Classification Confusion within NEFCLASS Caused by Feature Value Skewness in Multi-dimensional Datasets

Jamileh Yousefi[1] and Andrew Hamilton-Wright[1,2]
[1]*School of Computer Science (SOCS), University of Guelph, Guelph, Ontario, Canada*
[2]*Department of Mathematics and Computer Science, Mount Allison University, Sackville, NB, Canada*

Keywords: Fuzzy, Discretization, Neurofuzzy, Classification, Skewness, NEFCLASS.

Abstract: This paper presents a model for the treatment of skewness effects on the accuracy of the NEFCLASS classifier by changing the embedded discretization method within the classifier. NEFCLASS is a common example of the construction of a neuro-fuzzy system. The popular NEFCLASS classifier exhibits surprising behaviour when the feature values of the training and testing data sets exhibit significant skew. As skewed feature values are commonly observed in biological data sets, this is a topic that is of interest in terms of the applicability of such a classifier to these types of problems. From this study it is clear that the effect of skewness on classification accuracy is significant, and that this must be considered in work dealing with skewed data distributions. We compared accuracy of NEFCLASS classifier with two modified versions of NEFCLASS embedded with MME and CAIM discretization methods. From this study it is found the CAIM and MME discretization methods results in greater improvements in the classification accuracy of NEFCLASS as compared to using the original EQUAL-WIDTH technique NEFCLASS uses by default.

1 INTRODUCTION

Data distributions in machine learning, when they are discussed at all, are generally expected to have a symmetric distribution, with a central tendency, if not actually being normally distributed. When the feature values of data are skewed, however, issues arise having to do with the relative scarcity of sample values in the tails of the distribution relative to the abundance of data near the median.

Several studies (Chittineni and Bhogapathi, 2012; Liu et al., 2008; Mansoori et al., 2007; Tang and Chiu, 2004; Au et al., 2006; Peker, 2011; Changyong et al., 2014; Qiang and Guillermo, 2015) have examined the question of transforming the data when incorporating the distribution of input data into the classification system in order to more closely approximate normally distributed data. Very few studies (Mansoori et al., 2007; Liu et al., 2008; Zadkarami and Rowhani, 2010; Hubert and Van der Veeken, 2010) addressed these issues while focusing on some alternative to a data transformation approach.

Data transformation is a common preprocessing step to treat the skewness and improve dataset normality. However, in the biological and biomedical domain, data transformation interferes with the transparency of the decision making process, and can lead to the exclusion of important information from the decision making process, and affect the system's ability to correctly classify the case. Therefore, rather than transformation of the data to achieve a more normally distributed input, in this paper we directly investigate and report the NEFCLASS classifier's behaviour when dealing with distribution of data with skewed feature values.

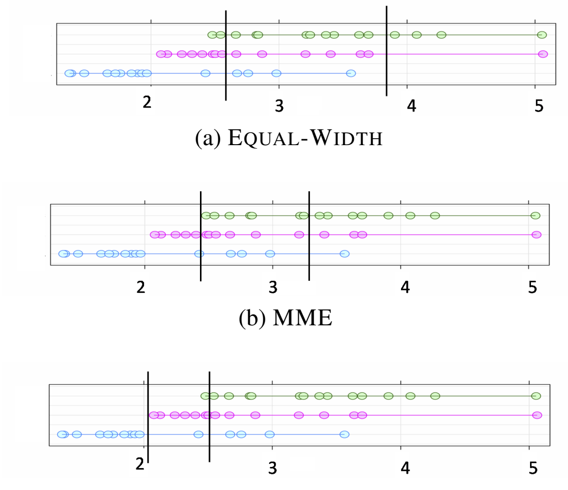
The choice of discretization technique is known to be one of the important factors that might affect the classification accuracy of a classifier. NEFCLASS classifiers use an EQUAL-WIDTH discretization method to divide the observed range of continuous values for a given feature into equally sized fuzzy intervals, overlapping by half of the interval width. EQUAL-WIDTH discretization does not take the class information into account, which, as we show here, results in a lower classification accuracy for NEFCLASS classifier than other techniques, especially when the feature values of the training and testing data sets exhibit significant skew. Dealing with skewness without performing a transformation will provide greater clarity in interpretation, and by extension better classification transparency, as the projection incurred by the transformation does not need to be taken into account

Yousefi, J. and Hamilton-Wright, A.
Classification Confusion within NEFCLASS Caused by Feature Value Skewness in Multi-dimensional Datasets.
DOI: 10.5220/0006033800210029
In *Proceedings of the 8th International Joint Conference on Computational Intelligence (IJCCI 2016) - Volume 2: FCTA*, pages 21-29
ISBN: 978-989-758-201-1

in interpretation.

We provide a study based on an easily reproducible synthetic data distributions, in order to allow deeper insights into the data analysis. We argue that the skewed data, in terms of feature value distribution, cause a higher misclassification rate in classification learning algorithms. We further argue that distribution sensitive discretization methods such as CAIM and MME result in greater improvements in the classification accuracy of the NEFCLASS classifier as compared to using the original EQUAL-WIDTH technique.

The next section of this paper contains a short review of the NEFCLASS classifier and three discretization methods that will be used to perform the classification task is presented. Section 3 describes the methodology of our study, and in section 4 the experimental results and statistical analysis are given. Finally, conclusions are presented.



(a) EQUAL-WIDTH

(b) MME

(c) CAIM

Figure 1: Three discretization techniques result in different intervals produced on the same three-class data set.

2 BACKGROUND

2.1 Discretization

A discretization process divides a continuous numerical range into a number of covering intervals where data falling into each discretized interval is treated as being describable by the same nominal value in a reduced complexity discrete event space. In fuzzy work, such intervals are then typically associated with the support of fuzzy sets, and the precise placement in the interval is mapped to the degree of membership in such a set.

Discretization methods are categorized into supervised and unsupervised algorithms. EQUAL-WIDTH intervals (Chemielewski and Grzymala-Busse, 1996), EQUAL-FREQUENCY intervals, such as *k*-means clustering (Monti and Cooper, 1999), and Marginal Maximum Entropy (Chau, 2001; Gokhale, 1999) are examples of algorithms for unsupervised discretization. Maximum entropy (Bertoluzza and Forte, 1985), ChiMerge (Kerber, 1992), CAIM (Kurgan and Cios, 2004), and URCAIM (Cano et al., 2016) are some examples of supervised algorithms that take class label assignment in a training data set into account when constructing discretization intervals.

In the following discussion, the three discretization methods, that we have chosen for the experiments are described. To demonstrate the partitioning we used a skewed dataset with 45 training instances and three classes, with sample discretizations shown Fig. 1. The subfigures within Fig. 1 each show the same data, with the green, red and blue rows of dots (top, middle and bottom) within each figure describing the data for each class in the training data.

2.1.1 EQUAL-WIDTH

The EQUAL-WIDTH discretization algorithm divides the observed range of continuous values for a given feature into a set number of equally sized intervals, providing a simple mapping of the input space that is created independent of both the distribution of class and of the density of feature values within the input space (Kerber, 1992; Chemielewski and Grzymala-Busse, 1996).

Fig. 1a demonstrates the partitioning using EQUAL-WIDTH intervals; note that there is no clear relation between classes and intervals, and that the intervals shown have different numbers of data points within each (21, 19 and 5 in this case).

2.1.2 MME

Marginal Maximum Entropy, or MME, based discretization (Chau, 2001; Gokhale, 1999) divides the dataset into a number of intervals for each feature, where the number of points is equal for all of the intervals, under the assumption that the information of each interval is expected to be equal. Fig. 1b shows the MME intervals for the example three-class dataset. Note that the intervals in Fig. 1b do not cover the same fraction of the range of values (*i.e.*, the widths differ), being the most dense in regions where there are more points. The same number of points (15) occur in each interval.

2.1.3 CAIM

CAIM (class-attribute interdependence maximization) discretizes the observed range of a feature into a class-based number of intervals and maximizes the inter-dependency between class and feature values (Kurgan and Cios, 2004). CAIM discretization algorithm divides the data space into partitions, which leads to preserve the distribution of the original data (Kurgan and Cios, 2004), and obtain decisions less biased to the training data.

Fig. 1c shows the three CAIM intervals for our sample data set. Note how the placement of the discretization boundaries is closely related to the points where the densest portion of the data for a particular class begins or ends.

2.2 NEFCLASS Classifier

NEFCLASS (Nauck et al., 1996; Nauck and Kruse, 1998; Klose et al., 1999) is a neuro-fuzzy classifier that tunes a set of membership functions that describe input data and associates these through a rule set with a set of potential class labels. Training is done in a supervised fashion based on a training data set.

Fig. 2 shows a NEFCLASS model that classifies input data with two features into two output classes by using three fuzzy sets and two fuzzy rules. Input features are supplied to the nodes at the bottom of the figure. These are then fuzzified, using a number of fuzzy sets. The sets used by a given rule are indicated by linkages between input nodes and rule nodes. If the same fuzzy set is used by multiple rules, these links are shown passing through an oval, such as the one marked “large” in Fig. 2. Rules directly imply an output classification, so these are shown by unweighted connections associating a rule with a given class. Multiple rules may support the same class, however that is not shown in this diagram. Rule weightings are computed based on the degree of association between an input fuzzy membership function and the rule (calculated based on the degree of association in the training data), as well as the level of activation of the fuzzy membership function, as is typical in a fuzzy system (Mendel, 2001, Chapter 1).

In Fig. 3a, a set of initial fuzzy membership functions describing regions of the input space are shown, here for a two-dimensional problem in which the fuzzy sets are based on the initial discretization produced by the EQUAL-WIDTH algorithm. As will be demonstrated, NEFCLASS functions work best when these regions describe regions specific to each intended output class, as is shown here, and as is described in the presentation of a similar figure in the classic work describing this classifier (Nauck et al., 1996, pp. 239).

As is described in the NEFCLASS overview pa-

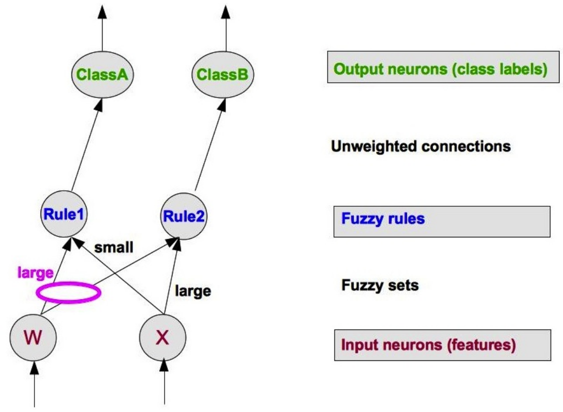
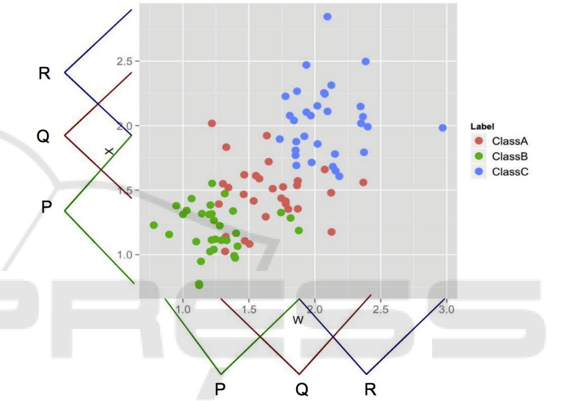
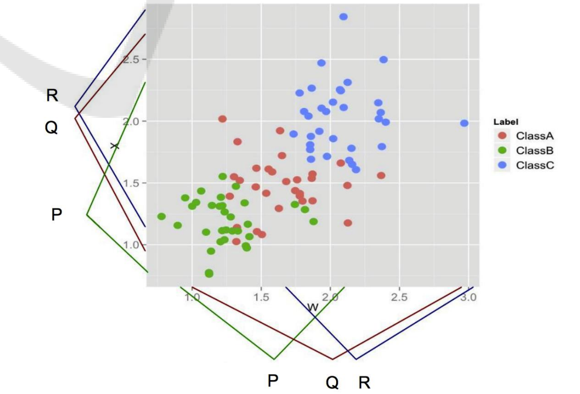


Figure 2: A NEFCLASS model with two inputs, two rules, and two output classes.



(a) Initial fuzzy set membership functions in NEFCLASS, produced using EQUAL-WIDTH discretization



(b) Results of tuning the above membership functions to better represent class/membership function information

Figure 3: Fuzzy membership functions before and after training data based tuning using the NEFCLASS algorithm.

per (Nauck and Kruse, 1998, pp. 184), a relationship is constructed through training data based tuning to maximize the association of the support of a single fuzzy set with a single outcome class. This implies both that the number of fuzzy sets must match the number of outcome classes exactly, and in addition, that there is an assumption that overlapping classes will drive the fuzzy sets to overlap as well.

Fig. 3a shows the input membership functions as they exist before membership function tuning, when the input space is partitioned into EQUAL-WIDTH fuzzy intervals.

Fig. 3b demonstrates that during the fuzzy set tuning process, the membership function is shifted and the support is reduced or enlarged, in order to better match the coverage of the data points belonging to the associated class, however as we will see later, this process is strongly informed by the initial conditions set up by the discretization to produce the initial fuzzy membership functions.

3 METHODOLOGY

This paper has two objectives. The first is to characterize how the NEFCLASS classification accuracy degrades as data skewness increases. The second is to evaluate alternative discretization methods to counteract the performance problems in skewed data domains. To support this second goal we will evaluate maximum marginal entropy (MME) (Chau, 2001; Gokhale, 1999) and the supervised, class-dependent discretization method CAIM (Kurgan and Cios, 2004).

We carried out two different set of experiments. In the first experiment, denoted as the effect of skewness of feature values, we evaluate unmodified NEFCLASS behaviour when dealing with different level of skewness.

In the second experiment, denoted as the effect of discretization, we investigate the classification accuracy of a modified NEFCLASS classifier upon employing the three different discretization techniques. The MME and CAIM methods are not part of the standard NEFCLASS implementation, therefore we implemented two modified versions of NEFCLASS classifier, each utilizing one of these two discretization methods. Experiments were then performed on synthesized datasets with different levels of feature values skewness.

Results from the experiments are presented in terms of misclassification rates, which are equal to the number of misclassified data instances divided by the total number of instances in the testing dataset.

3.1 Datasets

Three synthesized datasets were used for experiments. All the synthesized data sets used describe classification problems within a 4-dimensional data space containing distributions of data from three separate classes.

Our data was produced by randomly generating numbers following the F-distribution with different degrees of freedom chosen to control skew. The F-distribution (Natrella, 2003) has been chosen as the synthesis model because the degree of skew within an F-distribution is controlled by the pairs of degrees of freedom specified as a pair of distribution control parameters. This allows for a spectrum of skewed data distributions to be constructed. We designed the datasets to present different levels of skewness with increasing skew levels. Three pairs of degrees of freedom parameters have been used to generate datasets with different levels of skewness, including low, medium, and high-skewed feature values. After initial experiments datasets with degrees of freedom $(100, 100)$ was chosen to provide data close to a normal distribution, $(100, 20)$ provides moderate skew, and $(35, 8)$ provides high skew.

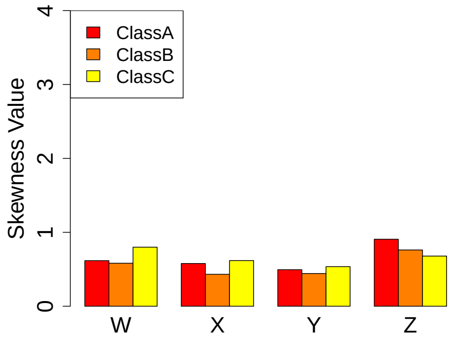
A synthesized data set consisting of 1000 randomly generated examples consisting of four-feature (W, X, Y, Z) F-distribution data for each of three classes was created. The three classes (ClassA, ClassB and ClassC) overlap, and are skewed in the same direction. The size of datasets were designed to explore the effect of skewness when enough data is available to clearly ascertain data set properties. Ten-fold cross validation was used to divide each dataset into training (2700) and testing (300 point) sets in which an equal number of each class is represented. We have taken care to ensure that all datasets used have a similar degree of overlap, and same degree of variability.

Fig. 4 shows the skewness of each dataset for each feature. From these figures one can see that the LOW-100,100 data is relatively symmetric, while the MED-100,20 and HIGH-35,8 data show an increasing, and ultimately quite dramatic, skew.

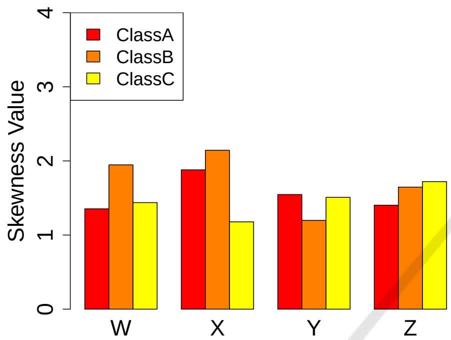
4 RESULTS AND DISCUSSION

4.1 The Effect of Skewness on Classification Accuracy

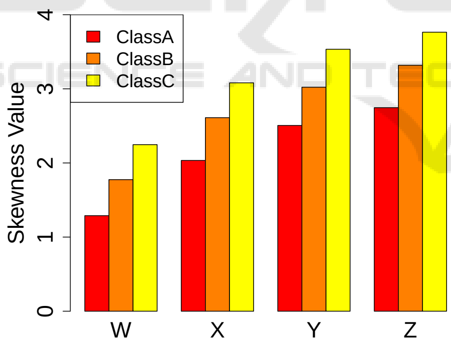
In this section, we discuss how feature value skewness affects the classification accuracy of the NEFCLASS



(a) LOW-100,100



(b) MED-100,20



(c) HIGH-35,8

Figure 4: Skewness by label and feature for the three synthetic datasets.

classifier. We begin by applying the NEFCLASS classifier to the synthesized datasets with our three different levels of skewness: low, medium, and high. Our null hypothesis is that the misclassification rate observed for NEFCLASS are equal in all three datasets.

As mentioned earlier, NEFCLASS will attempt to tune the fuzzy membership functions provided by the initial EQUAL-WIDTH discretization to associate the support of each fuzzy set unambiguously with a single class. As our skewed data sets overlap substantially, this is a difficult task for the NEFCLASS classifier.

Figs. 5 and 6 show the relationship between the distribution of input data values in feature X, and the placement of the final membership function by the NEFCLASS classifier when using the EQUAL-WIDTH discretization method for the three datasets LOW-100,100, MED-100,20 and HIGH-35,8.

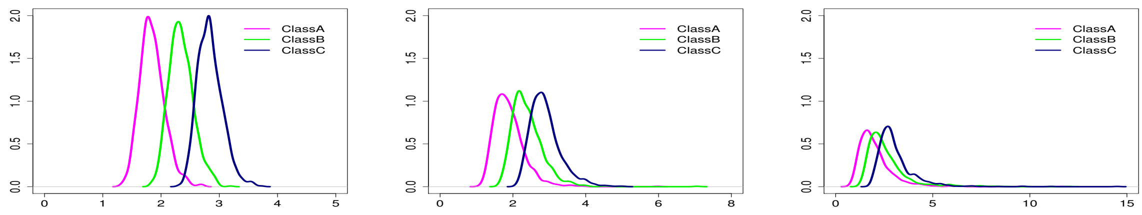
Figs. 5a, 5b, and 5c illustrate the density functions for the LOW-100,100, MED-100,20 and HIGH-35,8 data sets, respectively. As can be seen in 5a, the feature values in this case are centrally tended and generally symmetric, providing an approximation of a Normal distribution, though observable skew is still present, as shown in Fig. 4a. In Fig. 5b, tails are observable as the mean of feature values are pulled towards the right while the median values remain similar to those in 5a, for a moderately skewed distribution. In Fig. 5c, the feature values exhibit longer tails than normal, indicating a highly skewed distribution.

Fig. 5 and 6 allows comparison between a data distribution and the representation of fuzzy sets for this feature in this distribution, displayed immediately below. For example, Fig. 5a can be compared with Fig. 6a. Similar vertical comparisons are able to be made for Figs. 5b and 5c.

Fig. 6a illustrates the placement of initial fuzzy sets and final membership functions with the EQUAL-WIDTH discretization method for dataset LOW-100,100. As can be seen, the same number of fuzzy sets with equal support are constructed, and after tuning the membership functions are shifted and the supports are reduced or enlarged, in order to better match the distribution of class-specific feature values. The dotted line indicates the initial fuzzy sets and the solid line indicates the final placement of the fuzzy sets.

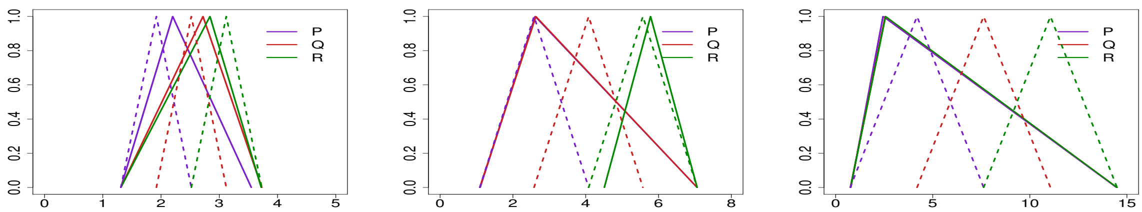
Fuzzy sets have been given the names "P", "Q" and "R" in these figures, rather than more traditional linguistic labels such as "small", "medium" and "large" because of the placement associations with the classes, and the assignment of class names to fuzzy sets lies within the NEFCLASS training algorithm, and is not under user control. For this reason, it is not possible to assign names *a priori* that have any linguistic meaning. The choices NEFCLASS makes in terms of which fuzzy set is used to represent a particular class is part of the underlying performance issue explored in this paper, as will be shown.

As one can see in comparing Fig. 6a with 5a, NEFCLASS has chosen to associate fuzzy set P with ClassB, set Q with ClassA, and fuzzy set R with ClassC based on this EQUAL-WIDTH initial discretization. The associations are not as clear with data exhibiting higher skew, as shown in Figs. 6b and 6c, in which NEFCLASS is attempting to set two, and then three, fuzzy membership functions to essentially the



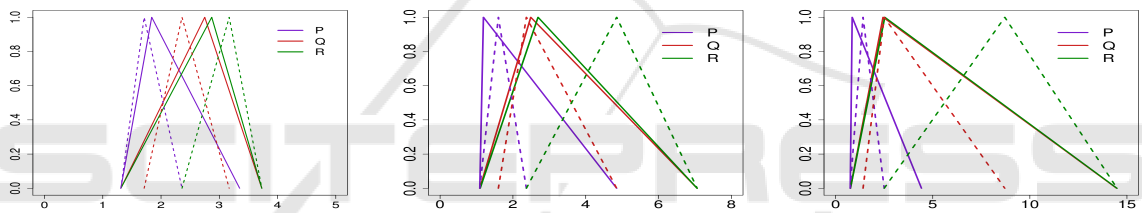
(a) Low-100,100 (b) Med-100,20 (c) High-35,8

Figure 5: Density plots for feature X for all datasets.



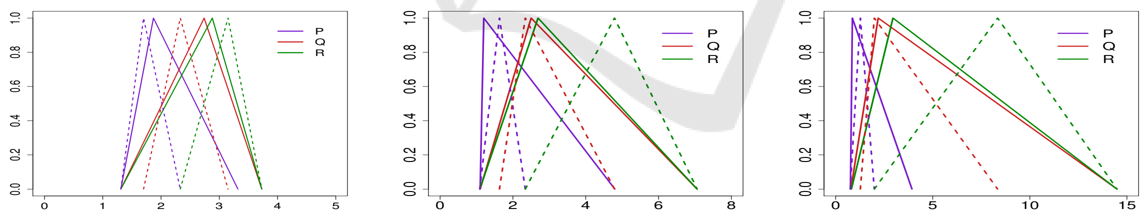
(a) Low-100,100 (b) Med-100,20 (c) High-35,8

Figure 6: Fuzzy Sets and Membership Functions constructed by Equal-Width for feature X of all datasets.



(a) Low-100,100 (b) Med-100,20 (c) High-35,8

Figure 7: Fuzzy Sets and Membership Functions constructed by MME for feature X of all datasets.



(a) Low-100,100 (b) Med-100,20 (c) High-35,8

Figure 8: Fuzzy Sets and Membership Functions constructed by CAIM for feature X of all datasets.

same support and with the same central peak. This, of course, limits or entirely destroys the information available to the rule based portion of the system, and is therefore unsurprisingly correlated with a higher number of classification errors.

The analysis of fuzzy sets and memberships for feature X in dataset Low-100,100 shows that all of three fuzzy sets are expanded in terms of their support. As shown in Fig. 6a, the support of fuzzy sets Q and R share identical support, as defined by the observed range of the training data. In addition, sets Q and R are defined by a nearly identical maximum point, rendering the distinction between them moot. Similarly, in Fig. 6b, we observe identical maxima and support for sets P and Q, however R is set apart with a different, but substantially overlapping, support and a clearly distinct maxima.

This confusion gets only more pronounced as the degree of skew increases, as is clear in Fig. 6c, representing the fuzzy sets produced for High-35,8, in which all fuzzy sets P, Q and R share identical support defined by the range of the observed data. In

Table 1: Mean and standard deviation of misclassification rates and median of number of rules for each classifier trained on three synthesized dataset.

	Discretization	Dataset		
		LOW-100,100	MED-100,20	HIGH-35,8
Mean and SD for Misclassification Rates	EQUAL-WIDTH	22.63 ± 1.07	65.30 ± 3.38	63.07 ± 6.77
	MME	25.67 ± 1.85	33.73 ± 1.64	42.30 ± 2.78
	CAIM	24.13 ± 2.72	34.30 ± 1.52	42.37 ± 3.08
Median for Number of Rules	EQUAL-WIDTH	49.00	34.50	15.00
	MME	44.00	50.00	46.00
	CAIM	53.50	51.50	45.00

Table 2: M-W-W results comparing the misclassification rates based on level of skew.

Discretization	LOW-100,100 *vs.* MED-100,20	LOW-100,100 *vs.* HIGH-35,8	MED-100,20 *vs.* HIGH-35,8
EQUAL-WIDTH	$2.9 \times 10^{-11}*$	$2.9 \times 10^{-11}*$	.9300
MME	$2.9 \times 10^{-11}*$	$2.9 \times 10^{-11}*$	.0001*
CAIM	$2.9 \times 10^{-11}*$	$2.9 \times 10^{-11}*$	.0001*

* significant at 99.9% confidence ($p < .001$)

addition, all the fuzzy sets in this example are defined by a nearly identical maximum point.

It is perhaps surprising that difference in the proportion of data in the tails of the distribution are not represented more directly here, however this is largely due to the fact that the EQUAL-WIDTH discretization technique is insensitive to data density, concentrating instead purely on range.

Table 1 reports the misclassification rates (as mean $\pm$ standard deviation), as well as the median number of fuzzy rules obtained by each classifier, using each discretization technique, and for each dataset. The results have been calculated over the 10 cross-validation trials. As shown in Table 1, LOW-100,100 achieved the lowest misclassification rate and the lowest variability. The results also show considerably larger variability in the misclassification rate of HIGH-35,8, compared to MED-100,20. The decrease in the number of rules produced using the EQUAL-WIDTH method shows that less information is being captured about the data set as skewness increases. As there is no reason to assume that less information will be required to make a successful classification, this decrease in the number of rules is therefore an contributing cause for the increase in the misclassification rate noted for EQUAL-WIDTH in Figs 9a.

Fig. 9 shows a graphical summary of the differences between the misclassification rates and number of rules observed for each discretization method.

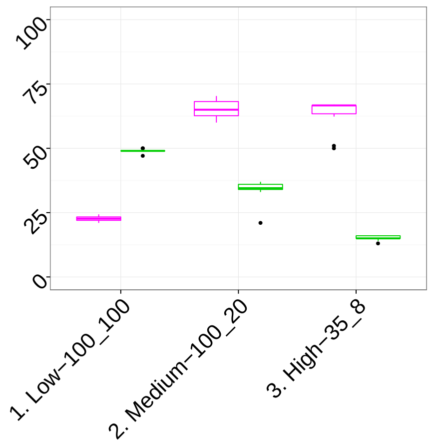
An exploration of the normality of the distribution of misclassification rates using a Shapiro-Wilks test found that a non-parametric test was appropriate in all cases. To explore the statistical validity of the differences between observed misclassification rates of NEFCLASS classifiers for different data sets and using different discretization techniques, Mann-Whitney-Wilcoxon (M-W-W) tests were performed.

By running a M-W-W test on the misclassification rates for each pair of skewed data sets, the results shown in Table 2 were obtained. The M-W-W test results shows that there is a significant difference for almost all levels of skewness, the only exceptions being the MED-100,20 and HIGH-35,8 distributions when the EQUAL-WIDTH discretization method has been used. We therefore reject the null hypothesis and conclude that the NEFCLASS classification accuracy was significantly affected by feature value skewness in the majority of cases.

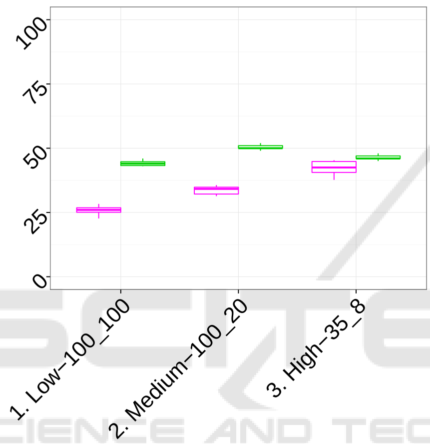
4.2 The Effect of Discretization Methods

In this section, we investigate how the choice of discretization method affects the classification accuracy of a NEFCLASS based classifier, when dealing with datasets with various degrees of skew. We compare the results for our new NEFCLASS implementations using MME and CAIM with the results of the default EQUAL-WIDTH discretization strategy. The null hypothesis is that there will be no difference in the observed misclassification rates.

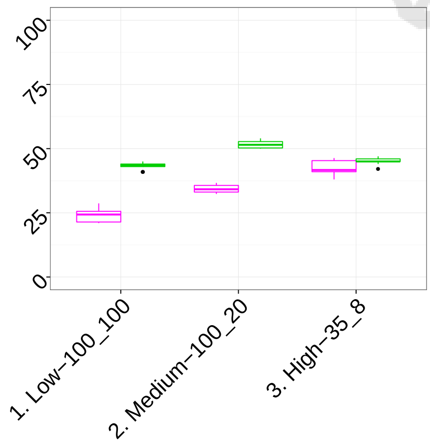
Fig. 7 illustrates the placement of initial fuzzy sets and final membership functions when using the MME discretization method for each of the three datasets. As can be seen in this figure, the same number of



(a) EQUAL-WIDTH



(b) MME



(c) CAIM

Figure 9: Summary of misclassification rates and number of rules for EQUAL-WIDTH, MME and CAIM.

fuzzy sets with unequal support are constructed. Similarly, in Fig. 8, the fuzzy membership functions produced using CAIM discretization are shown. The CAIM algorithm constructed three fuzzy sets for all features for our test data, however as the algorithm is free to choose a differing number of discretization intervals based on the observed data (Kurgan and Cios, 2004), for other applications this may vary from feature to feature. All fuzzy sets generated had distinct support, range, and a different centre value defining the triangular membership functions, though there was still a strong degree of similarity between some sets, as shown in the representative data displayed in Fig. 8.

As shown in Figs. 7 and 8, the final placement of support for fuzzy the first membership function, P, is different from the support for Q and R, in contrast with the very similar fuzzy set definitions of EQUAL-WIDTH shown in Fig. 6. This will preserve a greater degree of differentiation between the information captured in each fuzzy set. A comparison between the results for MME and CAIM for HIGH-35,8 (not shown graphically in the paper) indicated that the choice of the triangular membership functions produced by CAIM for HIGH-35,8 were slightly different, but the choice of the triangular membership functions produced by MME were nearly identical. As the difference between the support of the fuzzy membership functions and the centre points increases, the learning phase is more able to create meaningful new rules. This therefore leads to a lower number of classification errors. The larger number of rules generated by MME and CAIM for MED-100,20 and HIGH-35,8 is summarized in Table 1.

Table 3: M-W-W results comparing the misclassification rates based on discretization method.

Dataset	EQUAL-WIDTH *vs.* MME	EQUAL-WIDTH *vs.* CAIM	MME *vs.* CAIM
LOW-100,100	.0020∗	.2500	.1600
MED-100,20	.0001∗	.0001∗	.5000
HIGH-35,8	.0001∗	.0001∗	.9100

* significant at 99.9% confidence ($p < .001$)

As shown in Table 3, M-W-W results identify the significance of the difference in misclassification rates for EQUAL-WIDTH versus MME and CAIM at medium and high skew, where very low p values are computed. In the case of LOW-100,100, a significant difference is observed when comparing EQUAL-WIDTH to MME, however with a significantly greater p value (.002). Note that there is no significant difference in misclassification performance between MME and CAIM.

5 CONCLUSIONS

The results of this study indicate that the NEFCLASS classifier performs increasingly poorly as data feature value skewness increases. Further, this study indicates that the choice of initial discretization method affects the classification accuracy of NEFCLASS classifier, and that this effect is very strong in skewed data sets. Utilizing MME or CAIM discretization methods in the NEFCLASS classifier improved classifications accuracy.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the support of NSERC, the National Sciences and Engineering Research Council of Canada, for ongoing grant support.

REFERENCES

Au, W., Chan, K., and Wong, A. (2006). A fuzzy approach to partitioning continues attributes for classification. *IEEE Transactions on Knowledge and Data Engineering*, 18:715–719.

Bertoluzza, C. and Forte, B. (1985). Mutual dependence of random variables and maximum discretized entropy. *The Annals of Probability*, 13(2):630–637.

Cano, A., T., N. D., Ventura, S., and Cios, K. J. (2016). urcaim: improved caim discretization for unbalanced and balanced data. *Soft Computing*, 33:173–188.

Changyong, F., Hongyue, W., Naiji, L., Tian, C., Hua, H., Ying, L., and Xin, M. (2014). Log-transformation and its implications for data analysis. *Shanghai Arch Psychiatry*, 26(2):105–109.

Chau, T. (2001). Marginal maximum entropy partitioning yields asymptotically consistent probability density functions. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 23(4):414–417.

Chemielewski, M. R. and Grzymala-Busse, J. W. (1996). Global discretization of continuous attributes as preprocessing for machine learning. *International Journal of Approximate Reasoning*, 15:319–331.

Chittineni, S. and Bhogapathi, R. B. (2012). A study on the behavior of a neural network for grouping the data. *International Journal of Computer Science*, 9(1):228–234.

Gokhale, D. V. (1999). On joint and conditional entropies. *Entropy*, 1(2):21–24.

Hubert, M. and Van der Veeken, S. (2010). Robust classification for skewed data. *Advances in Data Analysis and Classification*, 4:239–254.

Kerber, R. (1992). ChiMerge discretization of numeric attributes. In *Proceedings of AAAI-92*, pages 123–12, San Jose Convention Center, San Jose, California.

Klose, A., Nürnberger, A., and Nauck, D. (1999). Improved NEFCLASS pruning techniques applied to a real world domain. In *Proceedings Neuronale Netze in der Anwendung*, University of Magdeburg. NN'99.

Kurgan, L. A. and Cios, K. (2004). CAIM discretization algorithm. *IEEE Transactions on Knowledge and Data Engineering*, 16:145–153.

Liu, Y., Liu, X., and Su, Z. (2008). A new fuzzy approach for handling class labels in canonical correlation analysis. *Neurocomputing*, 71:1785–1740.

Mansoori, E., Zolghadri, M., and Katebi, S. (2007). A weighting function for improving fuzzy classification systems performance. *Fuzzy Sets and Systems*, 158:588–591.

Mendel, J. M. (2001). *Uncertain Rule-Based Fuzzy Logic Systems*. Prentice-Hall.

Monti, S. and Cooper, G. (1999). A latent variable model for multivariate discretization. In *The Seventh International Workshop on Artificial Intelligence and Statistics*, pages 249–254, Fort Lauderdale, FL.

Natrella, M. (2003). *NIST SEMATECH eHandbook of Statistical Methods*. NIST.

Nauck, D., Klawonn, F., and Kruse, R. (1996). *Neuro-Fuzzy Systems*. John Wiley and Sons Inc., New York.

Nauck, D. and Kruse, R. (1998). NEFCLASS-X – a soft computing tool to build readable fuzzy classifiers. *BT Technology Journal*, 16(3):180–190.

Peker, N. E. S. (2011). Exponential membership function evaluation based on frequency. *Asian Journal of Mathematics and Statistics*, 4:8–20.

Qiang, Q. and Guillermo, S. (2015). Learning transformations for clustering and classification. *Journal of Machine Learning Research*, 16:187–225.

Tang, Y. and Chiu, C. (2004). Function approximation via particular input space partition and region-based exponential membership functions. *Fuzzy Sets and Systems*, 142:267–291.

Zadkarami, M. R. and Rowhani, M. (2010). Application of skew-normal in classification of satellite image. *Journal of Data Science*, 8:597–606.