Labeled Fuzzy Rough Sets Versus Fuzzy Flow Graphs

Alicja Mieszkowicz-Rolka and Leszek Rolka
Rzeszów University of Technology, Al. Powstańców Warszawy 8, 35-959 Rzeszów, Poland

Keywords: Information Systems, Fuzzy Sets, Rough Sets, Fuzzy Rough Sets, Fuzzy Flow Graphs.

Abstract: This paper presents the idea of labeled fuzzy rough sets which constitutes a novel approach to rough approximation of fuzzy information systems. The labeled fuzzy rough sets approach is compared with the fuzzy flow graph approach. The standard definition of fuzzy rough sets is based on comparing the elements of a universe by using a fuzzy similarity relation. This is a complex task, especially in the case of large universes. The idea of labeled fuzzy rough sets consists in comparison of elements of the universe to some ideals represented by linguistic values of attributes. Every element of the universe can be bound up with a linguistic label. Fuzzy rough approximations of any fuzzy set are obtained by describing its elements with the help of characteristic elements of linguistic labels. In this paper, new parameterized notions of the positive, boundary, and negative linguistic values are introduced.

1 INTRODUCTION

The fuzzy set theory (Zadeh, 1965), and the rough set theory (Pawlak, 1991) are two complementary paradigms, suitable for dealing with imperfect knowledge. Both theories were combined together (Dubois and Prade, 1992) in a fuzzy rough set approach which was further developed by other researchers (Radzikowska and Kerre, 2002). However, a significant problem in application of the standard fuzzy rough set approach is the size and complexity of obtained fuzzy similarity classes, in the case of large universes with many linguistic values of fuzzy attributes. In order to overcome this drawback, a novel way of analysis of fuzzy information systems was proposed recently (Mieszkowicz-Rolka and Rolka, 2016), which is called the labeled fuzzy rough set approach. The crucial point of this method consists in avoiding the determination of fuzzy similarity between particular elements of a universe. In the present paper, we propose and discuss a new parameterized version of that approach. We introduce new basic notions of the positive (dominating), boundary, and negative linguistic values. Determination of similarity classes, approximation of fuzzy sets, and generating of decision rules can be done easier with the help of linguistic labels. To show the effectiveness of the method, we give an illustrating computational example of analysis of a small fuzzy information system, with the goal of obtaining a set of fuzzy decision rules. Flow graph-based representation is yet another possibility of a formal description of (crisp) decision tables (Pawlak, 2005a; Pawlak, 2005b). A generalized fuzzy flow graph approach can be applied to evaluate the statistical properties of fuzzy information systems. As it can also be used for determining and evaluating the quality of fuzzy decision rules (Mieszkowicz-Rolka and Rolka, 2014), we performed a parallel computation for the example data. Finally, we compare and discuss results obtained with both approaches.

2 ANALYSIS OF FUZZY INFORMATION SYSTEMS

2.1 Fuzzy Rough Sets

The fundamental operation of the rough set theory is to find out classes of objects, being elements of a finite universe of discourse, which are indiscernible with respect to a subset of attributes. In the standard rough set theory proposed by Pawlak, the attributes of objects have always crisp values. Relationship between the classes of indiscernible elements of the universe can be used for discovering dependencies between attributes and for finding their reducts.

The attributes of elements of a universe U are commonly divided into a subset of condition attributes C and decision attributes D. In such a case, the rough set theory is suitable for evaluating the

Rolka, L. and Mieszkowicz-Rolka, A.
Labeled Fuzzy Rough Sets Versus Fuzzy Flow Graphs.
DOI: 10.5220/0006083301150120
In *Proceedings of the 8th International Joint Conference on Computational Intelligence (IJCCI 2016) - Volume 2: FCTA*, pages 115-120
ISBN: 978-989-758-201-1

quality of decision systems, detecting contradictions, reducing decision tables, and determining decision rules.

Another important and popular approach, developed in the last decades, is the fuzzy logic connected with the fuzzy set theory founded by Zadeh. There are many applications of fuzzy sets, e.g., in the form of fuzzy inference systems used in automatic control of technical plants.

The rough set theory and the fuzzy set theory address different aspects of uncertainty. It is possible to combine them together in one framework, but several generalizations and assumptions need to be done. First of all, a general form of a fuzzy information system FS (Mieszkowicz-Rolka and Rolka, 2016) is necessary that is defined as the 4-tuple

$$\mathrm{FS} = \langle U, Q, \mathbb{V}, f \rangle \tag{1}$$

where:

U – denotes a nonempty set, called the universe,
Q – is a finite set of fuzzy attributes,
$\mathbb{V}$ – is a set of linguistic values of fuzzy attributes, $\mathbb{V} = \bigcup_{q \in Q} \mathbb{V}_q$,
f – is an information function, $f\colon\ U \times \mathbb{V} \to [0,1]$, $f(x,V) \in [0,1]$, $\forall V \in \mathbb{V}$ and $\forall x \in U$.

In a fuzzy decision system, every element x of the universe U is described by fuzzy condition attributes $C = \{c_1, c_2, \ldots, c_n\}$, and fuzzy decision attributes $D = \{d_1, d_2, \ldots, d_m\}$. We denote by $\mathbb{C}_i = \{C_{i1}, C_{i2}, \ldots, C_{in_i}\}$ the family of linguistic values of the i-th condition attribute c_i, and by $\mathbb{D}_j = \{D_{j1}, D_{j2}, \ldots, D_{jm_j}\}$ the family of linguistic values of the j-th decision attribute d_j, where $i = 1, 2, \ldots, n,\ \ j = 1, 2, \ldots m$, respectively. Any element $x \in U$ possesses a membership degree in every linguistic value of all fuzzy attributes. The membership degree has a value in the interval $[0,1]$.

Furthermore, we require that the sum of membership degrees in all linguistic values of each particular fuzzy condition and decision attribute is equal to 1 for every element x of the universe U. This is a generalization of the property of crisp decision systems, in which every element $x \in U$ has a unique value of each attribute.

$$\mathrm{power}(\mathbb{C}_i(x)) = \sum_{k=1}^{n_i} \mu_{C_{ik}}(x) = 1\,, \tag{2}$$

$$\mathrm{power}(\mathbb{D}_j(x)) = \sum_{k=1}^{m_j} \mu_{D_{jk}}(x) = 1\,. \tag{3}$$

Fulfilment of the requirements (2) and (3), that we consider a fundamental property of a well-defined fuzzy inference system, is assumed in both the labeled fuzzy rough set approach and the fuzzy flow graph method.

The standard rough set theory is based on comparison of elements of the universe of a crisp information system that is performed with the help of an indiscernibility relation. This is a binary equivalence relation which is reflexive, symmetric, and transitive. In a fuzzy information system, a generalized counterpart in the form of a fuzzy similarity relation is applied. Determination of similarity between any two elements of a universe is not so straightforward and unambiguous as in the crisp case. The degree of similarity can be any value in the interval $[0,1]$. Furthermore, the obtained fuzzy similarity classes with respect to condition attributes need to be appropriately used in approximation of fuzzy similarity classes generated with respect to decision attributes. However, various fuzzy T-norm and implication operators can be used in computation (Mieszkowicz-Rolka and Rolka, 2004). Since there is no unique way to determine the fuzzy rough approximations, it seems that many degrees of freedom, which is a characteristic feature of the fuzzy set theory, is not always a preferable phenomenon. This is a motivation for the labeled fuzzy rough set approach. We want to simplify the method of comparing the elements of a fuzzy information system. Instead of using a fuzzy similarity relation, we want to act like a human expert, who does not perform a detailed comparison of particular objects. A human expert tries to assess how similar a new object is to a labeled prototype, which is a pattern that can be expressed with the help of linguistic values of attributes.

Hence, our task consists in discovering the labeled prototypes. We need to find out "active" linguistic values of attributes which are dominating in the inference process. To formalize this goal, we use a level β which satisfies the following inequality

$$0.5 < \beta \leqslant 1\,. \tag{4}$$

A selected value of the parameter β will be used in criteria for classifying particular linguistic values of attributes. Given a fuzzy information system FS, we define for any element x of the universe U and any fuzzy attribute $q \in Q$:

the set $\widehat{\mathbb{V}}_q(x) \subseteq \mathbb{V}_q$ of positive (or dominating) linguistic values

$$\widehat{\mathbb{V}}_q(x) = \{V \in \mathbb{V}_q : f(x,V) \geqslant \beta\}, \tag{5}$$

the set $\hat{\check{\mathbb{V}}}_q(x) \subseteq \mathbb{V}_q$ of boundary linguistic values

$$\hat{\check{\mathbb{V}}}_q(x) = \{V \in \mathbb{V}_q : 0.5 \leqslant f(x,V) < \beta\}, \tag{6}$$

and the set $\check{\mathbb{V}}_q(x) \subseteq \mathbb{V}_q$ of negative linguistic values

$$\check{\mathbb{V}}_q(x) = \{V \in \mathbb{V}_q : 0 \leqslant f(x,V) < 0.5\}. \tag{7}$$

Taking into account the constraints (2) and (3), we get the properties of the sets (5), (6), and (7):

1. the set $\widehat{\mathbb{V}}_q(x)$ of dominating linguistic values can contain at most one element,
2. the set $\widehat{\widehat{\mathbb{V}}}_q(x)$ of boundary linguistic values can contain at most two elements,
3. the set $\check{\mathbb{V}}_q(x)$ of negative linguistic values can contain at most $|\mathbb{V}_q|$ elements.

Every element x of the universe U is represented by a row in a decision table. Since we want to completely describe any element $x \in U$ in terms of linguistic values, we should determine a combination of the linguistic values that are dominating for the selected element x. In this way, we get a linguistic label of the element x. We define the set of linguistic labels $\widehat{\mathbb{E}}^P(x)$ of any element $x \in U$, for a subset of fuzzy attributes $P \subseteq Q$, as the cartesian product of the sets of dominating linguistic values $\widehat{\mathbb{V}}_p$, for $p \in P$

$$\widehat{\mathbb{E}}^P(x) = \prod_{p \in P} \widehat{\mathbb{V}}_p(x). \tag{8}$$

From definition (5) and property 1, we conclude that every element $x \in U$ can possess at most one linguistic label. We want to generate the family $\widehat{\mathbb{E}}^P$ of linguistic labels for the entire universe U. When the value of the parameter β is increased, the number of linguistic labels belonging to the family $\widehat{\mathbb{E}}^P$ can be only decreasing.

Each dominating linguistic label is bound up with a set, denoted by X_{E^P}, that represents a linguistic label $E^P \in \widehat{\mathbb{E}}^P$, for a subset of fuzzy attributes $P \subseteq Q$. This is a subset of those elements $x \in U$ which have the linguistic label $E^P \in \mathbb{E}^P$

$$X_{E^P} = \{x \in U : E^P \in \widehat{\mathbb{E}}^P(x)\}. \tag{9}$$

We call X_{E^P} the set of characteristic elements of the linguistic label $E^P \in \mathbb{E}^P$.

Linguistic label $E^P \in \mathbb{E}^P$ has the form of an ordered tuple of dominating linguistic values for all attributes $p \in P$

$$E^P = (\hat{V}_1, \hat{V}_2, \ldots, \hat{V}_{|P|}), \tag{10}$$

and the resulting membership degree of $x \in U$ in the linguistic label $E^P \in \mathbb{E}^P$ can be determined as follows

$$\mu_{E^P}(x) = \min(\mu_{\hat{V}_1}(x), \mu_{\hat{V}_2}(x), \ldots, \mu_{\hat{V}_{|P|}}(x)). \tag{11}$$

By finding the membership degree for all elements of a universe U in a linguistic label $E^P \in \mathbb{E}^P$, we get a fuzzy similarity class denoted by $\tilde{E}^P$

$$\tilde{E}^P = \{\mu_{E^P}(x_1)/x_1, \mu_{E^P}(x_2)/x_2, \ldots, \mu_{E^P}(x_N)/x_N\} \tag{12}$$

The lower and upper approximations of a set constitute basic notions of the rough set theory. There are many possibilities to define fuzzy rough approximations of a fuzzy set (Radzikowska and Kerre, 2002; Mieszkowicz-Rolka and Rolka, 2004). As we prefer to propose a simple method, which avoids problems with selecting appropriate fuzzy operators, we want to define the approximations by using characteristic elements of a fuzzy set to be approximated.

Given a fuzzy set A, we define the set X_A of characteristic elements of the set A as follows

$$X_A = \{x \in U : \mu_A(x) \geqslant 0.5\} \tag{13}$$

Now, we use the sets of the characteristic elements of linguistic labels to approximate the set of characteristic elements of a fuzzy set A. We define lower approximation $\underline{\mathbb{E}^P}(A)$ of a fuzzy A by the set of linguistic labels $\mathbb{E}^P$, which are obtained with respect to a subset of fuzzy attributes $P \subseteq Q$, as follows

$$\underline{\mathbb{E}^P}(A) = \bigcup_{E^P \in \mathbb{E}^P} \tilde{E}^P \colon X_{E^P} \subseteq X_A \tag{14}$$

Upper approximation $\overline{\mathbb{E}^P}(A)$ of a fuzzy set A by the set of linguistic labels $\mathbb{E}^P$, which are obtained with respect to a subset of fuzzy attributes $P \subseteq Q$, is defined as

$$\overline{\mathbb{E}^P}(A) = \bigcup_{E^P \in \mathbb{E}^P} \tilde{E}^P \colon X_{E^P} \cap X_A \neq \varnothing \tag{15}$$

In analysis of an information system given in the form of a decision table, two families of linguistic labels will be generated: $\widehat{\mathbb{E}}^C$ for the condition attributes C, and $\widehat{\mathbb{E}}^D$, for the decision attributes D, respectively. We need a convenient measure for evaluating the consistency of a fuzzy information system. Approximation quality $\gamma_C(\mathbb{E}^D)$ of the fuzzy similarity classes $\tilde{E}^D$ by the fuzzy similarity classes $\tilde{E}^C$ is defined as

$$\gamma_C(\mathbb{E}^D) = \frac{\mathrm{power}(\mathrm{Pos}_C(\mathbb{E}^D))}{\mathrm{card}(U)} \tag{16}$$

$$\mathrm{Pos}_C(\mathbb{E}^D) = \bigcup_{E^D \in \mathbb{E}^D} \underline{\mathbb{E}^C}(\tilde{E}^D) \tag{17}$$

The value of approximation quality $\gamma_C(\mathbb{E}^D)$ belongs to the interval $[0,1]$. It is decreasing in the case of inconsistency, i.e., when different decisions are taken for the same condition.

2.2 Fuzzy Flow Graphs

Another method for describing and analyzing of information systems utilizes the idea of flow graph (Pawlak, 2005a; Pawlak, 2005b). A decision flow graph has the form of layers of nodes, which represent particular values of condition and decision attributes, connected by branches. Every element of a universe, corresponding to a row of the decision table, flows through the graph by taking a unique path. The original flow graph concept of Pawlak was proposed for dealing with decision tables with crisp attributes. A generalized fuzzy flow graph approach (Mieszkowicz-Rolka and Rolka, 2006) can help to evaluate quality and statistical properties of fuzzy inference systems. It should be emphasized that this generalization is valid for the product T-norm operator only. Furthermore, the membership functions of the linguistic values for all attributes must satisfy the inequalities (2) and (3). In the case of information systems with fuzzy attributes, each element of the universe U can flow through more than one path in the flow graph.

A selected path in a fuzzy flow graph represents a single fuzzy decision rule. We denote by R_k the k-th decision rule from the set of r possible decision rules

$$\begin{array}{lll} R_k: & \text{IF} & c_1 \text{ is } C_1^k \text{ AND } c_2 \text{ is } C_2^k \ \ldots \text{ AND} \\ & & c_n \text{ is } C_n^k \\ & \text{THEN} & d_1 \text{ is } D_1^k \text{ AND } d_2 \text{ is } D_2^k \ \ldots \text{ AND} \\ & & d_m \text{ is } D_m^k \end{array} \tag{18}$$

where $k = 1, 2, \ldots, r$,
$C_i^k \in \mathbb{C}_i$, $i = 1, 2, \ldots, n$,
$C_j^k \in \mathbb{D}_j$, $j = 1, 2, \ldots, m$.

For every element x of the universe U, we can determine the degree of confirmation $\mathrm{cd}(x,k)$ of the decision rule R_k, by using a T-norm operator as follows

$$\mathrm{cd}(x,k) = \mathrm{T}(\mathrm{cda}(x,k), \mathrm{cdc}(x,k)), \tag{19}$$

where $\mathrm{cda}(x,k)$ denotes the confirmation degree of the decision rule's antecedent

$$\mathrm{cda}(x,k) = \mathrm{T}(\mu_{C_1^k}(x), \mu_{C_2^k}(x), \ldots, \mu_{C_n^k}(x)), \tag{20}$$

and $\mathrm{cdc}(x,k)$ is the confirmation degree of the decision rule's consequent, respectively

$$\mathrm{cdc}(x,k) = \mathrm{T}(\mu_{D_1^k}(x), \mu_{D_2^k}(x), \ldots, \mu_{D_m^k}(x)). \tag{21}$$

By computing the confirmation degrees (20), (21) and (19), for all elements of the universe U, we get the support set of the decision rule's antecedent

$$\begin{aligned} \mathrm{support}(\mathrm{cda}(x,k)) = \{&\mathrm{cda}(x_1,k)/x_1,\, \mathrm{cda}(x_2,k)/x_2, \\ &\ldots,\, \mathrm{cda}(x_N,k)/x_N\}, \end{aligned} \tag{22}$$

the support set of the decision rule's consequent

$$\begin{aligned} \mathrm{support}(\mathrm{cdc}(x,k)) = \{&\mathrm{cdc}(x_1,k)/x_1,\, \mathrm{cdc}(x_2,k)/x_2, \\ &\ldots,\, \mathrm{cdc}(x_N,k)/x_N\}, \end{aligned} \tag{23}$$

and the support of the decision rule R_k, respectively

$$\begin{aligned} \mathrm{support}(R_k) = \{&\mathrm{cd}(x_1,k)/x_1,\, \mathrm{cd}(x_2,k)/x_2, \\ &\ldots,\, \mathrm{cd}(x_N,k)/x_N\}. \end{aligned} \tag{24}$$

Finally, we can evaluate the quality of a decision rule R_k by taking into account the relative throughflow in the corresponding path of the flow graph. This is done by determining the fuzzy cardinality (power) of the obtained support sets. The certainty factor $\mathrm{cer}(R_k)$

$$\mathrm{cer}(R_k) = \frac{\mathrm{power}(\mathrm{support}(R_k))}{\mathrm{power}(\mathrm{support}(\mathrm{cda}(x,k)))} \tag{25}$$

expresses the determinism of the decision rule R_k, whereas the measure $\mathrm{strength}(R_k)$

$$\mathrm{strength}(R_k) = \frac{\mathrm{power}(\mathrm{support}(R_k))}{\mathrm{card}(U)} \tag{26}$$

indicates how many elements of the universe U flow through the selected path of the flow graph.

3 EXAMPLE

In the following, we perform an analysis of a fuzzy information system (Table 1) including a universe U of ten elements, which are described by three fuzzy condition attributes c_1, c_2, c_3, and one decision attribute d_1. All condition and decision attributes have three linguistic values. Observe, that for every element of the universe U, and every attribute, the constraints (2) and (3) are always satisfied.

Let us assume β equal to 0.55 in application of the labeled fuzzy rough set approach. Only positive linguistic values of attributes for each element of the universe U are taken into account. After inspecting the decision table, we get six linguistic labels with respect to the condition attributes C:

$$\begin{aligned} E_1^C &= (C_{11}, C_{21}, C_{32}), & E_2^C &= (C_{12}, C_{22}, C_{31}), \\ E_3^C &= (C_{13}, C_{23}, C_{33}), & E_4^C &= (C_{13}, C_{21}, C_{32}), \\ E_5^C &= (C_{11}, C_{21}, C_{31}), & E_6^C &= (C_{12}, C_{22}, C_{32}), \end{aligned}$$

and three linguistic labels with respect to the decision attributes D:

$$E_1^D = (D_{11}), \qquad E_2^D = (D_{12}), \qquad E_3^D = (D_{13}).$$

With each of these linguistic labels, a respective similarity class in the form of an appropriate fuzzy set is connected (Tables 2 and 3).

In the next step, we calculate the lower and upper approximations of every fuzzy similarity class to the linguistic labels for the decision attributes D, by the fuzzy similarity classes to the linguistic labels for the condition attributes C, according to the formulae (14) and (15). It turns out that for all decision similarity classes, the lower approximation is equal to the upper approximation. Hence, we get six certain decision rules, presented in Table 4, and denoted by

Table 1: Decision table with fuzzy attributes.

	C_1			C_2			C_3			D_1		
	C_{11}	C_{12}	C_{13}	C_{21}	C_{22}	C_{23}	C_{31}	C_{32}	C_{33}	D_{11}	D_{12}	D_{13}
x_1	0.75	0.25	0.00	0.90	0.10	0.00	0.00	0.80	0.20	0.85	0.15	0.00
x_2	0.35	0.65	0.00	0.15	0.85	0.00	0.90	0.10	0.00	0.10	0.90	0.00
x_3	0.00	0.25	0.75	0.00	0.20	0.80	0.00	0.25	0.75	0.00	0.10	0.90
x_4	0.00	0.45	0.55	0.60	0.40	0.00	0.30	0.70	0.00	0.00	0.20	0.80
x_5	0.80	0.20	0.00	1.00	0.00	0.00	0.00	0.75	0.25	0.90	0.10	0.00
x_6	0.20	0.80	0.00	0.10	0.90	0.00	1.00	0.00	0.00	0.05	0.95	0.00
x_7	0.00	0.10	0.90	0.00	0.10	0.90	0.00	0.00	1.00	0.00	0.00	1.00
x_8	0.00	0.10	0.90	0.00	0.25	0.75	0.00	0.10	0.90	0.00	0.15	0.85
x_9	0.90	0.10	1.00	0.85	0.15	0.00	0.90	0.10	0.00	1.00	0.00	0.00
x_{10}	0.25	0.75	0.00	0.00	0.90	0.10	0.10	0.90	0.00	0.00	0.90	0.10

Table 2: Fuzzy similarity classes to linguistic labels for the condition attributes C.

	E_1^C	E_2^C	E_3^C	E_4^C	E_5^C	E_6^C
x_1	0.75	0.00	0.00	0.00	0.00	0.10
x_2	0.10	0.65	0.00	0.00	0.15	0.10
x_3	0.00	0.00	0.75	0.00	0.00	0.20
x_4	0.00	0.30	0.00	0.55	0.00	0.40
x_5	0.75	0.00	0.00	0.00	0.00	0.00
x_6	0.00	0.80	0.00	0.00	0.10	0.00
x_7	0.00	0.00	0.90	0.00	0.00	0.00
x_8	0.00	0.00	0.75	0.00	0.00	0.10
x_9	0.10	0.10	0.00	0.00	0.85	0.10
x_{10}	0.00	0.10	0.00	0.00	0.00	0.75

Table 3: Fuzzy similarity classes to linguistic labels for the decision attributes D.

	E_1^D	E_2^D	E_3^D
x_1	0.85	0.15	0.00
x_2	0.10	0.90	0.00
x_3	0.00	0.10	0.90
x_4	0.00	0.25	0.75
x_5	0.90	0.10	0.00
x_6	0.05	0.95	0.00
x_7	0.00	0.00	1.00
x_8	0.00	0.15	0.85
x_9	1.00	0.00	0.00
x_{10}	0.00	0.10	0.90

$R_1,\ldots,R_6$ (+ in column LFRS). By applying the formulae (16) and (17), we determine the approximation quality, which has the value equal to 0.75. We see that the value of approximation quality is less than 1, although all decision rules are certain. This is caused by intersection of the membership functions of the neighbouring linguistic values of fuzzy attributes. Hence, the approximation quality can reach the value of 1 only in the case of a consistent crisp information system. The measure of approximation quality can also be used for determining which of the condition attributes could be removed from the information system. The results of attribute reduction are presented in Table 5. Only the attribute c_2 can be removed, and we obtain the same decision rules in the reduced information system. The approximation quality decreases, when we remove the remaining attributes. In other words, the number of certain decision rules becomes smaller. For example, after removing the attribute c_3, we get only three certain decision rules: $(C_{11},C_{21}) \rightarrow (D_{11})$, $(C_{13},C_{23}) \rightarrow (D_{13})$, and $(C_{13},C_{21}) \rightarrow (D_{13})$. Without the attribute c_3, the decision rules R_3 and R_5 become uncertain, and the decision rules R_1 and R_2 merge together. The results indicate that the attributes c_1 and c_3 are indispensable in the information system.

Another problem to be considered is the influence of the parameter β. The presented results were obtained for β equal to 0.55. When we increase β to 0.6, we observe that the linguistic label $E_4^C = (C_{13},C_{21},C_{32})$ disappears, and we obtain five certain decision rules $R_1,\ldots,R_5$.

In order to make a comparison, we generate decision rules with the help of the fuzzy flow graph approach. In this method, all combinations of linguistic values for all attributes are taken into account. Since the decision table contains four attributes, and each attribute possesses three linguistic values, there are 81 possible decision rules. Only part of these rules will be activated, for which the flow in the corresponding path is not equal to zero. We select the most significant decision rules by applying threshold values for the factors of certainty and strength. By setting the limit of the certainty factor to 0.7, we get 21 decision rules. Together with a limit value of the factor of strength equal to 1.7%, we eventually obtain eight decision rules $R_1,\ldots,R_8$. Table 4 contains the decision rules generated by the labeled fuzzy rough set (LFRS) and the fuzzy flow graph (FFG) approaches. Two additional rules R_7 and R_8, obtained with the fuzzy flow graph approach, are denoted by $-$ in column LFRS. As we can see, the rules R_7 and R_8 are composed of such linguistic values that do not form linguistic labels determined with the labeled fuzzy rough set approach. On the other hand, the decision rule R_6 has a low value of strength. This rule would be discarded, when we set a slightly higher threshold of the strength of rules.

Table 4: Decision rules.

Decision rule	LFRS	FFG	
		strength [%]	cer
R_1: $(C_{11}, C_{21}, C_{31}) \to (D_{11})$	+	6.94	0.92
R_2: $(C_{11}, C_{21}, C_{32}) \to (D_{11})$	+	10.76	0.88
R_3: $(C_{12}, C_{22}, C_{31}) \to (D_{12})$	+	11.52	0.85
R_4: $(C_{13}, C_{23}, C_{33}) \to (D_{13})$	+	17.31	0.92
R_5: $(C_{12}, C_{22}, C_{32}) \to (D_{13})$	+	6.55	0.79
R_6: $(C_{13}, C_{21}, C_{32}) \to (D_{13})$	+	1.73	0.75
R_7: $(C_{11}, C_{22}, C_{31}) \to (D_{12})$	−	4.14	0.70
R_8: $(C_{13}, C_{23}, C_{33}) \to (D_{13})$	−	3.63	0.89

Table 5: Approximation quality of the information system.

Removed attribute			
none	c_1	c_2	c_3
0.75	0.64	0.77	0.56

4 CONCLUSIONS

Both the labeled fuzzy rough set approach and the fuzzy flow graph method generate comparable results. Due to a new way of determination of fuzzy similarity classes, which helps to reduce the computational complexity of the standard fuzzy rough set algorithm, the labeled fuzzy rough set approach is a preferable method. It is also less computationally demanding in comparison with the fuzzy flow graph approach. By acting like a human expert, who takes into account only the most significant features of the observed phenomenon, a system of fuzzy decision rules can be easy obtained. The presented example of analysis of a simple information system with fuzzy attributes confirms the effectiveness of the developed method. Nevertheless, due to a detailed analysis of information system with a fuzzy flow graph representation, all possible decision rules can be taken into consideration, which can help to refine the fuzzy inference system. However, selecting suitable threshold values for the factors of certainty and strength of rules can be problematic. In future work, another variants of parameterized fuzzy rough set approach will be investigated.

REFERENCES

Dubois, D. and Prade, H. (1992). Putting rough sets and fuzzy sets together. In Słowiński, R., editor, *Intelligent Decision Support: Handbook of Applications and Advances of the Rough Sets Theory*, pages 203–232, Boston Dordrecht London. Kluwer Academic Publishers.

Mieszkowicz-Rolka, A. and Rolka, L. (2004). Fuzzy implication operators in variable precision fuzzy rough sets model. In Rutkowski, L. et al., editors, *Artificial Intelligence and Soft Computing — ICAISC 2004. Lecture Notes in Artificial Intelligence*, volume 3070, pages 498–503, Berlin Heidelberg New York. Springer-Verlag.

Mieszkowicz-Rolka, A. and Rolka, L. (2006). Flow graphs and decision tables with fuzzy attributes. In Rutkowski, L. et al., editors, *Artificial Intelligence and Soft Computing — ICAISC 2006. Lecture Notes in Artificial Intelligence*, volume 4029, pages 268–277, Berlin Heidelberg New York. Springer-Verlag.

Mieszkowicz-Rolka, A. and Rolka, L. (2014). Flow graph approach for studying fuzzy inference systems. *Procedia Computer Science*, 35:681–690. sciencedirect.com/science/journal/18770509/35.

Mieszkowicz-Rolka, A. and Rolka, L. (2016). A novel approach to fuzzy rough set-based analysis of information systems. In Świątek, J. et al., editors, *Information Systems Architecture and Technology. Knowledge Based Approach to the Design, Control and Decision Support*, volume 432 of *Advances in Intelligent Systems and Computing*, pages 173–183, Switzerland. Springer International Publishing.

Pawlak, Z. (1991). *Rough Sets: Theoretical Aspects of Reasoning about Data*. Kluwer Academic Publishers, Boston Dordrecht London.

Pawlak, Z. (2005a). Flow graphs and data mining. In Peters, J. F. et al., editors, *Transactions on Rough Sets III. Lecture Notes in Computer Science (Journal Subline)*, volume 3400, pages 1–36, Berlin Heidelberg New York. Springer-Verlag.

Pawlak, Z. (2005b). Rough sets and flow graphs. In Ślęzak, D. et al., editors, *Rough Sets, Fuzzy Sets, Data Mining, and Granular Computing. Lecture Notes in Artificial Intelligence*, volume 3641, pages 1–11, Berlin Heidelberg New York. Springer-Verlag.

Radzikowska, A. M. and Kerre, E. E. (2002). A comparative study of fuzzy rough sets. *Fuzzy Sets and Systems*, 126:137–155.

Zadeh, L. (1965). Fuzzy sets. *Information and Control*, 8:338–353.