Moving Bragg Grating Solitons in a Grating-assisted Coupler with Cubic-Quintic Nonlinearity

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Abstract: We analyze the existence of moving Bragg grating solitons in a semilinear coupled system where one core is equipped with a Bragg grating and has cubic-quintic nonlinearity and the other is linear. The system's linear spectrum contains three bandgaps, namely the upper, lower and central gaps. The bandgap edges shift with the soliton velocity (s) and group velocity mismatch term (c) for a given coupling coefficient (κ), and result in change in the spectral widths. Two families of moving Bragg grating solitons (referred to as Type 1 and Type 2) are found that fill the upper and lower gaps only. No moving solitons are found in the central gap. The border separating the two families depends on both c and s, and is determined numerically. We carried out systematic numerical stability analysis of the moving solitons and identified non-trivial stability borders in their parametric plane. The analysis also reveals that vast areas of stable Type 1 solitons exist in the system's parametric plane and that all Type 2 solitons are unstable.

1 INTRODUCTION

It is well known that the coupling between forward- and backward-propagating waves gives rise to a strong dispersion in fiber Bragg gratings (FBGs) that can be up to six orders of magnitude greater than that of the silica fiber (Desterke and Sipe, 1994). At sufficiently high intensities, the grating induced dispersion can be counterbalanced by nonlinearity resulting in the formation of Bragg grating (BG) solitons. One of the main features of these solitons is that their velocity can range from zero to the velocity of light in the medium (Aceves and Wabnitz, 1989; Christadoulides and Joseph, 1989; Neill and Atai, 2007; Mak et al., 2003). As a result, such solitons are considered as potential candidates for optical buffers, delay lines, optical memory elements and logic gates. Thus far, BG solitons with a velocity of $0.16c_0$ (where c_0 is the speed of light in the vacuum) have been demonstrated experimentally (Mok et al., 2006).

Owing to their potential applications, the existence and dynamics of BG solitons have been investigated extensively in a variety of settings and structures including quadratic nonlinearity (Mak et al., 1998b; Conti et al., 1997; He and Drummond, 1997), cubic-quintic nonlinearity (Atai and Malomed, 2001), Bragg gratings in sign-changing Kerr media (Atai and Malomed, 2002), semilinear dual-core system with Kerr nonlinearity (Atai and Malomed, 2000), coupled FBGs (Mak et al., 1998a; Mak et al., 2004; Tsofe and Malomed, 2007; Islam and Atai, 2015), grating superstructures (Mayteevarunyoo and Malomed, 2008), waveguide arrays (Dong et al., 2011), and photonic crystals (Skryabin, 2004; Atai et al., 2006).

Optical fiber couplers have received much attention over the past three decades due to their potential applications in signal processing and switching (Fraga et al., 2006; Kaup and Malomed, 1998; Jensen, 1982). In particular, couplers made of dissimilar cores (e.g. one nonlinear core coupled with a linear core) have shown to possess interesting switching characteristics (Atai and Chen, 1992; Atai, 1993). Therefore, one may anticipate that combination of such couplers and Bragg gratings lead to optical devices with novel switching capabilities. In this paper, we investigate the existence and stability of moving BG solitons in a grating-assisted coupler where one core has cubic-quintic nonlinearity and is equipped with a Bragg grating and the other core is linear.

2 THE MODEL

We consider a semilinear dual-core system, where one core is equipped with a Bragg grating and has cubic-quintic nonlinearity and the other core is linear. Start-

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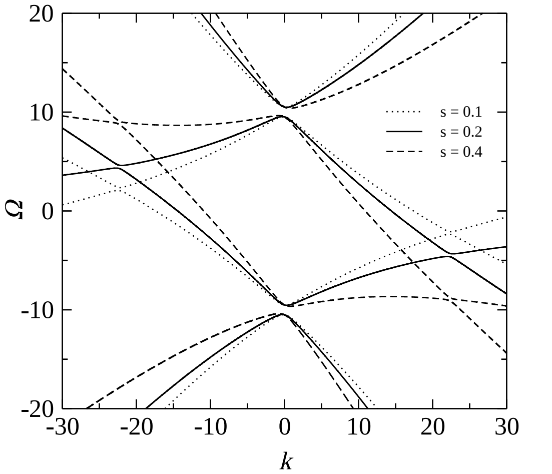

Figure 1: Dispersion curves in the moving frames for $\kappa = 10.0$, $c = 0.2$ with different values of soliton velocity s.

ing with the model of (Atai and Malomed, 2000), one can derive the following model that describes the propagation of light in such a system:

$$
\begin{aligned}
& iu_t + iu_x + \left[|v|^2 + \frac{1}{2}|u|^2\right] u - \\
& q\left[\frac{1}{4}|u|^4 + \frac{3}{2}|u|^2|v|^2 + \frac{3}{4}|v|^4\right] u + v + \kappa\phi = 0, \\
& iv_t - iv_x + \left[|u|^2 + \frac{1}{2}|v|^2\right] v - \\
& q\left[\frac{1}{4}|v|^4 + \frac{3}{2}|v|^2|u|^2 + \frac{3}{4}|u|^4\right] v + u + \kappa\psi = 0, \\
& i\phi_t + ic\phi_x + \kappa u = 0, \\
& i\psi_t - ic\psi_x + \kappa v = 0,
\end{aligned} \tag{1}
$$

where u and v are the forward- and backward-propagating waves in the nonlinear core, and ϕ and ψ are their counterparts in the linear core, respectively. $q > 0$ controls the strength of the quintic nonlinearity and κ denotes the coupling coefficient between the cores. Also, c represents the relative group velocity in the linear core (group velocity in the nonlinear core has been set to 1).

To determine the linear bandgaps within which moving solitons may exist, Eqs. (1) must be transformed into the moving coordinates, $\{X,T\} = \{x - st, t\}$, where s is the soliton velocity normalized such a way that $s = 1$ denotes the speed of light in the medium. Using this transformation, one can obtain the following system of equations:

$$
\begin{aligned}
& iu_T + i(1-s)u_X + \left[|v|^2 + \frac{1}{2}|u|^2\right] u - \\
& q\left[\frac{1}{4}|u|^4 + \frac{3}{2}|u|^2|v|^2 + \frac{3}{4}|v|^4\right] u + v + \kappa\phi = 0, \\
& iv_T - i(1+s)v_X + \left[|u|^2 + \frac{1}{2}|v|^2\right] v - \\
& q\left[\frac{1}{4}|v|^4 + \frac{3}{2}|v|^2|u|^2 + \frac{3}{4}|u|^4\right] v + u + \kappa\psi = 0, \\
& i\phi_T + i(c-s)\phi_X + \kappa u = 0, \\
& i\psi_T - i(c+s)\psi_X + \kappa v = 0.
\end{aligned} \tag{2}
$$

To determine the linear bandgap, $u, v, \phi, \psi \sim e^{ikX - i\Omega T}$ is substituted into Eqs. (2), which results in the following dispersion relation:

$$
\begin{aligned}
& \Omega^4 + 4ks\Omega^3 - \left[1 + 2\kappa^2 + \left(1 + c^2 - 6s^2\right)k^2\right]\Omega^2 - 2ks \\
& \left[1 + 2\kappa^2 + \left(1 + c^2 - 2s\right)k^2\right]\Omega + \left(c^2 - s^2 - 2c\kappa^2 - \right. \\
& \left. 2\kappa^2 s^2\right)k^2 + \kappa^4 + \left[c^2 - \left(1 + c^2\right)s^2 + s^4\right]k^4 = 0,
\end{aligned} \tag{3}
$$

where k represents the wave number, and Ω is the frequency in the moving frame and is related to the frequency in the lab frame by $\Omega(k) = \omega(k) - sk$. Figure 1 displays the dispersion diagrams corresponding to Eq. (3) for $\kappa = 10.0$, $c = 0.20$ with different soliton velocities. The spectrum analysis reveals that similar to the quiescent BG solitons with $c \neq 0$, moving spectrum contains three bandgaps: the upper, lower and central gaps. However, none of these three bandgaps are genuine as they overlap with one branch of continuous spectrum. The bandgap edges strongly depend on both c and s for a given κ. It is also found that increasing c results in the enlargement of both the upper and lower gaps. On the other hand, higher soliton velocities shrink the bandgap widths. Interestingly, unlike the coupled BG system with Kerr nonlinearity (Mak et al., 1998a), both the upper and lower gaps disappear above a critical soliton velocity (s_{cr}).

In order to determine the moving BG soliton solutions, we use the ansatz $\{u(X,T), v(X,T), \phi(X,T), \psi(X,T)\} = \{U(X), V(X), \Phi(X), \Psi(X)\}e^{-i\Omega T}$. Substitution of this expression into Eqs. (2) leads to a system of ordinary differential equations that is solved numerically for different system parameters. It is found that BG soliton solutions do not exist in the central gap. On the other hand, in the upper and lower gaps, similar to the case of single-core (Atai and Malomed, 2001) Bragg grating with cubic-quintic nonlinearity, two disjoint families of solitons (i.e. Type 1 and Type 2) are found. Type 1 and Type 2 moving solitons differ in their amplitudes, phase structures and parities. One noteworthy feature is that the borders separating the two families strongly

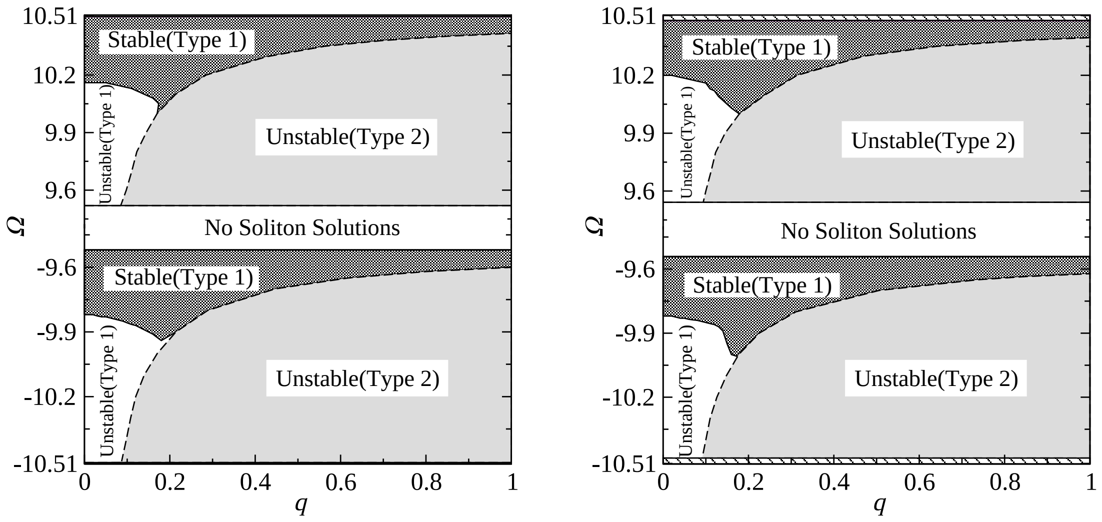

Figure 2: Stability diagrams of moving solitons plotted in the (q,Ω) plane for $\kappa = 10.0$ and $c = 0.2$ with velocities (a) $s = 0.1$ and (b) $s = 0.2$. The dashed curves represent the borders between Type 1 and Type 2 soliton families. Also, the regions represented by diagonal lines indicate the regions outside the upper and lower gaps, where solitons do not exist.

depend on both c and s, and numerical approach is used to determine the borders.

3 STABILITY OF MOVING BRAGG GRATING SOLITONS

To determine the stability of moving solitons, we have performed a systematic stability analysis by numerically solving Eqs. (2) using a split-step Fourier method. In all simulations, the soliton solutions of Eqs. (2) were propagated for $t = 2000$. It is found that intrinsic numerical noise was sufficient to trigger the instability development.

The results of the stability analysis are summarized in the (q,Ω) plane and a set of stability diagrams is displayed in Figure 2. Our results demonstrate that Type 2 moving solitons are always unstable. This is in stark contrast to the single-core BG model where it was shown that in certain parameter ranges stable Type 2 solitons exist (Atai and Malomed, 2001). On the other hand, vast regions exist where stable Type 1 moving solitons are found. The presence of quintic nonlinearity initially results in expansion of the stability regions. It is found that, in general, as the velocity of solitons increases the stable Type 1 regions in the upper and lower gaps shrink. In the examples shown in Figures 2 (a) and (b), the stability regions shrink by approximately 2% and 1% for the upper and lower gaps, respectively.

Examples of the evolution of both Type 1 and Type 2 solitons are shown in Figure 3. The instability development of Type 1 moving solitons can lead to several outcomes. Solitons far from the stability border radiate some energy and decay into radiation upon propagation (see Figure 3(b)). In certain parameter ranges, the instability development can result in splitting of moving solitons and formation of two moving solitons with different velcoties (see Figure 3(c)). An example of stable Type 1 soliton is displayed in Figure 3(a). As for Type 2 solitons, they are unstable and are completely destroyed upon propagation (see Figure 3(d)).

4 CONCLUSIONS

We have numerically investigated the bandgap and stability characteristics of moving solitons in a semilinear coupled system, in which one core is completely linear, and the other has cubic-quintic nonlinearity and is equipped with a Bragg grating. It is found that the model supports three bandgaps and moving soliton solutions exist only in the upper and lower bandgaps. The widths and edges of the bandgaps change with the system parameters, such as the group velocity mismatch term (c), soliton velocity (s) and coupling coefficient (κ) between the cores. Similar to the quiescent case, two families of solitons known as Type 1 and Type 2 were found in the (q,Ω) plane. The border separating the two families was determined numerically and found to be dependent on both c and s for a given κ. We have investigated stability of the moving solitons. The stability analysis

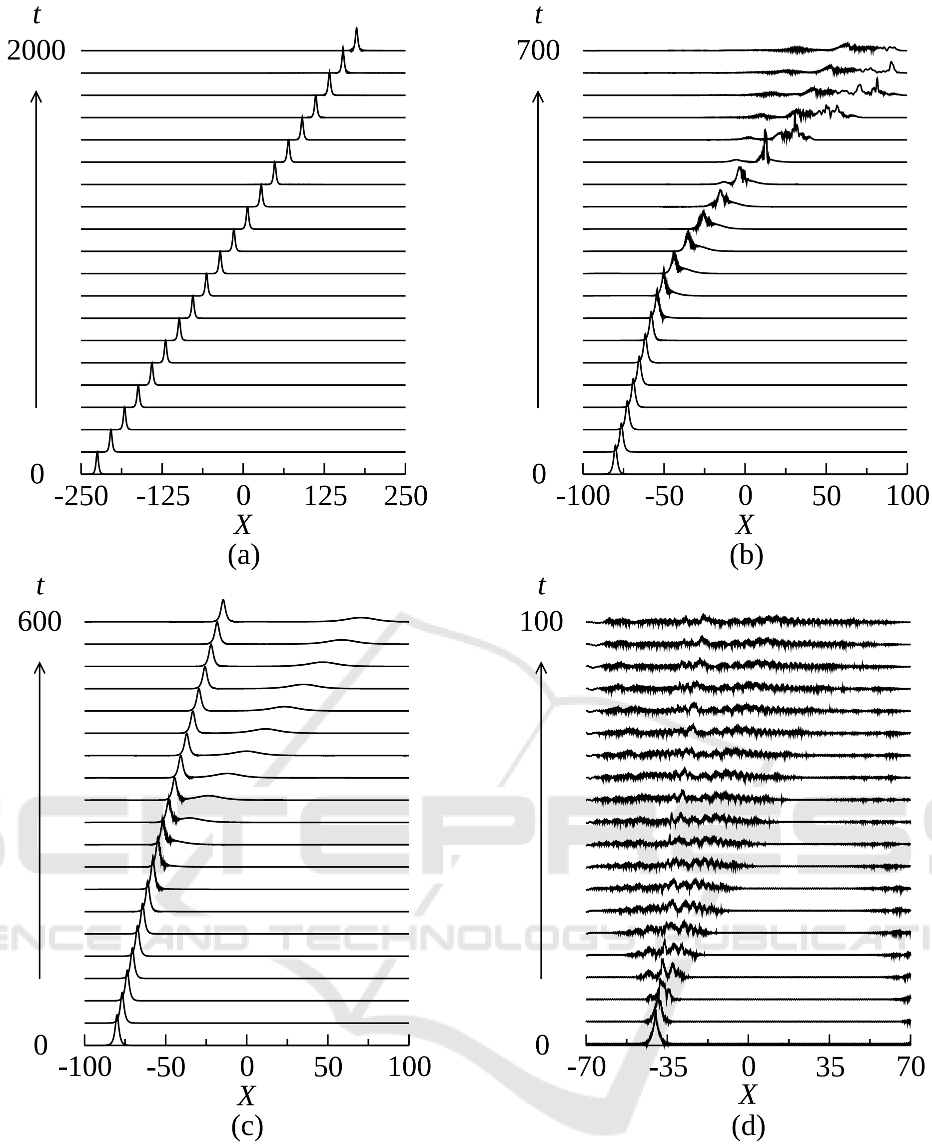

Figure 3: Examples of moving BG soliton evolution. (a) Stable Type 1 soliton with $\kappa = 10.0$, $c = 0.20$, $s = 0.20$, $q = 0.15$, $\Omega = 10.35$; (b) unstable Type 1 soliton with $\kappa = 5.0$, $c = 0.10$, $s = 0.10$, $q = 0.10$, $\Omega = 5.05$; (c) unstable Type 1 soliton with $\kappa = 10.0$, $c = 0.20$, $s = 0.10$, $q = 0.10$, $\Omega = 10.00$; and (d) unstable Type 2 soliton with $\kappa = 1.0$, $c = 0.10$, $s = 0.20$, $q = 0.90$, $\Omega = 1.35$.

demonstrates that Type 1 solitons can be stable but Type 2 solitons are always unstable. We have determined stability regions for Type 1 solitons in the (q, Ω) plane. We have also analyzed the effect of the variation of parameters on the size of the stable regions.

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