

# Linked Genes Migration in Island Models

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**Abstract:** Island Models (IMs) divide the whole population into many coevolving subpopulations, which periodically exchange fractions of their individuals. Some IMs, exchange probabilistic models built during the subpopulations evolution. The use of many coevolving subpopulations helps to preserve the population diversity, which makes it less likely to get stuck in the local optima. Another promising research direction in the Evolutionary Computation field is the Linkage Learning. The knowledge about gene dependencies can be used in many different ways that improve the overall method effectiveness. Therefore, this paper proposes the Gene Pattern Based Island Model (GePIM) that uses the multi-population nature of IMs to generate the linkage information. GePIM also introduces a new type of migration based on exchanging linked gene groups, instead of exchanging the whole individuals or probabilistic models.

## 1 INTRODUCTION

Evolutionary Algorithms (EAs) are well-known methods capable of solving hard computational problems. Many different EAs were proposed, all of them have their pros and cons. During the recent years, the research toward proposing universal and beneficial mechanisms for EAs has gained an increasing attention. Among these universal mechanisms, the coevolution of many subpopulations can be found (Chang, 2015; Dahzi et al., 2008; Fidrysiak and Przewozniczek, 2015; Fieldsend, 2014; Kurdi, 2016; Leitão et al., 2015; Kwasnicka and Przewozniczek 2011; Przewozniczek et al., 2015; Skolicki, 2008; Walkowiak et al., 2013; Zhang et al., 2007). Island Models (IMs) improve the performance of Genetic Algorithms (GAs) by better preservation of population diversity (Kurdi, 2016; Leitão et al., 2015; Skolicki and De Jong, 2007; Skolicki, 2008). IMs allow for decreasing the negative influence of pre-convergence (Kwasnicka and Przewozniczek, 2011; Watson and Pollack, 1999; Watson, 2006). Due to the higher diversity, more potentially valuable Building Blocks (BBs) remain in the population. Usually, the more BBs can be processed by a GA-based method, the greater is the chance of reaching the breakthrough and finding more valuable solutions than in the case of fast converging methods.

As stated above, in general, the better diversity preservation improves the overall GA effectiveness.

However, it does not seem enough to make the valuable BBs exist in a population (Kwasnicka and Przewozniczek, 2011; Skolicki, 2008). The method must also use mechanisms that allow for effective BBs processing. As shown in the literature, not all BBs are equally effectively processed by classical EA operators like crossover. Therefore, the Linkage Learning (LL) techniques became popular. The LL methods try to discover the possible gene dependencies during their run (so-called linkage discovery). The knowledge about linkage is used later on to improve the method effectiveness. For instance, during the crossover operation only linked genes can be exchanged between individuals instead of using classical single point or uniform crossover operators. Therefore, the LL techniques are often shown to be beneficial to many different EA types (Omidivar et al., 2014).

IMs use many coevolving subpopulations to improve the overall population diversity (Skolicki, 2008). To ensure the communication between separated subpopulations (called islands) the migration operator is used. During the migration, all subpopulations (isolated for the most of the method run) usually exchange individuals with one another. In some papers, the probabilistic models that were built by separate islands are exchanged (delaOssa et al., 2004; Muelas et al., 2014). The migration is, in fact, supposed to exchange BBs that are carried by migrated individuals or probabilistic models (Skolicki, 2008; Watson, 2006). The question is: if

we want to exchange BBs why don't we exchange BBs instead of exchanging other data structures that are supposed to carry them? Therefore, this paper proposes a new type of migration: the Linked Gene Groups Migration (LGGM). During the LGGM only linked gene groups (which are assumed to be BBs) are migrated. To show the potential of the new migration operator, the Gene Pattern Based Island Model (GePIM) that incorporates the LGGM and the LL techniques is proposed.

The remainder of the paper is organized as follows. In the second section, the related work is presented. The third section describes GePIM. The description of the performed experiments and their analysis is presented in Section 4. Finally, the last section concludes and summarizes the paper.

## 2 RELATED WORK

In the recent years, the LL became an important part of evolutionary computation research. The idea behind LL is to find potential gene dependencies during an EA run and use this knowledge to improve the overall method effectiveness. Many different ways have been proposed for linkage information gathering, storing, and use. Therefore, the next subsections contain a brief introduction to recent research on the LL and IMs.

### 2.1 Linkage Learning

One of the first classifications for the LL methods was proposed in (Chen et al., 2007a). The LL concerns three main fields: the way the good and the bad linkage are distinguished, linkage representation and linkage storage. According to the first classification field, the good and the bad linkage may be distinguished only on the base of fitness value (unimetric way) or some additional criteria may also be taken into consideration (multimetric way). The typical unimetric way of good and bad linkage distinguish may be found in Multi Population Pattern Searching Algorithm (MuPPetS) (Kwasnicka and Przewozniczek, 2011). The multimetric approach may be found in Bayesian Optimization Algorithm (BOA) (Pelikan et al., 1999; Pelikan et al., 2006). The linkage may be represented in a virtual or a physical way. If the linkage is represented by graphs, matrices, gene patterns (Kwasnicka and Przewozniczek, 2011; Pelikan et al., 1999; Pelikan et al., 2006), or other data structures then the virtual representation is used. If the linkage is being represented as physical

genes locations in the chromosome (i.e., genes that are close to one another are considered to be linked) then physical linkage representation is being used. The typical example of physical linkage representation is messy coding (Goldberg et al., 1993; Kwasnicka and Przewozniczek, 2011). Finally, the linkage may be stored in two different ways: centralized and distributed. The centralized way is used when linkage information is stored in some globally accessible database (i.e., the complete linkage information in the database may be accessed at any method operation). On the other hand, the messy coding is a typical example for distributed linkage information storing (each individual possess its own linkage information that is used only for the operations that include this individual).

Another classification of LL techniques, also called Decomposition Strategies, was proposed in (Yu et al., 2009). The three main linkage generation ways are pointed: perturbation, interaction adaptation, and model building. In (Omidivar et al., 2014) this list was supplemented by random methods. All techniques are discussed below. This paper defines another decomposition strategy which was not distinguished before although the methods that use it are present in the literature - the evolution results comparison.

**Perturbation.** These methods perturb the genotype. The fitness value changes caused by perturbations are analysed to detect the possible interactions between genes. The example of this strategy is the Probabilistically Complete Initialization (PCI) phase of fast messy GA (fmGA) (Goldberg et al., 1993) and Differential Grouping (Omidivar et al., 2014).

**Interaction Adaptation.** The methods that use this linkage discovery technique are capable of evolving the gene order in the chromosome. This decomposition strategy is used during the evolutionary process.

**Model Building.** These methods, also called Estimation of Distribution Algorithms (EDAs), construct the probabilistic model on the base of promising individuals in the population. The examples of such methods are BOA and hBOA (Pelikan et al., 1999; Pelikan et al., 2006).

**Random Methods.** These methods use the most simple linkage generation strategy – the linkage is generated randomly. The new linkage may be generated again after some evolutionary method iterations (in such case the quality of linkage is not controlled at all) (Yang et al., 2008). Another possibility is to generate new linkage information when the information used so far is found not useful

(Chen et al., 2007a; Fidrysiak and Przewozniczek, 2015). Note that such decomposition strategy is quite primitive, but still may be more effective than the use of typical linkage-blind operators like the uniform or the single point crossover operators (Przewozniczek, 2015).

**Evolution Results Comparison.** This decomposition strategy class was not distinguished before but seems necessary since the already proposed list does not cover all possibilities that may be found in the EA literature. The methods that use this decomposition strategy compare the individuals that are the results of different evolution processes. For instance, evolution results produced by various islands may be compared (Skolicki, 2008). Another way is used in MuPPetS (Kwasnicka and Przewozniczek, 2011; Przewozniczek et al., 2015; Walkowiak et al., 2013). In MuPPetS, a perturbation to a particular genotype is introduced. Then this perturbed genotype is optimized by an evolutionary process. The linkage information is generated on the base of differences between the genotype before perturbation and after evolutionary optimization of the perturbed genotype.

Linkage information is used in many different ways. The most common one is to improve the effectiveness of crossover operators (Chen et al., 2007a; Fidrysiak and Przewozniczek, 2015; Goldberg et al., 1993; Kwasnicka and Przewozniczek, 2011). Other ways may include the population initialization (Goldberg et al., 1993; Kwasnicka and Przewozniczek, 2011; Pelikan et al., 1999; Pelikan et al., 2006) and gene grouping in Cooperative Coevolution (Omidivar et al., 2014). Note that some of the proposed LL techniques may be hardly useful in practice. For instance, in (Omidivar et al., 2014) the proposed LL technique assumes that identified groups of genes are fully separable. It seems doubtful that a method built on such assumptions will be capable of effectively solving the real-life problems – usually, the BBs are not fully separable (Pelikan et al., 2006; Skolicki, 2008; Watson and Pollack, 1999).

## 2.2 Island Model

In IMs (Kurdi, 2016; Leitão et al., 2015; Skolicki and De Jong, 2007; Skolicki, 2008) the population is divided into subpopulations called islands. For each island (subpopulation), a separate evolutionary process is executed. The evolutionary operations are restricted to islands, so individuals from different islands cannot interact freely. The islands communicate with one another usually by migrating whole

individuals. Such model improves the diversity of the whole population and thus makes it less vulnerable to pre-convergence.

An interesting direction of research in the IM field is the hybridization of IM and EDAs (de la Ossa et al., 2004; Muelas et al., 2014). EDAs build the probabilistic models during their run which are used to generate offspring. The IM and EDA hybrids exchange probabilistic models instead of exchanging individuals. However, the question of how to effectively combine the exchanged models remains open. Therefore, in this paper, we concentrate on exchanging detected building blocks instead of exchanging models or individuals.

In some of the papers another way of understanding IMs may be found (Skolicki and De Jong, 2007; Skolicki, 2008). IMs may be interpreted as a two-level system, where islands are higher level individuals and interactions between them are a part of high-level evolution. This interpretation is close to the idea of Compositional Evolution (Watson, 2006) defined as “evolutionary processes involving the combination of systems or subsystems of semi-independently preadapted genetic material”. Skolicki (Skolicki, 2008) points out that lower evolution level of IMs is used to produce BBs while the higher level is used to exchange them. Therefore, IMs should be suitable to solve problems built from multiple subsolutions. Note that similar, two-level method construction, is becoming more and more popular and is typical not only for GA-based methods. For instance, MuPPetS (Kwasnicka and Przewozniczek, 2011; Przewozniczek et al., 2015; Przewozniczek, 2016) uses a dynamically changed number of messy individual subpopulations, which exchange data using LL. In (Alves, 2015; Kwasnicka and Przewozniczek, 2011; Kim and Choi, 2015) GA-based methods, with many coevolving subpopulations, were proposed. The idea of multiple subpopulations may also be found in papers concerning Particle Swarm Optimization (PSO) (Chang, 2015; Dahzi et al., 2008; Fidrysiak and Przewozniczek, 2015; Fieldsend, 2014; Zhang et al., 2007), Differential Evolution (DE) (Wang et al., 2015; Zavoianu et al., 2015), and others (Omidivar et al., 2014; Yang et al., 2008).

## 3 GENE PATTERN BASED ISLAND MODEL

In this section, the description of the proposed Gene Pattern Based Island Model (GePIM) is presented. As pointed in the former sections, the migration in

IMs is used to exchange the BBs between islands. Therefore, the main motivation behind GePIM is to exchange linked gene groups instead of exchanging individuals or probabilistic models.

### 3.1 Gepim Overview

The general GePIM procedure is presented in Figure 1. The method framework is typical for IMs. The main differences are the introduction of LL mechanisms and the use of linked genes.

```

1:  it ← 1;
2:  for each island:
3:      initialize(island);
4:      evaluate(island);
5:  while (!stopCondition):
6:      for each island:
7:          select(island);
8:          crossover(island);
9:          mutate(island);
10:         evaluate(island);
11:         if (it % retrievalFreq = 0):
12:             retrieveLinkage();
13:         if (it % migrationFreq = 0):
14:             migrateLinkedGeneGroups();
15:         for each island:
16:             evaluate(island);
17:         it ← it + 1;
18:  return bestIndividual;

```

Figure 1: The general GePIM procedure.

As shown in Figure 1, first all subpopulations (islands) are randomly initialized. Then all subpopulations are processed like in the standard GA (sGA). In the case of GePIM, the uniform crossover operator is used as it is not dependent on gene order. The probability of mutation is checked for every single gene separately. If the mutation occurs then the gene value is flipped. The linkage information retrieval and the LGGM are performed with frequencies defined by a user. These two operations are described in the next two subsections.

### 3.2 Linkage Information Retrieval

The linkage information is stored in a form of gene patterns (Kwasnicka and Przewozniczek, 2011). A gene pattern is a set of gene positions that are expected to be dependent on one another. A gene pattern of length  $l$  is denoted as  $\{p_1, p_2, \dots, p_l\}$ , where  $p_i$  is the  $i$ th gene position. For example, the gene pattern  $\{1, 4, 6\}$  marks three genes: first, fourth, and sixth. Here, gene patterns are created by comparing two individuals and selecting only those

genes that are different. Therefore, lengths of gene patterns are not fixed. For example, for the comparison of 001100 and 010101 individuals, the gene pattern is  $\{2, 3, 6\}$ , because the genotypes are different at the second, third, and sixth position.

Assuming that two individuals have different genotypes and are well evolved (they cannot be easily improved by a typical evolutionary process), some of their genotype differences may have a considerable impact on fitness. The existence of the specific good gene values only in one individual may be caused by difficulties in obtaining such sequence. It may be supposed that these genes depend on one another.

Therefore, in IMs, it seems reasonable to compare individuals from different islands to produce linkage. The objective is to compare individuals that are well evolved, so only the best individuals from each island are taken into account. For each island, two types of the best individual can be defined. The first is the current best individual (i.e., the best individual in the island's population). The other is the best individual found so far on a particular island. The current best individual and the best individual found so far can, but do not have to, be the same.

To retrieve the linkage information, comparisons between the best individuals from all islands are made. Each island provides two best individuals: current and the best found so far. All possible pairs are checked. Therefore,  $C_{2N}^2$  comparisons are made, where  $N$  is the number of islands. Each comparison produces a single gene pattern. Comparing two best individuals from the same island could provide a useful gene pattern since they may represent different local optima. A single gene pattern may contain gene positions from different BBs. Especially in the early method stage when all islands are the most diversified due to the random initialization. Thus, not every gene pattern generated in the above way will be a valuable one. Note that in every LL method the linkage information does not have to be perfect. It is as good as it improves the performance of the whole method. A wider discussion on this topic may be found in (Kwasnicka and Przewozniczek, 2011).

Each newly created gene pattern is added to a global gene patterns storage, called gene pattern pool. A size of a gene pattern pool cannot exceed a user defined maximum size. If a maximum size is reached then every new gene pattern replaces a randomly selected gene pattern from the gene pattern pool. The above mechanism of replacing randomly chosen gene patterns by new gene patterns

is adopted from (Kwasnicka and Przewozniczek, 2011). The motivation behind this mechanism is quite straightforward. It is very hard (if not impossible) to distinguish the linkage information that is useful on the current method stage, from the useless one. Therefore, it seems reasonable to assume that useful linkage information will be generated more times and will replace the useless one.

### 3.3 Linked Gene Groups Migration Operator

During a classic migration, individuals are migrated between islands. Note that even if migrated individuals provide some new BBs to an island, it may be hard to exchange them with other individuals. New BBs can be easily destroyed by classical crossover operators. For example, if the optimal problem solution is 11111111 then even if the population contains individuals 01010101 and 10101010, obtaining the individual representing the best solution is highly unlikely.

The above drawback can be avoided by migrating linked genes instead of individuals or probabilistic models. Genes marked by a single gene pattern are supposed to be linked. To perform the Linked Gene Groups Migration (LGGM), two islands are selected first. Then, the defined number of best-fitted individuals from both islands is selected. The individuals that migrate their genes are called ‘source individuals’ while the individuals from the island that receives the linked genes are called ‘receiving individuals’. The source and receiving individuals are paired in the way that the best source individual sends its genes to the best receiving individual, the second best source individual sends its genes to the second best receiving individual and so on. For each source-receiving individuals pair, the gene pattern is selected randomly from the gene pattern pool that contains gene patterns created during the linkage information retrieval phase. Finally, all genes from the source individual, marked by the chosen gene pattern replace proper genes in the receiving individual. For example, if the 01010101 is the receiving individual, the 10101010 is the source individual and the {1, 3, 5, 7} gene pattern was selected for this migration then after such operation the genotype of receiving individual will be as follows: 11111111.

### 3.4 Summary

GePIM is an example of IM, which uses the LL

technique. The linkage information is used during (and only during) the LGGM operation. This improves the method effectiveness. The method is not dependent on gene order since the uniform crossover operator is used.

The GePIM classification as an LL method is as follows. GePIM uses the unimodal way of good and bad linkage distinguish, the linkage information representation is virtual and is stored in a centralized way. The evolution results comparison is used as a decomposition strategy.

## 4 THE RESULTS

In this section, the results of performed experiments are presented. In the first subsection the competing methods choice is presented, then the problems used for tests and the stop condition are discussed. The tuning procedure, obtained results, and their discussion are described in the latter subsections. All methods were coded in C++. Whenever it was possible, the methods shared the same pieces of code. All experiments were conducted on HP Elite Desk800 3.4 GHz 8GB RAM server with Intel Core i7-4770 CPU and Windows 7 64-bit installed. For each test case, ten independent runs were executed. Complete results, source codes of all competing methods, and configuration files are available at: [http://mp2.pl/download/ai/20160531\\_gepim.zip](http://mp2.pl/download/ai/20160531_gepim.zip).

### 4.1 Competing Methods Choice

Four methods were chosen to compete with GePIM. The classical Island Model (IM) was chosen to check how significant is the improvement caused by changes proposed in this paper. sGA was chosen to check if the difference between the simple and more evolved methods is significant. Both, classical IM and sGA use (same as GePIM) the uniform crossover operator and the gene flip mutation checked for every gene separately. Finally, BOA (Pelikan et al., 1999) and MuPPetS (Kwasnicka and Przewozniczek, 2011) methods were chosen as the literature review points them as highly effective ones. BOA is effective when used for solving the deceptive functions concatenations, while MuPPetS was shown capable of effective solving both: theoretical (Kwasnicka and Przewozniczek, 2011) and practical problems (Przewozniczek, 2015; Walkowiak et al., 2013).

BOA (Pelikan et al., 1999) is an LL method that builds a Bayesian network to represent gene dependencies. At each iteration, a Bayesian network

is generated on the base of a set of best individuals. In the next iteration, new individuals are created on the base of Bayesian network. BOA was shown to use a relatively low number of Fitness Function Evaluations (FFE) when compared to other evolutionary methods. However, in the case of BOA, the main part of computation load is consumed not for the fitness value computation, but on Bayesian tree generation. In this paper the same Bayesian network construction algorithm as in (Kwasnicka and Przewozniczek, 2011; Pelikan et al., 1999) is used. The time complexity of this algorithm is  $O(k^2kn^2N + kn^3)$ , where  $n$  is the problem size,  $N$  is the size of the dataset, and  $k$  is the maximum allowed indegree (the tree depth).

MuPPetS (Kwasnicka and Przewozniczek, 2011) is a relatively new proposition of LL method. It uses a flexible number of coevolving virus populations. The viruses are messy-coded individuals (Goldberg et al., 1993; Kwasnicka and Przewozniczek, 2011), but purpose and way of use are different than in the classical messy coded individuals case. The number of virus subpopulations is increased when the method is stuck and decreased after reaching a breakthrough. This feature makes the method capable of automatically adjusting itself to the current evolution state. The linkage information is extracted with the use of evolution results comparison strategy.

## 4.2 The Test Problems and Stop Condition

As discussed in the previous subsection the computation load used by BOA is mainly dependent on Bayesian network construction, not on fitness value computation. Therefore, FFE is not a fair computation measure for BOA. A detailed analysis of the dependency between FFE and the computation time used by BOA may be found in (Kwasnicka and Przewozniczek, 2011). Therefore, in this paper, the computation time (7200 seconds) was used as a stop condition. The stop condition was checked after each method iteration. Thus, the overall computation time could be slightly greater than 7200 seconds. The time-based stop condition favors the methods that spend most of the computation load for fitness value computation. The proper analysis of FFE and computation time dependency is given at the end of Section 4.4.

All experiments were executed in a repeatable environment without any other resource consuming processes running. Three different kinds of test problems were chosen: the deceptive functions

concatenations, the Knapsack, and the MAX-2SAT problem.

### 4.2.1 Mixed Deceptive Functions Concatenations

Eight different deceptive functions were used to build the deceptive functions concatenations. They are presented in Table 1. The value of the deceptive function is dependent on unitation  $u$  (the number of '1's in the genotype).

Table 1: Used deceptive functions definitions.

$u$	3-bit (3l)	3-bit (3lh)	3-bit (3h)	3-bit (3hh)	5-bit (5l)	5-bit (5lh)	5-bit (5h)	5-bit (5hh)
0	0.33	0.98	3.33	9.80	0.4	0.99	4	9.88
1	0.17	0.49	1.67	4.90	0.3	0.74	3	7.41
2	0	0	0	0	0.2	0.49	2	4.94
3	1	1	10	10	0.1	0.25	1	2.47
4	NA	NA	NA	NA	0	0	0	0
5	NA	NA	NA	NA	1	1	10	10

To each deceptive functions concatenation, the tale function was added. The tale is the OneMax function and is defined as follows:  $Tale(length) = u/length$ , where  $u$  is the unitation and  $length$  is the tale function gene number. The test cases used in the experiments are defined in Table 2 according to Table 1 and the tale definition. The number of bits necessary to encode the complete problem solution was 600 for all used concatenations.

Table 2: Used deceptive functions concatenations.

TC no.	Definition
1	$50*3l + 30*5l + Tale(300)$
2	$50*3lh + 30*5lh + Tale(300)$
3	$15*3l + 35*3h + 9*5l + 21*5h + Tale(300)$
4	$15*3lh + 35*3hh + 9*5lh + 21*5hh + Tale(300)$
5	$60*5l + Tale(300)$
6	$60*5lh + Tale(300)$
7	$18*5l + 42*5h + Tale(300)$
8	$18*5lh + 42*5hh + Tale(300)$

The definitions of the above test cases were adopted from (Kwasnicka and Przewozniczek, 2011). Note that frequently, when the deceptive functions concatenations are used, the glued deceptive blocks are identical (Goldberg et al., 1993; Pelikan et al., 1999; Pelikan et al., 2006). Such a

practice may be found surprising since deceptive functions concatenations are supposed to mimic the existence of BBs in the problem. It seems a reasonable assumption that, usually, a problem is built from many BBs (Watson and Pollack, 1999; Watson, 2006). Nevertheless, the assumption that these BBs are identical seems unjustified. Therefore, mixed deceptive functions concatenations were proposed (Kwasnicka and Przewozniczek, 2011). The test problems include a tale which is a part that should be easy to optimize by any GA-based method. Note that it was shown that some of the methods (e.g., fnGA) considered effective in solving the problems built from deceptive functions concatenations are ineffective when the deceptive blocks are not identical (Kwasnicka and Przewozniczek, 2011).

#### 4.2.2 The Knapsack Problem

The Knapsack problem is a common tool used to test the effectiveness of methods that use the binary coding. Most of the test instances of the knapsack problem can be solved in pseudo-polynomial time using dynamic programming, but it is possible to generate instances that are hard to solve (Pisinger, 2005). In the performed experiments, only such hard-to-solve instances were used. The set of hard instances and their solutions was downloaded from [http://www.diku.dk/~pisinger/largecoeff\\_pisinger.tgz](http://www.diku.dk/~pisinger/largecoeff_pisinger.tgz). For each instance the solution time that indicates its hardness was provided. Six of the hardest instances corresponding to the greatest value of the solution time were chosen for tests: “knapPI\_3\_500\_10000000\_76” (test case (TC) no.: 9; bit length: 500), “knapPI\_3\_500\_10000000\_92” (10; 500), “knapPI\_3\_1000\_10000000\_73” (11; 1000), “knapPI\_4\_500\_10000000\_13” (12; 500), “knapPI\_4\_1000\_10000000\_22” (13; 1000), “knapPI\_4\_1000\_10000000\_49” (14; 1000).

#### 4.2.3 The MAX-2SAT Problem

The MAX-2SAT problem is the specific kind of the MAX-SAT problem, because the given conjunctive normal form (CNF) formula is the conjunction of clauses of two literals. Instances of the MAX-2SAT problem can be generated using a planted solution model (Watanabe and Yamamoto, 2010). In this model, for each instance, a solution that is very likely to be optimal is provided. Thus, a large number of instances can be created and experimentally checked if they are hard to solve. Four instances were generated with the use of planted solution model and used during the

experiments: test case no. 15 ( $p=0.1243$ ;  $r=0.0311$ ; bit length: 500), 16 ( $p=0.1492$ ;  $r=0.0373$ ; 500), 17 ( $p=0.0553$ ;  $r=0.0138$ ; 1000), 18 ( $p=0.0691$ ;  $r=0.0173$ ; 1000). The  $p$  and  $r$  variable values reported for the MAX-2SAT problem instances are the parameters used to generate the instances by the planted model.

### 4.3 The Tuning Procedure

All methods were tuned with the use of the same tuning procedure. The initial settings were proposed, on the basis of the literature review. Then, each parameter was optimized separately in a greedy way – if a parameter change improves the results then the change is accepted. The tuning was made for the following deceptive functions concatenations: 5, 6, 7, 8. These functions are supposed to be the most challenging test cases to solve. Therefore, if the tuning procedure causes each method to propose the results of quality as good as possible for these problems, then the methods should also perform well for the other test cases. During tuning, two independent runs were executed, for each test case. Similar tuning procedure can be found in (Kwasnicka and Przewozniczek, 2011, Przewozniczek et al., 2015).

The complete initial and final configurations of all competing methods are given in Tables 3, 4, 5, 6, and 7. The parameter tuning order was the same as the parameter order in the tables.

Table 3: GePIM configuration.

Parameter	Initial value	Final value
Crossover	0.6	0.3
Population size per island	400	200
Number of islands	10	30
Migration frequency	200	50
Number of migrating Individuals	40	40
Gene pattern pool size	300	300
Linkage information retrieval freq.	100	500
Mutation	1 / length	1 / (3 * length)

As presented in Table 3, the most significant change of initial GePIM configuration made by tuning was the increase of the LGM frequency and decrease the frequency of linkage gathering. It seems that it is more beneficial for GePIM to exchange BBs more often and collect the linkage information when the current and the overall best

individual are well evolved, which should increase the gathered linkage quality.

Table 4: MuPPetS configuration.

Parameter	Initial value	Final value
Gene pattern pool size	200	200
Minimal pattern size	3	3
Virus generation number	5	10
Virus population per competitive template	400	180
Cut probability	0.05	0.16
Splice probability	0.5	0.16
Mutation	0.1	0.02
Remove gene probability	0.1	0.02
Add gene probability	0.1	0.02

Table 5: BOA configuration.

Parameter	Initial value	Final value
Population size	80000	50000
Bayesian tree level	4	4
Parents percentage	50	40
Offspring percentage	50	60

Table 6: Classical IM configuration.

Parameter	Initial value	Final value
Crossover	0.7	0.5
Population size per island	400	400
Number of islands	10	50
Migration frequency	200	50
Number of migrating individuals	40	40
Mutation	1 / length	1 / length

Table 7: sGA configuration.

Parameter	Initial value	Final value
Crossover	0.7	0.4
Population size	400	800
Mutation	1 / length	2 / length

#### 4.4 The Methods Comparison

As mentioned, for each test case ten independent runs were performed. In Table 8 the average solution quality for each test case is presented. In the case of deceptive functions concatenations, the quality measure was the best solution unitation percentage:  $UnitPerc(X_{best}) = u(X_{best})/len$ , where  $X_{best}$  is the best-

found individual,  $u(X_{best})$  is its unitation, and  $len$  is the genotype length. Unitation percentage informs how similar is the best-found individual to the global optimum and is typical for deceptive functions concatenations (Fidrysiak and Przewozniczek, 2015; Goldberg et al., 1993; Kwasnicka and Przewozniczek, 2011; Pelikan et al., 2006). For the Knapsack and the MAX-2SAT problems, the quality measure was the proportion of the best-found solution and the global optimum values. Due to the space limitations, the standard deviation was not reported. However, for all the test cases the standard deviation was rather low (usually below 0.01). The average time presented in Table 9 informs how fast the final solution was found.

Table 8: The average solution quality comparison.

TC no.	GePIM [%]	Classical IM [%]	sGA [%]	BOA [%]	MuPPetS [%]
1	99.92	87.87	98.62	<b>100.00</b>	<b>100.00</b>
2	99.25	63.75	58.55	97.90	<b>99.50</b>
3	99.50	87.90	98.28	<b>100.00</b>	99.70
4	<b>99.45</b>	63.12	58.31	95.20	99.20
5	<b>100.00</b>	75.36	98.63	<b>100.00</b>	<b>100.00</b>
6	97.25	52.26	50.07	<b>100.00</b>	98.30
7	<b>100.00</b>	76.07	86.16	<b>100.00</b>	<b>100.00</b>
8	97.75	53.33	50.50	94.20	<b>98.10</b>
9	<b>100.00</b>	<b>100.00</b>	99.96	<b>100.00</b>	99.99
10	<b>100.00</b>	<b>100.00</b>	99.98	<b>100.00</b>	99.99
11	<b>100.00</b>	<b>100.00</b>	99.96	99.01	99.95
12	<b>100.00</b>	<b>100.00</b>	<b>100.00</b>	<b>100.00</b>	<b>100.00</b>
13	<b>100.00</b>	<b>100.00</b>	<b>100.00</b>	99.99	<b>100.00</b>
14	<b>100.00</b>	<b>100.00</b>	<b>100.00</b>	99.99	99.96
15	<b>100.00</b>	<b>100.00</b>	99.91	<b>100.00</b>	99.93
16	<b>100.00</b>	<b>100.00</b>	99.95	<b>100.00</b>	99.93
17	<b>100.00</b>	<b>100.00</b>	99.95	86.70	99.96
18	<b>100.00</b>	99.99	99.94	86.10	99.99

For deceptive functions concatenations Classical IM and sGA performed worse than all other three methods for every single instance. BOA for two cases proposed the solutions of relatively low quality (for test case number 8 and 4 the average solution quality is only 94.2 and 95.2 respectively). Therefore, the best methods for this problem class are GePIM and MuPPetS, while MuPPetS is slightly better. For the Knapsack and the MAX-2SAT problems, all methods report high quality-results,

Table 9: The average time comparison.

TC no.	GePIM [s]	Classical IM [s]	sGA [s]	BOA [s]	MuPPetS [s]
1-8	300	2624	3040	3622	4886
9-14	643	2151	1890	4410	2534
15-18	1035	3253	2941	5485	427

but GePIM and classical IM are the best. Note that BOA reported results of very low quality for 17th and 18th test case. To encode solutions to these test cases 1000 genes is necessary. For such a large gene number, the computation load spent by BOA for building the Bayesian network (it is built at every iteration) rises significantly and makes the method ineffective. Similar situation, in which BOA was unable to handle the problems encoded with large gene number was already observed and discussed in (Kwasnicka and Przewozniczek, 2011).

The worst solution quality for each test case type group is reported in Table 10. The comparison confirms that GePIM and MuPPetS are the only two methods that guarantee the highest solution quality for all problem types.

Table 10: The comparison of the worst solution quality for each test case type.

	GePIM [%]	Classical IM [%]	sGA [%]	BOA [%]	MuPPetS [%]
Dec. func.	95.00	51.67	49.83	90.00	<b>96.00</b>
Knap sack	<b>100.00</b>	<b>100.00</b>	99.95	98.89	99.93
MAX-2SAT	<b>100.00</b>	99.98	99.77	85.96	99.77

To confirm the statistical significance of solution quality differences the Wilcoxon statistical test was used. The p-values reported by this test are presented in Table 11. The tests were performed on the results of all runs, rounded if necessary, to the sixth most significant position. The results obtained confirm the analysis presented above – GePIM significantly outperforms all competing methods except MuPPetS for the deceptive functions concatenations case.

In Figure 2 the dependency between the computation time and FFE for all competing methods is presented. The comparison is done for deceptive functions concatenations test cases (80 runs per method). Similar comparisons may be found in (Cai and Wang, 2015; Kwasnicka and Przewozniczek, 2011; Przewozniczek et al., 2015; Saha et al., 2010, Suganthan et al., 2005).

Table 11: The solution quality comparisons on the base of p-values reported by Wilcoxon test.

Null hypothesis: GePIM...		better or equal	worse or equal	equal
Classical IM	DF	<b>1</b>	0	0
	KS	0.50	0.50	<b>1</b>
	M2S	<b>0.85</b>	0.16	0.33
sGA	DF	<b>1</b>	0	0
	KS	<b>1</b>	0	0
	M2S	<b>1</b>	0	0
BOA	DF	0.59	0.41	<b>0.83</b>
	KS	<b>0.99</b>	0	0
	M2S	<b>1</b>	0	0
MuPPetS	DF	0.24	<b>0.76</b>	0.49
	KS	<b>1</b>	0	0
	M2S	<b>1</b>	0	0

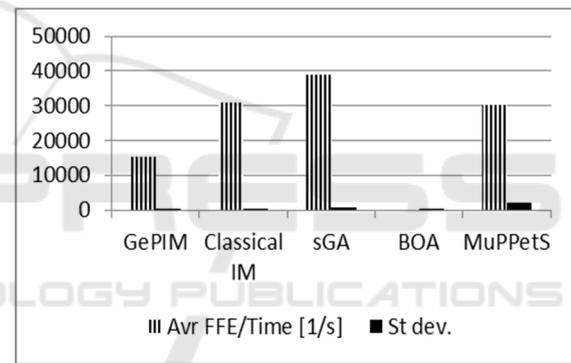


Figure 2: Average FFE per second for deceptive functions concatenations.

As shown in Figure 2 the dependency between FFE and computation time is highly repeatable – the standard deviation value is low. The sGA is capable of doing the highest FFE number per second. This is an expected result since sGA does not do many computation load consuming operations other than fitness calculation. The high number of FFE per second is also understandable for MuPPetS and Classical IM. GePIM does about 50% of FFE done by Classical IM. This difference is a result of frequent migrations, which require many sorting operations. Finally, the average number of FFE per second for BOA is about 30 times lower than in the GePIM case, which confirms the previous reasoning that the FFE number is not a suitable computation load measure for BOA.

## 5 CONCLUSIONS AND FURTHER WORK

In this paper, the proposition of the Linked Gene Groups Migration for Island Models was presented. The GePIM method, an IM using the proposed LGGM was shown to be an effective tool when compared to other evolutionary methods. Despite its simplicity, GePIM was able to compete successfully with MuPPetS and BOA methods.

The main fields that should be concerned in the future work are as follows:

- the application of GePIM to other problems than those considered in this paper,
- further LGGM development,
- employing in GePIM other LL techniques than used in this paper,
- combining the LGGM, the LL and dynamic subpopulation number control (Przewozniczek, 2016).

The further research in the above directions should allow proposing new and more effective evolutionary methods.

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