Multiobjective Adaptive Wind Driven Optimization

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Abstract: In this work, we introduce a new nature-inspired multiobjective numerical optimization algorithm where Pareto dominance is incorporated into Adaptive Wind Driven Optimization for handling multiobjective optimization problems and named as Multiobjective Adaptive Wind Driven Optimization (MO-AWDO) method. This new approach utilizes an external repository of air parcels to record the non-dominated Pareto-fronts found at each iteration via the fast non-dominated sorting algorithm, which are then utilized in the velocity update equation of the AWDO for the next iteration. The performance of the MO-AWDO is tested on five different numerical test functions with two objectives and results indicate that the MO-AWDO offers a very competitive approach compared to well-known methods in the published literature even performing better than NSGA-II for ZDT4 test function.

1 INTRODUCTION

Evolutionary algorithms (EA) and nature-inspired optimization methods like Genetic Algorithms (GA), Particle Swarm Optimization (PSO), Ant Colony Optimization (ACO), etc. were successfully utilized since their introduction to the literature as single objective optimization algorithms. To handle multi-objective functions, variants of these algorithms were proposed and they were shown to be very effective (Deb, 2002; Zitzler, 2000; Coello, 2004; Coello, 2007). The primary goal of these multi-objective optimization algorithms is to identify the Pareto-optimal front solutions as diversely as possible. To achieve this, different methods were proposed such as archiving the solutions over iterations, preserving elitism, implementing crowding distance, utilizing adaptive grids, introducing new operators into existing methods or hybridization of multiple EAs and many others.

In this work, we are introducing a new population based multi-objective optimization method, where Pareto dominance is incorporated into the Adaptive Wind Driven Optimization. At each iteration Pareto-fronts are identified using the fast non-dominated sorting algorithm and stored in an external population. At each iteration, each particle utilizes one of the randomly selected members of the external repository to update its velocity vector and then the position of the particle is updated accordingly. Such an external population provides a diverse set of solutions on the non-dominated Pareto-front that the rest of the population can utilize to follow and to update their location on the search domain.

The rest of this paper is structured as follows. The second section introduces the Wind Driven Optimization (WDO) (Bayraktar, 2010) algorithm and discusses the update equations. The third section describes the Adaptive WDO (AWDO) technique (Bayraktar, 2015) and the fourth section describes the newly introduced multiobjective AWDO algorithm (MO-AWDO) in detail. The fifth section demonstrates the efficient implementation of the MO-AWDO on five numerical benchmark functions. Conclusions and recommendations for future work are presented in the last two sections.

2 WIND DRIVEN OPTIMIZATION

The Wind Driven Optimization (WDO) algorithm was first introduced in (Bayraktar, 2010) as an efficient population-based and nature-inspired global

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optimization algorithm. The WDO is inspired by the motion of wind in atmosphere and is derived from the atmospheric dynamics equations in hydrostatic equilibrium (Bayraktar, 2013). The movement of wind, or in other words, the movement of an infinitesimally small air parcel in wind, can be explained via Euler description, where one can derive the position and velocity of the air parcel from various forces that are exerted on the air parcel by utilizing Newton's second law of motion. While the WDO algorithm tries to stay true to the real physical equations, certain assumptions and simplifications are made to achieve an efficient numerical optimization algorithm mapped to a search space with *N*-dimensions. Details of the WDO can be found in (Bayraktar, 2013), hence we will only briefly describe the position and velocity update equations below:

$$u_n = (1-\alpha)u_c - g x_c + \left|\frac{i-1}{i}\right| RT(x_{max} - x_c) + \frac{c * u_c^{otherd}}{i} \quad (1)$$

where u_n is the updated velocity for next iteration and, u_c is the velocity at the current iteration. The x_c term represents the current position of the air parcel in the search space and x_{max} represents the best position found so far during the search. u_c^{otherd} is velocity at another dimension affecting the velocity update in dimension, *d*. Air parcels are ranked by their pressure value, i.e. cost function value, among themselves, where *i* represents the rank of the air parcels within the population. Let us call this ranking as the population ranking. Low-pressure value, i.e. low cost, indicates a good solution and high-pressure value indicates a bad solution. Other terms in equation 1 are the inherent coefficients of the classical WDO algorithm and are preset by the user, which allow users to tune them if needed (Bayraktar, 2013). These terms are friction coefficient, α, gravitational constant, *g*, Coriolis constant, *c*, to represent the rotation of the Earth, universal gas constant, *R*, and temperature, *T*, which can be combined into single coefficient term of *RT*. Air parcels' position is bounded within the range of [-1, 1] before the position vector is linearly scaled to the upper and lower bounds of the optimization problem. Updated velocity is limited to a value of $V_{max} = \pm |0.5|$, if it becomes larger than the V_{max}.

After the new velocity, u_n, is computed the position is updated by the position update equations:

$$x_n = x_c + u_n \times \Delta t \quad (2)$$

where x_n is the updated position of an air parcel, that is the sum of the current position vector, x_c, and updated velocity, u_n, with the assumption that time step is set to unity, $\Delta t = 1$. Using equations 1 and 2, the position of the air parcel changes at each iteration on the search domain. The WDO algorithm terminates either when a predetermined level of pressure value is achieved or when the maximum number of iterations is exhausted.

3 ADAPTIVE WIND DRIVEN OPTIMIZATION

The inherent terms of the velocity update equations in the classical WDO, namely, *α, g, c, and RT*, must be determined by the user, which provides the flexibility to tune the algorithm performance per optimization problem at hand. A numerical study is conducted in (Bayraktar, 2013) to recommend the best value ranges for these terms. However, such flexibility brings a challenge to novice users and selecting the most appropriate values for the inherent terms becomes a burden. To eliminate algorithms dependency on user input, Adaptive Wind Driven Optimization (AWDO) algorithm was introduced in (Bayraktar, 2015).

The AWDO utilizes an existing optimization algorithm, namely, Covariance Matrix Adaptation Evolutionary Strategy (CMAES) as a block-box solver to select the inherent terms. At each iteration, pressure values are calculated for each parcel by the WDO and these values are passed on to CMAES as cost values so that CMAES can choose a new set of values for the inherent terms, *α, g, c, and RT*, based on the cost from the WDO. This creates a four-dimensional optimization problem for CMAES with the same population size as the WDO, and CMAES does not make any cost function calls since it utilizes the pressure values computed by the WDO. Because the inherent terms are chosen adaptively by CMAES, there is no need to preset them at initialization removing the burden on user and creating a parameter free adaptive wind driven optimization method.

4 MULTIOBJECTIVE ADAPTIVE WIND DRIVEN OPTIMIZATION ALGORITHM

The cost function of WDO (or AWDO) was originally designed for single objectives while one can also optimize multiobjective problems through implementing a weighted sum cost function (Komurcu, 2011; Bayraktar 2011). However, instead of a weighted sum of multiple objectives, one can aim to find the Pareto-optimal solutions, which are the best solutions to the problem but are not better than each other. Multiobjective evolutionary algorithms have shown to be effective in finding multiple Pareto-optimal solutions in one single run since they utilize large populations (Deb, 2001; Fonseca, 1993; Zitzler, 1998). Similarly, we introduce a new population-based multiobjective optimization method for AWDO utilizing fast-nondominated sorting method (Deb, 2002) to identify Pareto-front solutions and an external population to archive the non-dominated fronts. We will refer to this method as Multiobjective Adaptive Wind Driven Optimization (MO-AWDO).

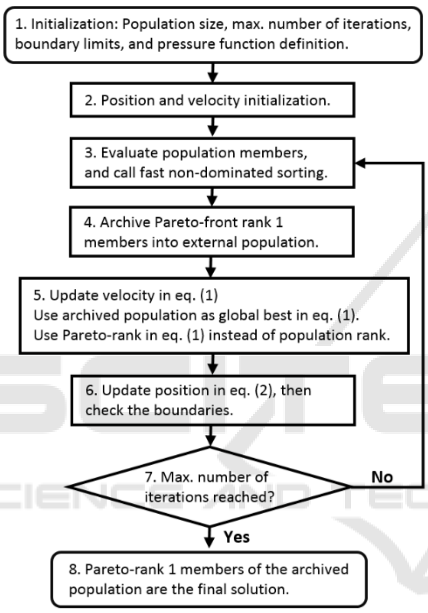

Figure 1: Flowchart of the Multiobjective Adaptive Wind Driven Optimization Algorithm (MO-AWDO).

In MO-AWDO method, the maximum velocity is bounded by $V_{max} = \pm |0.5|$ but chosen adaptively by the AWDO in addition to the four inherent terms mentioned in the previous section. The MO-AWDO flowchart is shown in Figure 1, where the algorithm starts with randomly initializing the position and velocity vectors. Then, at each iteration, pressure functions are evaluated for each member in the population. Based on the two cost functions per multiobjective problem, the fast non-dominated sorting algorithm determines the Pareto-fronts among the current population members, i.e. each member is assigned a Pareto-front number based on the sorting. This Pareto-front rank information for each parcel is used in equation 1 in place of *i*. At each iteration, members with Pareto-front rank one are added to the external population archive and then the archived population also goes through the fast non-dominated sorting. The members of the archived population with Pareto-front rank one then become the ones selected for the x_{max} in equation 1, simply because they represent the global best solutions found so far with the non-dominated Pareto-fronts. Once velocity is updated with the modifications described above, then the position is updated as shown in equation 2. Next, boundaries are checked along with the termination criterion. If the termination criterion is met, the algorithm terminates with results of Pareto-front rank one of the archived population as final best results.

5 NUMERICAL RESULTS

In this section, we describe and utilize five test functions to demonstrate the performance of the MO-AWDO algorithm. These standard numerical functions are selected from published literature (Deb, 2002; Zitzler, 2000; Coello, 2004) and many others can be found in the literature. We picked five representative functions with different dimensions and properties to be tested here. All of these five problems have two objective functions and only Kita's function comes with constraints. All cases were run with a population size of 100 air parcels for maximum number of 250 iterations totaling a maximum of 25,000 function evaluations as in (Deb, 2002) to compare.

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5.1 Schaffer's Function

The Schaffer's function is a convex problem and it is the simplest out of all problems presented here, such that the number of decision variables is only n=1. The variable bounds are set to be within [-10^3, 10^3], where

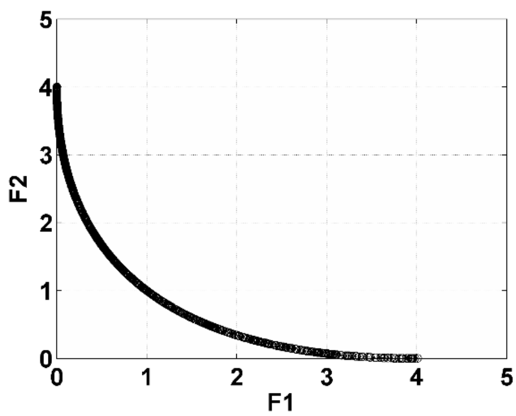


Figure 2: Pareto front produced by MO-AWDO for the Schaffer's Function shown with circles. The true Pareto front is shown as a continuous line.

the optimal solutions are within the range of $x \in [0, 2]$. The two cost functions for the Schaffers' function are:

$$F_1(x)=x^2, \text{ and } F_2(x)=(x-2)^2 \quad (3)$$

Figure 2 shows the results of the MO-AWDO from the archived Pareto population at the end of the maximum number of iterations. The true Pareto front for the Schaffer's function is illustrated with a continuous line on the same figure as well.

5.2 Kita's Function

The Kita's function is a constrained multi-objective function with number of decision variables of n=2. The variable bounds are limited to be within [0, 7]. The two cost functions to be maximized are shown below along with constraints:

$$F_1(x_1, x_2) = -x_1^2+x_2 \quad \text{and} \quad F_2(x_1, x_2) = (x_1/2) + x_2+1. \quad (4)$$

subject to

$$\frac{x_1}{6} + x_2 \leq \frac{13}{2}, \quad \frac{x_1}{2} + x_2 \leq \frac{15}{2}, \quad 5x_1 + x_2 \leq 30,$$

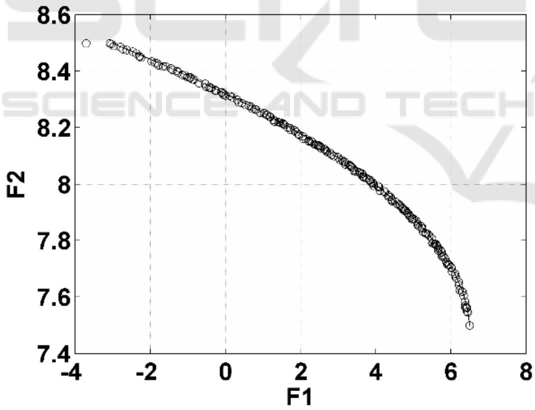


Figure 3: Pareto front produced by MO-AWDO for the Kita's Function shown with circles. The true Pareto front is shown as a continuous line.

Since MO-AWDO is designed to minimize the pressure (i.e. cost function), we simply took the negative of the pressure for the Kita's function to be minimized. Constraints are handled at pressure computation so that if any of the three constraints are violated, the pressure is penalized by setting it to be a very large value, i.e. 1e-5. Such high pressure encourages the particles to stay away from the constraints and converge on the Pareto front.

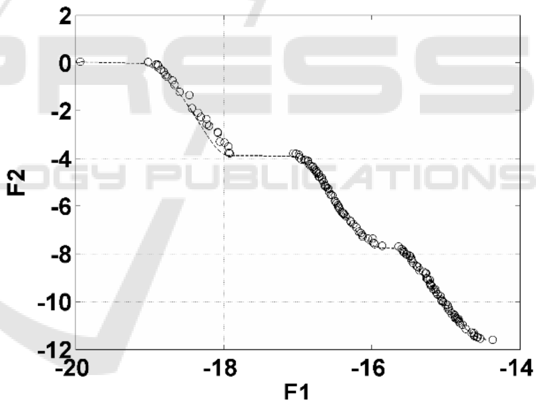
Figure 3 shows the results of the MO-AWDO from the archived Pareto population at the end of the last iteration. The true Pareto front for the Kita's function is illustrated with a continuous line on the same figure along with the results.

5.3 Kursawe's Function

The Kursawe's function is a nonconvex multi-objective function with number of decision variables of n=3. The variable bounds are set to be within [-5, 5]. The two cost functions to be minimized are:

$$F_1(x) = \sum_{i=1}^{n-1}\left(-10\exp\left(-0.2\sqrt{x_i^2 + x_{i+1}^2}\right)\right)$$
$$F_2(x) = \sum_{i=1}^{n}(|x_i|^{0.8} + 5\sin(x_i^3)) \quad (5)$$

Figure 4 shows the results of the MO-AWDO from the archived Pareto population at the end of the maximum number of iterations. The true Pareto front for the Kursawe's function is illustrated with a continuous line on the same figure along with the results. The MO-AWDO converges to the true Pareto front finding diverse solutions including the extreme points.



Figures 4: Pareto front produced by MO-AWDO for the Kursawe's Function shown with circles. The true Pareto front is shown as a continuous line.

5.4 ZDT1 Function

The ZDT1 function is a convex multi-objective function with number of decision variables of n=30. The variables are bounded within [0, 1] and the two cost functions to be minimized are:

$$F_1(x) = x_1 \quad and \quad F_2(x) = g(x)[1 - \sqrt{(x_1/g(x))}] \quad (6)$$

where,

$$g(x) = 1 + 9(\textstyle\sum_{i=2}^{n} x_i)/(n-1)$$

Figure 5 shows the results of the MO-AWDO from the archived Pareto population at the end of the maximum number of iterations. The true Pareto front for the ZDT1 function is illustrated with a continuous line on the same figure along with the results. The MO-AWDO converges to the true Pareto front finding diverse solutions including the extreme points.

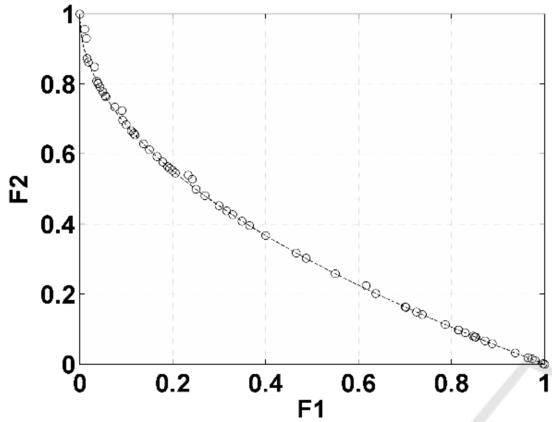


Figure 5: Pareto front produced by MO-AWDO for the ZDT1 Function shown with circles. The true Pareto front is shown as a continuous line.

5.5 ZDT4 Function

The ZDT4 function is a nonconvex multi-objective function with number of decision variables of n=10. The variable bounds are $x_1 \in [0, 1]$, and $x_i \in [-5, 5]$ for $i=2,\ldots,n$. The two cost functions to be minimized are:

$$F_1(x) = x_1 \quad and \quad F_2(x) = g(x)[1 - \sqrt{(x_1/g(x))}] \tag{7}$$

where,

$$g(x) = 1 + 10(n-1) + \sum_{i=2}^{n} (x_i^2 - 10\cos(4\pi x_i))$$

The ZDT4 function has 21^9 different local Pareto-optimal fronts (Zitzler, 2000), and only one of them is the global Pareto-optimal front. This challenging problem has been studied in (Deb, 2002) and they demonstrated that NSGA-II, and other MO-algorithms compared in their paper needed a population of 500 members ran for 250 iterations to be able find the global Pareto-optimal front. On the other hand, MO-AWDO can easily find the global Pareto-optimal front with a population of 100 air parcels within 100 iterations as shown in Figure 6, providing 10x speed up in convergence.

Figure 6 shows the results of the MO-AWDO from the archived Pareto population at the end of the maximum of 100 iterations using only 100 members. The true global Pareto-optimal front for the ZDT4 function is shown with a continuous line on the same figure. The MO-AWDO converges to the true Pareto front finding diverse solutions including the extreme points.

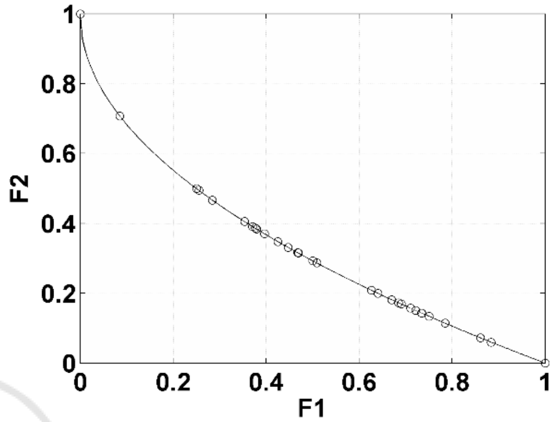


Figure 6: Pareto front produced by MO-AWDO for the ZDT4 Function shown with circles. The true global Pareto-optimal front is shown as a continuous line.

6 CONCLUSIONS

In this work, we introduced the Multiobjective Adaptive Wind Driven Optimization (MO-AWDO) algorithm and successfully demonstrated its efficient performance on five different numerical multi-objective benchmark functions with different dimensions and properties from published literature. The MO-AWDO combines the fast non-dominated sorting method with the Adaptive Wind Driven Optimization to identify the Pareto-fronts at each iteration and archives them in an external population. At each iteration, randomly selected archived non-dominated Pareto-optimal solutions are utilized as the global best solutions in the velocity update equation of the AWDO, providing elitism while preserving diverse non-dominated Pareto-fronts. Successful demonstration of the MO-AWDO shows that it can outperform well-known multi-objective algorithms like NSGA-II on difficult problems like ZDT4.

7 FUTURE WORK

As future work, we aim to improve how the MO-AWDO handles the archived population in terms of size and diversity so that it can record the most diverse Pareto-fronts with minimum number of members reducing memory requirements as iterations progresses. Also, extension of MO-AWDO to handle many-objective functions are also planned as future work.

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