presenting our results, let us give a brief refresher on possibilistic lightweight ontologies and on merging under commensurability assumption.

2 PRIORITIZED LIGHTWEIGHT ONTOLOGIES

In this section we briefly introduce the logical-based formalism used to represent prioritized ontologies.

2.1 Lightweight Description Logics: DL-Lite

One of the well-known description logics for querying data is *DL-Lite* (Calvanese et al., 2007). This is due to the so-called first-order rewritability property that separates the TBox and the ABox when reasoning. Such property guarantees a very low computational complexity for query answering. This makes *DL-Lite* well-suitable for applications that use large volume of data. The following briefly reviews the core fragment of all the *DL-Lite* family, lightweight ontologies, called *DL-Lite$_{core}$*. However, results of this paper are valid for *DL-Lite$_R$* and *DL-Lite$_F$*, two important fragments of the *DL-Lite* family.

2.1.1 Syntax and Semantics

A standard *DL-Lite* knowledge base K_s=$\langle T_s, A_s\rangle$ is composed of a set of atomic concepts (*i.e.* unary predicates), a set of atomic roles (*i.e.* binary predicates) and a set of individuals (*i.e.* constants). Complex concepts and roles are built as follows:

$$B \longrightarrow A|\exists R \qquad R \longrightarrow P|P^{-} \qquad C \longrightarrow B|\neg B$$

where A (*resp.* P) is an atomic concept (*resp.* role). B (*resp.* C) is called basic (*resp.* complex) concept and role R is called basic role. The TBox T_s includes a finite set of inclusion assertions of the form $B \sqsubseteq C$ where B and C are concepts. The ABox A_s contains a finite set of assertions on atomic concepts and roles of the form $A(a)$ and $P(a,b)$ where a and b are two individuals.

The semantics of a *DL-Lite* knowledge base is given in term of first order logic interpretations. An interpretation $I = (\Delta^I, .^I)$ consists of a non-empty domain Δ^I and an interpretation function $.^I$ that maps each individual a to $a^I \in \Delta^I$, each A to $A^I \subseteq \Delta^I$ and each role P to $P^I \subseteq \Delta^I \times \Delta^I$. Furthermore, the interpretation function $.^I$ is extended in a straightforward way for complex concepts and roles: $(\neg B)^I = \Delta^I \setminus B^I$, $(P^-)^I = \{(y,x)|(x,y) \in P^I\}$ and $(\exists R)^I = \{x|\exists y$ s.t. $(x,y) \in R^I\}$. An interpretation I is said to be a model of a concept inclusion axiom, denoted by $I \models B \sqsubseteq C$, iff $B^I \subseteq C^I$. Similarly, we say that I satisfies a concept (*resp.* role) assertion, denoted by $I \models A(a)$ (*resp.* $I \models P(a,b)$), iff $a^I \in A^I$ (*resp.* $(a^I, b^I) \in P^I$). An interpretation I is said to be a model of $K = \langle T, A\rangle$, denoted by $I \models K_s$, iff $I \models T_s$ and $I \models A_s$ where $I \models T_s$ (*resp.* $I \models A_s$) means that I is a model of all axioms in T_s (*resp.* A_s). A knowledge base K_s is said to be consistent if it admits at least one model, otherwise K_s is said to be inconsistent. A *DL-Lite* TBox T is said to be incoherent if there exists at least a concept C such that for each interpretation I which is a model of T, we have C^I=$\emptyset$.

2.1.2 Query Answering

A query is a first-order logic formula, denoted q=$\{\vec{x}\,|\phi(\vec{x})\}$, where $\vec{x}$=$(x_1,...,x_n)$ are free variables, n is the arity of q and atoms of $\phi(\vec{x})$ are of the form $A(t_i)$ or $P(t_i,t_j)$ with $A \in N_C$ and $P \in N_R$ and t_i, t_j are terms, *i.e.* constants of N_I or variables. When $\phi(\vec{x})$ is of the form $\exists\vec{y}.conj(\vec{x},\vec{y})$ where $\vec{y}$ are bound variables called existentially quantified variables, and $conj(\vec{x},\vec{y})$ is a conjunction of atoms of the form $A(t_i)$ or $P(t_i,t_j)$ with $A \in N_C$ and $P \in N_R$ and t_i, t_j are terms, then q is said to be a conjunctive query (CQ). When n=0, then q is said to be a boolean query (BQ). A BQ with no bound variables is said to be a ground query (GQ). Lastly, when q contains only one atom with no free variables, then it is said to be an instance query (IQ) (*i.e.* instance checking). For a BQ q, we have $I \models q$ iff $(\phi)^I = true$ and $K \models q$ iff $\forall I{:}I \models K, I \models q$. For a CQ q with free variables $\vec{x}=(x_1,...,x_n)$, a tuple of constants $\vec{a}=(a_1,...,a_n)$ is said to be the certain answer for q over K if the BQ $q(\vec{a})$ obtained by replacing each variable x_i by a_i in $q(\vec{x})$, evaluates to true for every model of K. Hence CQ answering can be reduced to BQ answering.

2.2 Possibilistic DL-Lite

Let $\mathcal{L}$ be the *DL-Lite* description language described in the previous section (Section 2.1). A prioritized (or weighted) DLs knowledge base is made by a set of DL axioms where each axiom is attached with a weight that reflects its certainty/priority. In general, the higher is the weight of an axiom the more the axiom is important. Handling priorities can be conveniently and efficiently dealt in the possibility theory framework (Dubois and Prade, 1988). Recently, an extension of *DL-Lite* to the possibility theory framework has been proposed in (Benferhat and Bouraoui, 2015). In this paper we use this framework to encode available knowledge.

A possibilistic *DL-Lite* knowledge base $K=\{(\phi, w_\phi) : 1..n\}$, denoted by *DL-Lite*$^\pi$, is a set of weighted axioms of the form (ϕ, w_ϕ) where ϕ is either a *TBox* or an *ABox* axiom and $w_\phi \in]0,1]$ is the degree of certainty or priority of ϕ. The weighted axiom (ϕ, w_ϕ) means that the certainty degree of ϕ is at least equal to w_ϕ. When $\forall (\phi_i, w_{\phi_i}) \in K$, we have w_{ϕ_i}=1 then the classical *DL-Lite* knowledge base, as recalled in Section 2.1, is recovered. The inconsistency degree of a *DL-Lite*$^\pi$ knowledge base K, denoted by $Inc(K)$, is defined as follow:

$$Inc(K) = \max\{w_{\phi_i} : K_{\geq w_{\phi_i}} \text{is inconsistent}\}$$

Where $K_{\geq \alpha} = \{\phi_i : (\phi_i, w_{\phi_i}) \in K \text{ and } w_{\phi_i} \geq \alpha\}$ is composed of axioms having a weight greater than α. Besides, $Inc(K) = 0$ if $\{\phi_i : (\phi_i, w_{\phi_i}) \in K$ is consistent$\}$.

The semantics of *DL-Lite*$^\pi$ knowledge bases is given by the concept of a possibility distribution, denoted by π. This latter is a mapping from a set of *DL-Lite* interpretations Ω (namely, $I = (\Delta, .^I) \in \Omega$) to the unit interval $[0,1]$.

Definition 1. *The possibility distribution induced from a DL-Lite*$^\pi$ *is defined as follows:* $\forall I \in \Omega : \pi_K(I) =$

$$\begin{cases} 1 \ \ if \ \forall \ (\phi_i, w_{\phi_i}) \in K, I \models \phi_i \\ 1 - max\{w_{\phi_i} : (\phi_i, w_{\phi_i}) \in K \ \ I \not\models \phi_i\} \ \ otherwise \end{cases}$$

Interpretations which have possibility degrees equal to 1 are the most preferred ones since they are models of $\{\phi_i : (\phi_i, w_{\phi_i}) \in K\}$. For countermodels, an interpretation I is considered as preferred to an interpretation I', if the highest axiom falsified by I is less important than the highest axiom falsified by I'.

It can be shown that $Inc(K) = 1 - \underset{I \in \Omega}{max}\{\pi_K(I)\}$. For more details on possibilistic *DL-Lite*; see (Benferhat and Bouraoui, 2015). Finally, given K a possibilistic *DL-Lite* knowledge base, a conjunctive query q is said to be a consequence of K iff q follows from $\{\phi : (\phi, w_\phi) \in K, w_\phi > Inc(K)\}$ using standard *DL-Lite* reasoner.

Throughout this paper, we assume that the TBox is coherent and fully certain and only assertional facts (ABoxes) may be somewhat certain.

3 EGALITARIAN MERGING UNDER COMMENSURABILITY ASSUMPTION

This section briefly reviews egalitarian merging of possibilistic *DL-Lite* knowledge bases in the case where uncertainty scales used by the different sources are commensurable, namely when all sources share the same meaning of uncertainty scales.

Let $A = \{A_1, ..., A_n\}$ be a set of n prioritized ABoxes issued from n distinct sources, and let T be a common *DL-Lite* TBox representing the integrity constraint (ontology) to be satisfied during the merging process. We suppose that each ABox is consistent with T. Let $\pi_1, ..., \pi_n$ be the possibility distributions provided by the n sources of information, namely a π_i denotes the possibility distribution associated with each $K_i = \langle T, A_i \rangle$ a *DL-Lite* knowledge base.

Given n commensurable ABoxes, the merging process aims to compute a new *DL-Lite*$^\pi$ knowledge base, denoted by $\Delta_T(A)$, where T is the integrity constraint and A is an ABox representing the result of the fusion of these ABoxes. In the literature, different methods for merging have been proposed. In this section, we perform merging of A_1,...,A_n with respect to T using *min*-based merging operator. This operator is often seen as an example of egalitarian merging and is used when distinct sources that provide information are assumed to be dependent. We first introduce the notion of *profile* associated with an interpretation I, denoted by $\nu(I)$, and defined by

$$\nu(I) = \langle \pi_1(I), ..., \pi_n(I) \rangle.$$

Namely, $\nu(I)$ represents the possibility values of an interpretation I with respect to each source. From a semantics point of view, the result of merging is a possibility distribution π_Δ obtained in two steps:

i the possibility degrees $\pi_i(I)$'s are first combined with a merging operator (here we use the minimum operator), and

ii the interpretations having highest degrees are considered as models of the result of merging (*i.e.* the resulting *DL-Lite* knowledge base $\Delta_T(A)$).

This leads to define a total pre-order relation, denoted by $\leq_{min}$, between interpretations as follows: an interpretation I is preferred to another interpretation I' if the minimum element of the profile of I is higher than the minimum element of the profile of I'. Formally:

Definition 2 (Definition of $\leq_{Min}$). *Let $A = \{A_1, ..., A_n\}$ be a set of ABoxes and T be an ontology. Let $\{\pi_1, ..., \pi_n\}$ be the possibility distributions associated with $\{K_1 = \langle T, A_1\rangle, ..., K_n = \langle T, A_n\rangle\}$. Let I and I' be two interpretations and $\nu(I)$ and $\nu(I')$ be their associated profiles. Then:*

$$I' \leq^A_{min} I \iff Min(\nu(I)) > Min(\nu(I'))$$

where

$$Min(\nu(I)) = Min\{\pi_i(I) : i \in \{1, ..., n\}\}.$$

The result of the merging $\Delta^{min}_T(A)$ is a *DL-Lite*$^\pi$ knowledge base whose models are interpretations which are models of T and which are maximal with respect to $\leq_{Min}$. More formally:

Definition 3 (Min-Based Merging Operator). *Let $A = \{A_1, ..., A_n\}$ be a set of ABoxes and T be an ontology. Let $\{\pi_1, ..., \pi_n\}$ be the possibility distributions associated with $\{K_1 = \langle T, A_1\rangle, ..., K_n = \langle T, A_n\rangle\}$. The result of merging is a DL-Lite$^\pi$ knowledge base, denoted by $\Delta^{min}_T(A)$ is such that its models are defined by:*

$$Mod(\Delta^{min}_T(A)) = \{I \in Mod(T) : \nexists I' \in Mod(T), I \leq^A_{Min} I'\}$$

Example 1. *Let $T = \{A \sqsubseteq B,\ B \sqsubseteq \neg C\}$ be a TBox, where the certainty degree of each axioms is set to 1 (the weights 1 in axioms pf T are omitted for sake of simplicity). Let us consider the following set of ABox to be linked to T: A_1=$\{(A(a), .6),\ (C(b), .5)\}$, A_2=$\{(C(a), .4),\ (B(b), .8),\ (A(b), .7)\}$. Table 1 considers an example of four interpretations. It gives the possibility degrees associated with $\langle T, A_1, A_2\rangle$ using Definition 1 and the result of combining these four interpretations with the minimum operator.*

Table 1: Example of merging of possibility distributions using min-based operator.

I	$.^I$	π_{A_1}	π_{A_2}	$\pi_\Delta(A)$
I_1	A={a},B={a},C={b}	1	.2	.2
I_2	A={},B={},C={a,b}	.4	.2	.2
I_3	A={a,b},B={a,b},C={}	.5	.6	**.5**
I_4	A={b},B={b},C={a}	.4	1	.4

From a syntactic point of view, the *min*-based merging operator, denoted by $\Delta^{min}_T(A)$ is simply the union of all ABox that are above the inconsistency degree. More formally:

Definition 4. *Let $A = \{A_1, ..., A_n\}$ be a set of ABoxes and T be an ontology. Then:*

$$\Delta^{min}_T(A) = \{\phi_{ij} : (\phi_{ij}, w_{\phi_{ij}}) \in \langle T, A_1 \cup ... \cup A_n\rangle \text{ and } w_{\phi_{ij}} > Inc(\langle T, A_1 \cup ... \cup A_n\rangle)\}$$

where $Inc(\langle T, A_1 \cup ... \cup A_n\rangle)$ is defined in Section 2.2

Proposition 1. *Let $A = \{A_1, ..., A_n\}$ be a set of ABoxes and T be an ontology. Let $\pi_\Delta(A)$ be the possibility distribution associated. Then $\Delta^{min}_T(A)$ represents the result of merging.*

The following definition introduces query answering using min-based merging operator under commensurability assumption.

Definition 5. *A query q is said to be a egalitarian consequence relation of $\langle T, A_1, ..., A_n\rangle$ iff q follows from $\Delta^{min}_T(A)$ using standard DL-Lite (see Section 2.1).*

Example 2 (continued). *At syntactic level, we have $\Delta^{min}_T(A)$=$\langle T, \{(A(a), .6), (C(b), .5), (C(a), .4), (B(b), .8),\ (A(b), .7)\}\rangle$. We have $Inc(\Delta^{min}_T(A))$=.5 and $\Delta^{min}_T(K) = T, \{(A(a), .6), (B(b), .8), (A(b), .7)\}$.*

4 EGALITARIAN MERGING UNDER INCOMMENSURABILITY ASSUMPTION

The min-based merging operator presented in the previous section is based on the assumption that all the sources providing the ABoxes use the same scale to encode uncertainties between facts. In Example 1, when dealing with assertions, we assumed that the weights attached to a fact $\phi \in A_i$ can be compared with the weight associated with $\varphi \in A_j$ with $j \neq i$. In this section, we analyse the situation where the sources are incommensurable, namely the weights used between ABoxes assertions are not commensurable.

A natural way to tackle the incommensurability assumption is to use the notion of *"compatible scales"* on existing scales used by each source. An uncertainty scale is said to be compatible with all sources if it preserves the original total pre-orders between assertions inside each ABox.

The concept of compatible scale is very natural and has been used in different settings. Intuitively, when one has to deal with an imprecise or an unknown variables, then using compatible scales consists in considering all possible values of the variables. This is the case with interval-based probability (Augustin et al., 2003; Wallner, 2007) or possibility distributions (Benferhat et al., 2011), where for each event instead of specifying a single probability degree, one specifies an interval. A compatible scale in this case is a value of the interval. In our framework, assume that one has two sources $A_1 = \{(\phi, w_\phi), (\varphi, w_\varphi)\}$ and $A_2 = \{(\psi, w_\psi)\}$ each of them provides some uncertain facts. If the sources are incommensurable (do not share the same meaning of the scale), then a compatible scale, denoted by $\mathcal{R}$, is any re-assignement of uncertainty degrees of ϕ, φ and ψ such that $\mathcal{R}(\phi) > \mathcal{R}(\varphi)$ (if $w_\phi > w_\varphi$). Namely, the only requirement is that we preserve the relative plausibility-ordering inside each assertional base A_i.

Definition 6 (Compatible Scales)**.** *Let $A = \{A_1, ..., A_n\}$ be a set of ABoxes where $A_i = \{(\phi_j, w_{\phi_j})\}$. Then a compatible uncertainty scale $\mathcal{R}$ is defined by:*

$$\mathcal{R}: \quad A_1 \cup ... \cup A_n \rightarrow]0,1] \qquad (\phi_j, w_{\phi_j}) \mapsto r_{\phi_j}$$

An uncertainty scale $\mathcal{R}$ is said to be compatible with A iff: $\forall A_i \in A$, $\forall (\phi, w_\phi) \in A_i$, $(\varphi, w_\varphi) \in A_i$, $w_\phi \leq w_\varphi$ iff $r_\phi \leq r_\varphi$.

Example 3 (Example Continued)**.** *Let us consider again the following set of ABox to be linked to T given in Example 1: A_1=$\{(A(a),.6),(C(b),.5)\}$, $A_2 = \{(C(a),.4),(B(b),.8),(A(b),.7)\}$. The following table gives three examples of uncertainty scales.*

Table 2: Examples of uncertainty scales.

	ϕ_j	w_{ϕ_j}	$\mathcal{R}^1_{\phi_j}$	$\mathcal{R}^2_{\phi_j}$	$\mathcal{R}^3\phi_j$
A_1	$A(a)$	.6	.5	.4	.6
	$C(b)$	.5	.2	.7	.5
A_2	$C(a)$	.4	.3	.3	.4
	$B(b)$	.8	.7	.6	.8
	$A(b)$	.7	.4	.2	.7

The scaling $\mathcal{R}^1$ is a compatible one because it preserves the total pre-order inside each ABox. However, the scaling $\mathcal{R}^2$ is not a compatible one since it inverses priorities inside A_1 and A_2. $\mathcal{R}^3$ is the compatible scale where the same uncertainty degrees are used.

Given a compatible scales $\mathcal{R}$, we denote by $A_i^{\mathcal{R}}$ the assertional base obtained from A_i by replacing each assertion (ϕ_j, w_{ϕ_j}) by (ϕ_j, r_{ϕ_j}). Similarly, we denote by $A^{\mathcal{R}}$ the set obtained from A by replacing each A_i in A by $A_i^{\mathcal{R}}$. According to Example 3, it is clear that the set of compatible uncertainty scales is not unique. Let us denote by $\mathcal{R}(A)$ the set of compatible uncertainty scales associated with $A = \{A_1, ..., A_n\}$. Note that the concept of compatible scale has been used in the propositional logic setting. However, the computational complexity of reasoning process is very hard (at least Δ^2_p), even for simple knowledge bases like Horn clauses.

Now, given the set of all compatible scales $\mathcal{R}(A)$, different possibilities may exist in order to merge the ABoxes. For instance, one can only select one scale to perform merging (a credulous merging) or one can consider all the compatible ranking in $\mathcal{R}(A)$ to define result of merging (skeptical merging). We first consider the case where all compatible uncertainty scales are used to perform merging. When considering the set of all compatible scales, an interpretation I is said to be more plausible than I', if for each compatible scale $\mathcal{R} \in \mathcal{R}(A)$, I is considered more plausible than I' using Definition 2 . More precisely,

Definition 7. *Let $A = \{A_1, ..., A_n\}$ be a set of DL-Lite$^\pi$ ABoxes and $\mathcal{R}(A)$ be the set of all compatible scalings associated with A. Let I and I' be two interpretations. Then:*

$$I' <^A_\forall I \text{ iff } \forall \mathcal{R} \in \mathcal{R}(A),\ I' \leq^{A^{\mathcal{R}}}_{min} I$$

where $\leq^{A^{\mathcal{R}}}_{min}$ is the result of applying Definition 2 on $A^{\mathcal{R}}$.

Definition 7 is illustrated by Figure 1 where m represents the size of $\mathcal{R}(A)$ (which may be infinite).

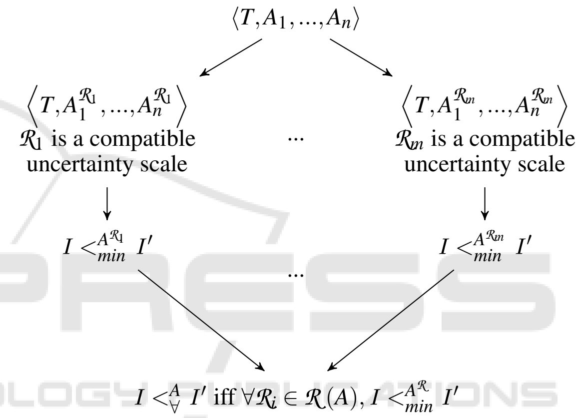

Figure 1: Merging process using the notion of compatible scale.

According to Definition 7, models of the result of merging the ABoxes (using compatible scales), $\Delta^\forall_T(A)$ are those which are models of T and minimal for $<^A_\forall$:

$$Mod(\Delta^\forall_T(A)) = \{I \in Mod(T) : \nexists I' \in Mod(T),\ I' <^A_\forall I\}.$$

The following example illustrates the fusion process based on all compatible uncertainty scales.

Example 4 (continued)**.** *Let us consider again the following set of ABoxes given in Example 1: A_1=$\{(A(a),.6), (C(b),.5)\}$, A_2=$\{(C(a),.4), (B(b),.8), (A(b),.7)\}$. Let us consider the scaling $\mathcal{R}_1$ defined: $A_1^{\mathcal{R}_1} = \{(A(a),.8),(C(b),.4))\}$ and $A_2^{\mathcal{R}_1} = \{(C(a),.2),(B(b),.9),(A(b),.6)\}$. And the scaling $\mathcal{R}_2$ defined: $A_1^{\mathcal{R}_2} = \{(A(a),.4),(C(b),.2))\}$ and $A_2^{\mathcal{R}_2} = \{(C(a),.3),(B(b),.6),(A(b),.5)\}$. Both of them are compatible scale since they preserve for each assertional base certainty degrees of assertions. Table 3 gives an example of four interpretations I_1-I_4 and presents their profile for each uncertainty scale.*

Table 3: Merging under two compatible scales.

I	$\nu_{A^{\mathcal{R}_1}}(I)$	Min	$\nu_{A^{\mathcal{R}_2}}(I)$	Min
I_1	$<1,.1>$	.1	$<1,.4>$	.4
I_2	$<.2,.1>$	.1	$<.6,.4>$	.4
I_3	$<.6,.8>$	**.6**	$<.8,.7>$	**.7**
I_4	$<.2,1>$	.2	$<.6,1>$	.6

Note that in both uncertainty scales $\mathcal{R}_1$ and $\mathcal{R}_2$, I_3 is the preferred one. In fact, whatever is the considered compatible scale, it will be the preferred one. Hence, it can be shown that if we consider all the compatible scales, I_3 will represent the result of merging under the incommensurability assumption.

Once preferred models are computed, query answering from a set of uncertain ABoxes under incommensurability assumption, is given as follows:

Definition 8. *Let $A = \{A_1, ..., A_n\}$ be a set of ABoxes and T be an ontology. A query $q(\vec{x})$ is said to be consequence of A under incommensurability assumption if $\forall I, I \in Mod(\Delta_T^{min}(A^{\mathcal{R}})), I \models q(\vec{x})$.*

Said differently, a query follows from $\langle T, A_1, ..., A_n\rangle$ under incommensurability assumption if and only if for each compatible uncertainty scale $\mathcal{R}$, $q(\vec{x})$ follows from $\left\langle T, A_1^{\mathcal{R}}, ..., A_n^{\mathcal{R}}\right\rangle$ under a commensurability assumption.

Example 5 (Example Continued)**.** *From Example 4, we have $Mod(\Delta_T^{min}(A^{\mathcal{R}}))=\{I_3\}$ where $A^{I_3} = \{a, b\}$, $B^{I_3} = \{a, b\}$ and $C^{I_3} = \{\}$. Let $q_1(x) \leftarrow A(x) \wedge B(x)$ be a conjunctive query. One can easily check that $< b >$ is an answer of $q_1(x)$ using $\Delta_T^{min}(A^{\mathcal{R}})$. Similarly, let $B(a)$ be an instance query, one can check that $B(a)$ follows from $\Delta_T^{min}(A^{\mathcal{R}})$.*

5 A POLYNOMIAL ALGORITHM FOR QUERY ANSWER UNDER INCOMMENSURABILITY ASSUMPTION

In the previous section, we have seen that a query follows from $\langle T, A_1, ..., A_n\rangle$ under incommensurability assumption if and only if for each compatible uncertainty scale $\mathcal{R}$, q follows from $\left\langle T, A_1^{\mathcal{R}}, ..., A_n^{\mathcal{R}}\right\rangle$ under a commensurability assumption.

The problem is that the number of compatible uncertainty scales may be infinite. As we will show in this section, there is no need to explicitly state all these compatible uncertainty scales. In fact, query answering under the incommensurability assumption can be achieved in a polynomial time. For the sake of simplicity, we will illustrate our approach for the instance checking problem. However, result of this section can be generalized to general conjunctive query. More precisely, we provide an algorithm that implements Definition 8 when q is of the form $A(a)$ or $P(a, b)$ where A is a concept, P is a role and a, b are individuals. In the following, we simply write $X(z)$ instead of $A(a)$ or $P(a, b)$ to denote an instance fact.

We first recall in standard *DL-Lite* that in order to check whether an instance query of the form $X(z)$ is inferred from a standard (one consistent source) *DL-Lite* knowledge base (*i.e.* $K \models X(z)$), we first add to K the assumption that $X(z)$ is false. This is encoded by the following statements: $\{Y \sqsubseteq \neg X, Y(z)\}$ where Y is a new concept not appearing in K. Then we check if the augmented knowledge base is consistent or not. If it is inconsistent then $X(z)$ holds from K. Otherwise $X(z)$ does not follow from K.

The aim of this section is to adapt this reasoning process when the assertional bases come form several incommensurable sources. Recall that we are interested in checking if $X(z)$ holds from $K = \{T, A_1, ..., A_n\}$. A preliminary step of the algorithm consists in computing:

- the set of conflict $C(K)$ and
- the set of conflicts of the augmented KB $K' = \langle T \cup \{Y \sqsubseteq \neg X\}, A_1 \cup \ldots \cup A_n \cup \{Y(z)\}\rangle$.

Let us denote by

$$\mathcal{F}_{X(z)} = \{\phi \in A_1 \cup \ldots \cup A_n : (\phi, Y(z)) \in C(K')\}$$

the set of all assertional facts from $A_1 \cup ... \cup A_n$ that directly contradict the assumption that $X(z)$ is false.

To check if $X(z)$ is a consequence of K, we first need to see if there exists a new conflict $(\phi, Y(z))$ in $C(K')$. Namely, we need to see whether there exists a fact that contradicts the assumption that $X(z)$ is false. If such a conflict does not exist then $X(z)$ is not a consequence from K. More formally,

Proposition 2. *Let $K = \langle T, A_1, ..., A_n\rangle$ be a DL-Lite knowledge base issued from different sources. Let $K' = \langle T \cup \{Y \sqsubseteq \neg X\}, A_1 \cup ... \cup A_n \cup \{Y(z)\}\rangle$ be the augmented knowledge base by the assumption that $X(z)$ is false. If $C(K)$=$C(K')$ (namely $\mathcal{F}_{X(z)} = \emptyset$) then $X(z)$ is not a consequence using Definition 8 of K.*

Proof. The proof is immediate. Assume that $\mathcal{F}_{X(z)} = \emptyset$ or similarly $C(K) = C(K')$. Indeed if $X(z)$ is a consequence of K using Definition 8, this means that there exists a compatible uncertainty scale $\mathcal{R}$ such that $X(z)$ is a consequence of $K^{\mathcal{R}} = \left\langle T, A_1^{\mathcal{R}}, ..., A_n^{\mathcal{R}}\right\rangle$ using the incommensurability assumption. Hence, there exists a consistent subset $K_1 = \langle T, B\rangle$, of $K^{\mathcal{R}}$ with $B \subseteq \{\phi : (\phi, w_\phi) \in A_1^{\mathcal{R}} \cup ... \cup A_n^{\mathcal{R}})\}$ s.t $X(z)$ follows from K in a standard way. This means that $K' = \langle T \cup \{Y \sqsubseteq \neg X\}, B \cup \{Y(z)\}\rangle$ is inconsistent. This means that there exists a conflict that involves

$Y(z)$ (a new conflict), but this contradicts the fact that $C(K') = C(K)$. □

The following lemma simply states that if two elements are conflicting then they necessarily belong to two different assertional bases.

Lemma 1. *Let* $(\alpha,\beta) \in C(K)$. *Then* α *and* β *belongs to two different assertional bases, namely* $\exists i, j$ *such that* $\alpha \in A_i, \beta \in A_j$ *and* $i \neq j$.

The proof of this lemma follows from the fact that it is assumed that for each assertional base A_i, $\langle T, A_i\rangle$ is consistent. We now analyze the general case where $C(K) \neq C(K')$. Namely, there exists a new conflict arising by adding the assumption that $X(z)$ is false. For sake of simplicity, if $\mathcal{R}$ is a compatible scale and if ϕ and φ are two assertions then we simply write $\phi^{\mathcal{R}} > \varphi^{\mathcal{R}}$ (*resp.* $\phi^{\mathcal{R}} = \varphi^{\mathcal{R}}$, $\phi^{\mathcal{R}} < \varphi^{\mathcal{R}}$) to denote that the uncertainty degree of ϕ is more (*resp.* equal, less) plausible than the certainty of φ using compatible scale $\mathcal{R}$. Recall that when $X(z)$ is a consequence of K then, by Definition 8, for all compatible scales and for all conflicts $(\alpha,\beta) \in C(K)$, one should have: $\phi^{\mathcal{R}} > min(\alpha^{\mathcal{R}}, \beta^{\mathcal{R}})$, said differently $\phi^{\mathcal{R}} > \alpha^{\mathcal{R}} or \phi^{\mathcal{R}} > \beta^{\mathcal{R}}$.

The following proposition gives, in case where $C(K') \neq C(K)$, the two cases needed to check whether $X(z)$ is a consequence of K.

Proposition 3. *Let* $(\alpha,\beta) \in C(K)$ *and* $(\phi, Y(z))$ *be a new conflict. Let* i, j *be two integers such that* $\alpha \in A_i$ *and* $\beta \in A_j$ *(with* $i \neq j$*). Then*

1. *if* $\phi \notin A_i$ *and* $\phi \notin A_j$. *Then* $X(z)$ *is not a consequence of* $\langle T, A_1, ..., A_n\rangle$.
2. *if* $\phi \in A_i$ *or* $\phi \in A_j$ *then:*
   - *if* $\phi \in A_i$ *and* $\phi \leq \alpha$ *(resp.* $\phi \in A_j$ *and* $\phi \leq \beta$*), then X(z) is not a consequence of* $\langle T, A_1, ..., A_n\rangle$.
   - *if* $\phi \in A_i$ *and* $\phi > \alpha$ *(resp.* $\phi \in A_j$ *and* $\phi > \beta$*), then* $X(z)$ *is a consequence of* $\langle T, A_1, ..., A_n\rangle$.

On the basis of Propositions 2 and 3, we are now ready to provide a polynomial algorithm (Algorithm 1) that implements Definition 8.

Algorithm 1 can be improved where rather to consider $C(K)$ one can only use $Reduce(C(K))$ defined by $Reduce(C(K)) = \{(\alpha,\beta) : (\alpha,\beta) \in C(K), \nexists(\alpha',\beta')$ such that $\alpha' > \alpha$ and $\beta' > \beta$ with $\alpha, \alpha' \in A_i, \beta, \beta' \in A_j\}$. Clearly the computational complexity of Algorithm 1 is polynomial. Indeed, Steps 1 and 4 are polynomial since computing the set of conflicts in *DL-Lite* (*core*, *F* and *R*) is polynomial (Calvanese et al., 2007). Steps 2 and 3 are trivially polynomial, as well as steps 5-7, 9-20. Step 4 is a loop with a maximal $|A_1 \cup ... \cup A_n|$ iterations. Step 8 is another loop with a maximal of n^3 where n is the maximal size of an ABox. Hence, the whole algorithm is polynomial.

Algorithm 1: Query answering for egalitarian incommensurable merging.

Input: $K = \langle T, A_1, \ldots, A_n\rangle$, $X(z)$.
Output: Yes (1) if X(z) is a consequence, no (0) otherwise.
1: $C \leftarrow$ conflict of K
2: $K' \leftarrow \langle T \cup \{Y \sqsubseteq \neg X\}, A_1 \cup \ldots \cup A_n \cup \{Y(z)\}\rangle$.
3: $\mathcal{F}_{X(z)} \leftarrow \{\alpha : (\alpha, Y(z) \in$ Conflict of $K'\}$.
4: **while** $\mathcal{F}_{X(z)} \neq \emptyset$ **do**
5: $\quad bool \leftarrow (1)$
6: $\quad \phi \leftarrow \phi \in \mathcal{F}_{X(z)}$
7: $\quad \mathcal{F}_{X(z)} \leftarrow \mathcal{F}_{X(z)} \setminus \{\phi\}$
8: $\quad$ **for all** $(\alpha,\beta) \in C$ **do**
9: $\quad\quad i \leftarrow$ the assertional base that contains α
10: $\quad\quad j \leftarrow$ the assertional base that contains β
11: $\quad\quad k \leftarrow$ the assertional base that contains ϕ
12: $\quad\quad$ **if** $k \neq i$ and $k \neq j$ **then**
13: $\quad\quad\quad bool \leftarrow 0$
14: $\quad\quad$ **else**
15: $\quad\quad\quad$ **if** $k = i$ and ϕ is less certain than α in A_i **then**
16: $\quad\quad\quad\quad bool \leftarrow 0$
17: $\quad\quad\quad$ **else**
18: $\quad\quad\quad\quad$ **if** $k{=}j$ and ϕ is less certain than β is A_j **then**
19: $\quad\quad\quad\quad\quad bool \leftarrow 0$
20: $\quad$ **if** $bool == 1$ **then return** 1
21: **return** 0

Example 6. *Continue with Example 4 and consider again* $B(a)$ *as instance query. We have* $C(K) = \{\{(A(a), .6), (C(a), .4)\}, \{(C(b), .5), (B(b), .8)\}, \{(C(b), .5), (A(b), .7)\}\}$ *and* $reduce(C) = \{\{(A(a), .6), (C(a), .4)\}, \{(C(b), .5), (B(b), .8)\}\}$. *After adding the assumption that* $B(a)$ *is false, we have* $\mathcal{F}_{B(a)} = \{(A(a), .6)\}$. *By taking* $\phi \leftarrow (A(a), .6)$ *and* $\{(A(a), .6), (C(a), .4)\} \in C'$, *it is easily to check that* $bool \leftarrow 1$. *Similarly, by considering* $\{(C(b), .5), (B(b), .8)\} \in C'$. *One can verify that* $bool \leftarrow 1$. *Hence* $K \models B(a)$ *under egalitarian incommensurable merging.*

6 SELECTING ONE NORMALIZED COMPATIBLE SCALE

Using the set of all compatible scales may lead to a very cautious merging operation. One way to get rid of incommensurability assumption is to use some normalization function in the spirit of the ones used in clustering methods for gathering attributes having incommensurable domaines. Let A_i be an ABox and

$\mathcal{W}(A_i)$ be the set of different certainty degrees used in A_i. Let $Min(\mathcal{W}(A_i))$ and $Max(\mathcal{W}(A_i))$ be respectively the minimum and maximum certainty degrees associated with assertional facts in $\mathcal{W}(A_i)$. Then an example of normalization function is $\forall \phi_j \in A_i$:

$$N(w_{\phi_j}) = \frac{w_{\phi_j} - (Min(\mathcal{W}(A_i) - \varepsilon)}{Max(\mathcal{W}(A_i)) - (Min(\mathcal{W}(A_i)) - \varepsilon)} \quad (1)$$

Where w_{ϕ_j} is a certainty degree belonging to $\mathcal{W}(A_i)$ and ε is a very small number (lower than $Min(\mathcal{W}(A_i))$. ε is added to avoid to have null degrees in possibilistic *DL-Lite* knowledge base. The main advantage of only having one normalization function is that one can have an immediate syntactic counterpart. More precisely, it is enough to replace for each fact (ϕ_j, w_{ϕ_j}) by $(\phi_j, N(w_{\phi_j}))$ where $N(w_{\phi_j})$ is the normalization function given by Equation 1.

Example 7 (Example Continued)**.** *From Example 1, we have* $A_1=\{(A(a),.6),\ (C(b),.5)\}$*,* $A_2=\{(C(a),.4),\ (B(b),.8),\ (A(b),.7)\}$*. We have* $Min(A_1) = .5$*,* $Min(A_2) = .4$*,* $MaX(A_1) = .6$ *and* $Max(A_2) = .8$*. Let* $\varepsilon = .01$*, then applying Equation 1 on* A_1 *and* A_2*, gives:* $A_1=\{(A(a),1),\ (C(b),.09)\}$*, and* $A_2=\{(C(a),0,02),\ (B(b),1),\ (A(b),.75)\}$*.*

Once the syntactic computation of normalized assertional bases is done, it is enough the reuse merging of commensurable possibilistic knowledge bases for query answering recalled in Section 2.2.

Example 8 (Example Continued)**.** *From Example 7, we have* $\Delta_T^{min}(A) = \langle T, \{(A(a),1),(C(b),.09),(C(a),.02),(B(b),1),(A(b),.75)\}\rangle$*. We have* $Inc(\Delta_T^{min}(A))$*=.09 and* $\Delta_T^{min}(K)$*=*$T,\{(A(a),1),(B(b),1),(A(b),.75)\}$*. Consider now* $q_1(x) \leftarrow A(x) \wedge B(x)$ *and* $q_2 \leftarrow B(a)$*, queries given in Example 5. One can check that* $< b >$ *is an answer of* $q_1(x)$ *from the and* $B(a)$ *holds from the resulting knowledge bases.*

7 CONCLUSIONS

This paper dealt with the problem of merging possibilistic *DL-Lite* assertional bases under the incommensurability assumption. The main result of the paper is that query answering is achieved in a polynomial time. This is a nice feature comparing for instance with merging merging within propositional setting where the problem is intractable even for simple knowledge bases such as horn clauses.

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