Mueller Matrix Polarimetry by Means of Azimuthally Polarized Beams and Adapted Commercial Polarimeter

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Abstract: A simple method for Mueller matrix polarimetry is proposed. The experimental set up is based on using an azimuthally polarized input beam, which presents all possible linearly polarized states across its transverse section, and an adapted commercial light polarimeter for analyzing the polarization state of the output beam. It will be shown that by measuring the Stokes parameters at only three different positions across the output beam section, the complete Mueller matrix of linear deterministic samples can be easily determined.

1 INTRODUCTION

A useful way to represent the polarization state of light is by means of Stokes parameters (Goldstein, 2003; Chipman, 2010; Azzam, 2016) that can be arranged in a 4×1 vector. In general, the polarization state of an incident beam impinging onto a specimen changes due to the interaction of light with the sample. The polarization state changes in a way that can be described by a 4×4 real matrix known as Mueller matrix. Different techniques have been used for determining the Mueller matrix of a specimen. It is necessary to generate light with different polarization states and to measure the output polarization states. In general, this process involves a set of at least 16 measurements (Chipman, 2010). Generally, uniformly polarized light across the transverse section is used as input light on the sample and a polarization analyzer is used to determine the output polarization state after the specimen.

Recently, light with non uniform polarization state across its transverse section have been proposed for simultaneuosly generating several states of polarization. This kind of beams can be used for Mueller matrix polarimetry with a reduced number of measurements (Tripathi and Toussaint, 2009; Kenny et al., 2011). However, a problem that arise when using a general nonuniformly polarized beam is that the polarization distribution across the section of the incident light changes after propagation and, in many cases, it is not easy to study how it evolves (Martínez-Herrero and Mejías, 2010; Santarsiero et al., 2013; de Sande et al., 2012). In this work we propose two improvements: the first one is to use an azimuthally polarized beam (APB) (Gori, 2001; Zhan, 2009; Ramírez-Sánchez et al., 2009) to simultaneously generate all possible linearly polarized states at once; the second one is to measure the output polarization state at only three points of the transverse cross section of the output beam. To our knowledge, there is only an experimental determination of Mueller matrices using an APB as polarization state generator, but the polarization state analyzer is completely different (de Sande et al., 2017).

APB is a particular case of a more general kind of beams, the so called spirally polarized beams that were introduced several years ago (Gori, 2001) and has been experimentally synthesized and applied in different areas (Zhan, 2009; Ramírez-Sánchez et al., 2009; Ramírez-Sánchez et al., 2010). The polarization characteristics of an APB are invariant upon free propagation, then the sample under test can be placed at any plane along the beam (de Sande et al., 2017). It is important to note that, although the polarization state map of an APB changes after passing through a sample, the output polarization map remains invariant in propagation for homogeneous linear and deterministic samples. Then, the polarization analyzer of the output beam can be positioned at any plane beyond the sample.

Circular or elliptical states of polarization are not generated in an APB. However, the three probing

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method (Oberemok and Savenkov, 2002) can be employed for obtainig a 4×3 partial Mueller matrix. This can be accomplished by measuring, by means of a commercial light polarimeter, the Stokes parameters at only three different positions across the transverse section of the output light. This is sufficient to obtain a 4×3 partial Mueller matrix of the specimen under test. Taking into account the symmetry constraints of the Mueller matrices (Hovenier, 1994) for the case of linear deterministic samples (Simon, 1990; Gil, 2007), it is possible to recover the complete sample's Mueller matrix.

After this Introduction, the theoretical foundations of the proposed method are described in the next Section. Afterwards, an experimental application of this method and the obtained results are presented and, finally, the main findings of this work are summarized in the Conclusions Section.

2 THEORETICAL BASIS

The Stokes parameters are measurable quantities that describe the polarization state of a beam propagating along a given direction (Born and Wolf, 1980; Chipman, 2010; Goldstein, 2003; Azzam, 2016). Although it is usual to deal with uniformly polarized light, in general, the Stokes parameters are functions of the position vector $\mathbf{r} = (r, \theta)$ in the transverse plane of the beam. They can be arranged in a four dimensional vector as

$$\mathbf{S}(r,\theta) = \begin{bmatrix} S_0(r,\theta) \\ S_1(r,\theta) \\ S_2(r,\theta) \\ S_3(r,\theta) \end{bmatrix} \tag{1}$$

where $S_0(r,\theta)$ represents the intensity of the beam at the point $\mathbf{r}$, $S_1(r,\theta)$ is equal to the difference between the amount of light linearly polarized at 0 and at $\pi/2$, $S_2(r,\theta)$ is analogous to $S_1(r,\theta)$ but considering linear polarization states at $\pi/4$ and $-\pi/4$, and finally, $S_3(r,\theta)$ is the difference of right and left circular polarized intensity at such point.

The polarization state of light changes when it passes through an optical system or sample. The relation between the output light Stokes vector, $\mathbf{S}^{out}$ and the input Stokes vector, $\mathbf{S}^{in}$, is described by

$$\mathbf{S}^{out}(r,\theta) = \widehat{M}\mathbf{S}^{in}(r,\theta) \,, \tag{2}$$

where

$$\widehat{M} = \begin{pmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ m_{10} & m_{11} & m_{12} & m_{13} \\ m_{20} & m_{21} & m_{22} & m_{23} \\ m_{30} & m_{31} & m_{32} & m_{33} \end{pmatrix} \tag{3}$$

is the 4×4 Mueller matrix representing the polarization changes produced by the interacting object.

To experimentally determine the 16 elements, m_{ij}, $i, j = 0, 1, 2, 3$, of the sample's Mueller matrix, at least 16 measurements have to be done. Usually, a polarization state generator that produce at least four states whose Stokes vectors are linearly independent is used to modify the polarization state of the probing beam and the projection of the output light onto 4 polarization states with linearly independent Stokes vector are measured for obtaining the complete Mueller matrix (Azzam, 2016; Chipman, 2010; Goldstein, 2003).

Azimuthally polarized beams are nonuniformly totally polarized beams, whose electric field vector at each point is directed along the azimuthal direction. For these beams, the Stokes vector is given by (Gori, 2001; Ramírez-Sánchez et al., 2009)

$$\mathbf{S}_{APB} = I(r) \begin{bmatrix} 1 \\ -\cos(2\theta) \\ -\sin(2\theta) \\ 0 \end{bmatrix} , \tag{4}$$

where $I(r,\theta)$ is the irradiance of the incident beam, which must be zero at the center because the polarization is not defined at that point.

By measuring the output intensity (first Stokes parameter $S_0^{out}(r,\theta)$) at different points of the output beam cross section (at three different θ angles) and measuring the polarization state of the input beam at the same positions, the following set of linear equations can be written

$$\begin{bmatrix} S_0^{out}(r_0,\theta_0) \\ S_0^{out}(r_1,\theta_1) \\ S_0^{out}(r_2,\theta_2) \end{bmatrix} = \widehat{W} \begin{bmatrix} m_{00} \\ m_{01} \\ m_{02} \end{bmatrix} , \tag{5}$$

where $\widehat{W}$ is the polarimetric measurement matrix given by

$$\widehat{W} = \begin{bmatrix} S_0^{in}(r_0,\theta_0) & S_1^{in}(r_0,\theta_0) & S_2^{in}(r_0,\theta_0) \\ S_0^{in}(r_1,\theta_1) & S_1^{in}(r_1,\theta_1) & S_2^{in}(r_1,\theta_1) \\ S_0^{in}(r_2,\theta_2) & S_1^{in}(r_2,\theta_2) & S_2^{in}(r_2,\theta_2) \end{bmatrix} . \tag{6}$$

Note that $S_3^{in}(r,\theta) = 0$ for any point at the cross section of an APB.

By properly selecting three different points (r_k, θ_k), in such a way that the input polarization states are represented by three linearly independent Stokes vector, the elements m_{0j} with $j = 0, 1, 2$ can be obtained by inverting Eq. (5).

In a similar way, if the second, third, and forth Stokes parameters of the output beam (S_1^{out}, S_2^{out}, and S_3^{out}, respectively) are measured at the same points of the transverse section of the beam, the elements m_{1j}, m_{2j}, and m_{3j} respectively, can be obtained from the following equation:

$$\begin{bmatrix} S_i^{out}(r_0,\theta_0) \\ S_i^{out}(r_1,\theta_1) \\ S_i^{out}(r_2,\theta_2) \end{bmatrix} = \widehat{W} \begin{bmatrix} m_{i0} \\ m_{i1} \\ m_{i2} \end{bmatrix}, \quad (7)$$

where $i = 1,2,3$. It must be noted that the polarimetric measurement matrix $\widehat{W}$ is the same for obtaining the four rows of the Mueller submatrix formed by the first three columns.

Minimization of the condition number of the polarimetric measurement matrix gives the optimum positions for measuring the output beam Stokes parameters (Peinado et al., 2015; Oberemok and Savenkov, 2002; Zallat et al., 2006). The optimum positions when using input linear polarization correspond to points where the input polarization linear states have azimuths equally spaced by $\pi/3$ (Oberemok and Savenkov, 2002) and where the intensity of the input beam is close to its maximum.

Several symmetry constraints hold for the Mueller matrix elements (Hovenier, 1994) of linear deterministic samples (Simon, 1990). They can be used to recover the last column of the sample Mueller matrix (Swami et al., 2013), however, the way these authors propose to recover the Mueller matrix could be cumbersome when experimental errors are accounted for. In the present work, we minimize a cost function formed as the sum of the squares of the left hand side of Eqs. (42), (43) and the remaining 28 relations represented by the pictograms of Fig. 1 in Hovenier, 1994.

3 APPLICATION AND EXPERIMENTAL RESULTS

In order to test the proposed system we have used an APB synthesized by means of an Arcoptix crystal polarization converter (*PC*) as shown in the set up of Fig. 1. A He-Ne laser stabilized in intensity and frequency is used as light source. A linear polarizer P_1 selects the incident state of polarization, which must be linearly polarized along a direction parallel to the *PC* axis, in order for the *PC* to work properly (Ramírez-Sánchez et al., 2009; Ramírez-Sánchez et al., 2010). A microscope objective O_1 and a lens L_1 are used to expand the beam impinging onto the *PC*. After the *PC*, the beam is spatially filtered and collimated by means of a microscope objective, a 25μm diameter pinhole, and a converging lens (O_2, *PH*, and L_2, respectively).

The propagated light is analyzed by means of a commercial light measuring polarimeter (Thorlabs TXP polarimeter) that can be positioned at any plane

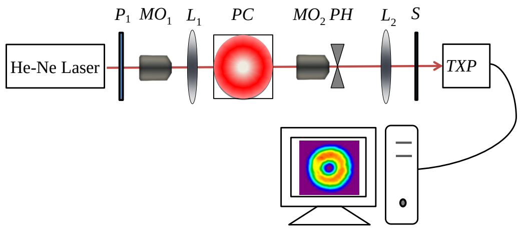

Figure 1: Proposed experimental setup. *P*: linear polarizers; *MO*: microscope objectives; *L*: lenses; *PC*: polarization converter; *PH*: Pinhole; *S*: sample; *TXP*: light polarimeter.

after the sample due to the invariability of the polarization distribution after the sample. This light polarimeter (*TXP*), mounted on a $XY-$micropositioner stage, has been modified by inserting a 500 μm diameter pinhole at its entrance in order to select a small enough area where the polarization state could be considered as nearly uniform. By means of the *TXP* light polarimeter, the Stokes parameters of the input and output beams are measured at three different points located at three equally spaced by $\pi/3$ directions and at a distance from the center of the beam where the intensity is near to its maximum (see Table 1 and Table 2).

As homogeneous, deterministic and transparent test sample we have used a quarter-wave plate (QWP) with its axes at 0 and at $+\pi/6$ relative to the x axis. The Stokes parameters of the output beam were measured by means of the modified *TXP* polarimeter at the same three points than those where the input beam has been previously characterized (see Table 1 and Table 2). By using Eq. (7) with the four measured Stokes parameters $S_i^{out}(r_k,\theta_k)$ with $i = 0,1,2,3$, at three different points $k = 0,1,2$, the first three columns of the sample's Mueller matrix are obtained. As it has been already commented, when dealing with linear deterministic samples, as is the present case, the last column can be recovered by imposing several symmetry constraints that the sample Mueller matrix elements obey (Hovenier, 1994).

Figure 2 shows the experimental values measured for a QWP with its axes at two different angles, 0 and at $+\pi/6$, relative to the x axis. As it can be noted in Fig. 3, the absolute value of the differences between theoretical ideal QWP Mueller matrix elements and the corresponding experimentally obtained values are very small (less than 0.025), so the proposed method is suitable for characterizing linear deterministic samples.

Table 1: Measured Stokes vectors, normalized to the maximum input intensity, of the input and output beam at three different positions for the case of a QWP at 0.

Position	(r_0, θ_0)	$(r_1, \theta_0 + \pi/3)$	$(r_2, \theta_0 - \pi/3)$
Input beam	$\mathbf{S}^{in} = \begin{bmatrix} 1.0000 \\ -0.9367 \\ -0.0048 \\ -0.0188 \end{bmatrix}$	$\mathbf{S}^{in} = \begin{bmatrix} 0.9014 \\ 0.4422 \\ 0.7676 \\ 0.0565 \end{bmatrix}$	$\mathbf{S}^{in} = \begin{bmatrix} 0.8280 \\ 0.4062 \\ -0.7040 \\ -0.0265 \end{bmatrix}$
Output beam	$\mathbf{S}^{out} = \begin{bmatrix} 0.9841 \\ -0.9504 \\ 0.0011 \\ 0.0477 \end{bmatrix}$	$\mathbf{S}^{out} = \begin{bmatrix} 0.9207 \\ 0.4345 \\ 0.0020 \\ -0.8036 \end{bmatrix}$	$\mathbf{S}^{out} = \begin{bmatrix} 0.8703 \\ 0.4430 \\ -0.0019 \\ 0.7321 \end{bmatrix}$

Table 2: Measured Stokes vectors, normalized to the maximum input intensity, of the input and output beam at three different positions for the case of a QWP at $\pi/6$.

Position	(r_0, θ_0)	$(r_1, \theta_0 + \pi/3)$	$(r_2, \theta_0 - \pi/3)$
Input beam	$\mathbf{S}^{in} = \begin{bmatrix} 0.9607 \\ -0.8818 \\ 0.0003 \\ -0.0201 \end{bmatrix}$	$\mathbf{S}^{in} = \begin{bmatrix} 1.0000 \\ 0.4815 \\ 0.8331 \\ 0.0153 \end{bmatrix}$	$\mathbf{S}^{in} = \begin{bmatrix} 0.9971 \\ 0.4841 \\ -0.8380 \\ -0.0234 \end{bmatrix}$
Output beam	$\mathbf{S}^{out} = \begin{bmatrix} 0.9508 \\ -0.2254 \\ -0.4091 \\ -0.7624 \end{bmatrix}$	$\mathbf{S}^{out} = \begin{bmatrix} 0.9767 \\ 0.4691 \\ 0.8107 \\ -0.0004 \end{bmatrix}$	$\mathbf{S}^{out} = \begin{bmatrix} 0.9736 \\ -0.2355 \\ -0.4193 \\ 0.8265 \end{bmatrix}$

Figure 2: Experimental Mueller matrix for a QWP with its axes at 0 (left) and rotated $\pi/6$ (right) relative to the x axis.

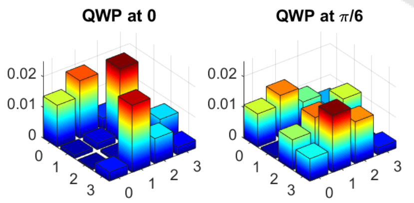

Figure 3: Absolute values of the differences between theoretical and experimental Mueller matrix elements for a QWP with its axes at 0 (left) and rotated $\pi/6$ (right) relative to the x axis.

4 CONCLUSIONS

An effective and easy method to obtain the Mueller matrix of linear deterministic samples is proposed and experimentally tested. The method is based on using azimuthally polarized beams as a continuous polarization generator and a commercial adapted polarimeter. This kind of beams presents invariant polarization characteristics under free propagation. Moreover, when this kind of beams passes through a linear deterministic system or sample, the output polarization distribution also remains invariant under free propagation. Then, both the sample and the light measuring polarimeter can be placed at any plane along the beam propagation. Measurement of the Stokes parameters at only three different points suffices for obtaining the first three columns of the sample's Mueller matrix. The well known constraints for the Mueller matrix elements in the case of linear deterministic samples are exploited to recover the complete Mueller matrix. Experimental results confirm the validity of the proposed method.

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