Output Tracking Control for Networked Control Systems

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Abstract: This paper aims to compare alternative time delay relaxations for a class of nonlinear systems controlled via network and described by Takagi-Sugeno fuzzy models. In this regard, three alternatives were proposed and compared with a very recent relaxation proposed in the literature. Basically, the changes are made at two strategic points. The first point is the Lyapunov functional proposed and the second one is related to the introduction of different integral inequalities conditions. A numerical example of a network-based fuzzy tracking control systems is presented to highligth the advantages of the alternatives relaxations.

1 INTRODUCTION

The usual communication network architecture for control systems established during the past decades is point to point communication, ie, connection between the plant, sensors and actuators is made via a physical medium, for example, a cable. However, the increasing complexity of control systems is leading this architecture to reach its limits. Because of this, more and more communication architectures are being replaced by one in which all communication is done through a common communication medium for all equipments.

The introduction of this type of architecture can increase the efficiency, flexibility and reliability of these systems and reduce installation and maintenance costs. Networked Control Systems are currently in evidence, as they provide the control system with features such as cost reduction, easy maintainable and increases flexibility and agility (with regard to possible adaptations and modifications). These characteristics become more important when the complexity of control systems increases.

A classic control structure (point to point) considers that the means of communication between the components are ideal, i.e. there is no loss or delay in the transmitted information. A networked control system should take into account the characteristics of the physical environment in which the information circulates, because it will influence the system dynamics. The following characteristics of the physical environment can be listed:

- **Bandwidth:** the network may have a limitation in data transmission capacity, limiting the information that travels over the network;
- **Packet Loss:** the network has information loss, ie, the information sent may not reach their destiny;
- **Delay:** the information takes time to reach your destiny, therefore, a network delay should be considered.

At the beginning, the NCSs operated using a periodic triggered control method (also called time-triggered control). In this triggering method, a fixed sample interval should be selected to guarantee a desired performance under worst case conditions such as external disturbances, uncertainties, time-delays and so on. Therefore, this kind of triggering leads to sending many “unnecessary” sampling signals through the network, which incur in high utilization of the communication bandwidth Yue et al. (2013). In order to eliminate this problem, it was recently replaced by the event-triggered control method. This method provides a useful way of determining when the sampling action is carried out, which guarantees that only “necessary” state signals will be send out to the controller Yue et al. (2013), which reduces the utilization of the communication bandwidth. Albert (2004) presents a comparison between event-

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triggered and time-triggered concepts from the control theory point of view. Hu et al. (2012); Yue et al. (2013) an $\mathcal{H}_\infty$ tracking controller for NCS with event-triggering sampling method. Zhang et al. (2015); Jia et al. (2009); Tseng et al. (2001) design a $\mathcal{H}_\infty$ controller for output tracking for a T-S fuzzy system using the event-triggered method.

In the last years, more and more efforts have been done to derive new powerful convex stability conditions including alternative integral inequalities as, for example, the Wirtinger based integral inequality that has been proven to encompass the standard Jensen integral inequality Seuret and Gouaisbaut (2013). In Feng and Zheng (2016) the stability analysis problem of Takagi-Sugeno fuzzy systems with time-varying delay is investigated, utilizing the Wirtinger inequality and the reciprocally convex combination technique. Souza et al. (2014) presents new less conservative stability conditions analysis for Takagi-Sugeno fuzzy systems subject to interval time-varying and based on an appropriate Lyapunov functional selection combined with an integral inequality choice. Park et al. (2015a) introduced a Wirtinger based double integral inequality and new stability conditions have been obtained. In Sun et al. (2009) a new delay-dependent stability is obtained in terms of LMI by constructing a Lyapunov functional and using integral inequalities without introducing any free-weighting matrices.

The main objective of this paper is to derive less conservative conditions for stability and stabilization gathering those new relaxations based on integral inequalities and modifications to the Lyapunov functional. The idea is to present a state-of-the-art of recent conditions to the problem of output tracking control for networked Takagi-Sugeno fuzzy models with event-triggered control. In order to do this, the Lyapunov functional has been modified to include a triple integral term as proposed in Zhang et al. (2015) and its effect has been analysed. The analysis also includes the new class of integral inequaliteis proposed in Park et al. (2015) and the so-called auxiliary functions.

Notation: The notation considered in this paper is standard. $sym\{X\}$ denotes $X + X^T$. "$\otimes$" represents the Kronecker product for matrices. The term "$*$" indicates a term induced by symmetry in a matrix.

2 PROBLEM FORMULATION

This paper deals with nonlinear systems described by T-S fuzzy models. Consider the system described as follows:

Plant Rule $\mathcal{R}^i$: if $\theta_1(t)$ is M_{i1} and $\theta_2(t)$ is M_{i2} and ... and $\theta_g(t)$ is M_{ig}, then:

$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) + E_i w(t) \\ y(t) = C_i x(t) \end{cases} \tag{1}$$

where $i = 1, 2, \ldots, r$, and r denotes the number of if-then rules; $x(t) \in \mathbb{R}^n$ is the vector of state variables; $u(t) \in \mathbb{R}^m$ is the vector of control input, $w(t) \in \mathbb{R}^p$ is the vector of exogenous inputs and $y(t) \in \mathbb{R}^r$ is the vector of controlled variables. $\theta_j (j = 1, 2, \ldots, g)$ are the premise variables, $\theta(t) = [\theta_1(t), \theta_2(t), \ldots, \theta_g(t)] \in \mathbb{R}^g$ is a vector composed by stacking the premise variables. $M_{ij} (i = 1, 2, \ldots, r)(j = 1, 2, \ldots, g)$ are the fuzzy sets. A_i, B_i, C_i and E_i are the system matrices with appropriate dimensions.

By making use of a center-average defuzzifier, product fuzzy inference and singleton fuzzifier, the T-S fuzzy model is inferred as:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i [A_i x(t) + B_i u(t) + E_i w(t)] \\ y(t) = \sum_{i=1}^{r} \mu_i [C_i x(t)] \end{cases} \tag{2}$$

This membership functions satisfy the following properties:

$$\mu_i \in [0, 1], \quad \sum_{i=1}^{k} \mu_i = 1, \quad \sum_{i=1}^{k} \dot{\mu}_i = 0. \tag{3}$$

We also consider the following reference model

$$\begin{cases} \dot{x}_r(t) = A_m x_r(t) + B_m r(t) \\ y(t) = C_m x_r(t) \end{cases} \tag{4}$$

An event-triggered (ET) scheme is introduced to decide wheter or not the sample-data should be transmitted. Figure 1 shows the configuration of the network-based fuzzy tracking control system.

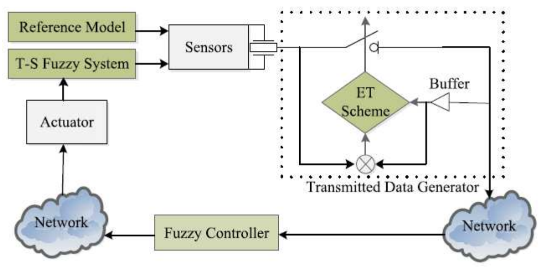

Figure 1: Configuration of the system (Zhang et al., 2015).

The initial triggering instant is defined as 0, i.e. $t_k h = 0$, when $k = 0$. $t_k \in \mathbb{N}$ is the triggering instant and h is the samplig period. The next triggering instant is defined by the following:

$$t_{k+1} = t_k h + \inf_{i \in \mathbb{N}} \left\{ ih \left| \left\| W^{\frac{1}{2}} \left[\xi(t_k h + ih) - \xi(t_k h)\right] \right\| - \varepsilon \left\| W^{\frac{1}{2}} \xi(t_k h) \right\| > 0 \right\}, \ \forall k \in \mathbb{N} \quad (5)$$

where $\xi(t)$ will be defined in 8.

The parameters ε ($0 < \varepsilon < 1$) and matrix W ($W > 0$) determinate how frequently and how much sample-data should be transmitted. In Yue et al. (2013); Peng et al. (2013); Hu et al. (2012) the parameter W and controller parameters are determined by some LMI-based criteria for a given ε.

Clearly the fuzzy system and the fuzzy controller operate asynchronously due to the event-triggering process and the network-induced delays in data transmission.

The control rule of the state feedback controller is defined as follows:

If $\theta_1(t_k h)$ is M_{i1} and $\theta_2(t_k h)$ is M_{i2} and ... and $\theta_g(t_k h)$ is M_{ig}, then:

$$u(t) = F_{1i} x(t_k h) + F_{2i} x_r(t_k h) \quad (6)$$

Then, the fuzzy controller is defined as:

$$u(t) = \sum_{i=1}^{r} \mu_i(\theta(t_k h)) \left[F_{1i} x(t_k h) + F_{2i} x_r(t_k h)\right], \quad t \in [t_k h + \tau_{tk}, t_{k+1} h + \tau_{tk+1},) \quad (7)$$

where $\tau_{tk} = \tau_{tk}^{sc} + \tau_{tk}^{ca}$. τ_{tk}^{sc} is the delay corresponding to the time that the information takes to travel between the system and the controller. τ_{tk}^{ca} is the delay corresponding to the time that the information takes to travel between the controller and the actuator. Other delays such as the time the computer takes to calculate the controller gains are ignored.

Let $e(t) = y(t) - y_r(t)$ and $\varepsilon_k(t - \tau(t)) = \xi(t - \tau(t)) - \xi(t_k h)$. Using the system model and the reference model, an augmented system can be obtained as follows:

$$\begin{cases} \dot{\xi}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i \mu_j [\bar{A}_i \xi(t) + \bar{B}_i \bar{F}_j \xi(t - \tau(t)) \\ \qquad\qquad - \bar{B}_i \bar{F}_j \varepsilon_k(t - \tau(t)) + \bar{E}_i \bar{\omega}(t)] \\ e(t) = \sum_{i=1}^{r} \mu_i [\bar{C}_i \xi(t)] \end{cases} \quad (8)$$

with $\xi(t - \tau(t))$, the augmented state, and $\varepsilon_k(t - \tau(t))$ satisfying:

$$\left\| W^{\frac{1}{2}} \varepsilon_k(t - \tau(t)) \right\| \leq \sigma \left\| W^{\frac{1}{2}} \xi_k(t - \tau(t)) - \varepsilon_k(t - \tau(t)) \right\| \quad (9)$$

where σ is a positive scalar and

$$\bar{A}_i = \begin{bmatrix} A_i & 0 \\ 0 & A_m \end{bmatrix}, \bar{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix}, \bar{E}_i = \begin{bmatrix} E_i & 0 \\ 0 & B_m \end{bmatrix},$$

$$\bar{C}_i = \begin{bmatrix} C_i & -C_r \end{bmatrix}, \bar{F}_i = \begin{bmatrix} F_{1i} & F_{2i} \end{bmatrix},$$

$$\mu_i = \mu_i(\theta(t)), \quad \mu_i^{(k)} = \mu_i(\theta(t_k h)) \geq 0,,$$

$$i = 1, 2, \ldots, r, \textstyle\sum_{i=1}^{r} \mu_i^{(k)} = \sum_{i=1}^{r} \mu_i = 1, \ \forall k \in \mathbb{N},$$

$$\bar{\omega}(t) = \begin{bmatrix} \omega^T(t) & r^T(t) \end{bmatrix} \quad (10)$$

where F_{1i} and F_{2i} are the fuzzy control gains.

Zhang et al. (2015) designs a network-based fuzzy tracking controller such that the output of the fuzzy system follows the output of the reference model as close as possible by taking into consideration the deviation bounds of asynchronous normalized membership functions, which are described by:

$$|\mu_i(\theta(t)) - \mu_i(\theta(t_k h))| \leq \delta_i, \ i = 1, 2, \ldots, r, \ \forall k \in \mathbb{N} \quad (11)$$

where δ_i are given positive constants. In order to obtain the δ_i bounds, the Lemmas available in: Zhang et al. (2015); Peng et al. (2013); Zhang and QL.Han (2013) are introduced. In this scenario, three different alternatives are proposed in our paper to be analyzed and compared with the solution proposed by Zhang et al. (2015). The alternatives are listed below:

- **Alternative 1:** Change the Lyapunov functional in Zhang et al. (2015), by adding a tripple integral term and use Lemmas 1, 2, 3 and 4;
- **Alternative 2:** Maintain the Lyapunov functional in Zhang et al. (2015) and use (Park et al., 2015b, Lemma 5.1) to modify the integral inequalities;
- **Alternative 3:** Change the Lyapunov functional in Zhang et al. (2015) by adding a tripple integral term to the funcional and use the integral inequalities proposed by (Park et al., 2015b, Lemma 5.1).

For that, we consider the following Lyapunov functional:

$$V(t, \xi_t) = \sum_{i=0}^{7} V_i(t, \xi_t) \quad (12)$$

$$V_0(t, \xi_t) = \begin{bmatrix} \xi(t) \\ \int_{t-\tau_m}^{t} \xi(s) ds \end{bmatrix}^T P \begin{bmatrix} \xi(t) \\ \int_{t-\tau_m}^{t} \xi(s) ds \end{bmatrix}$$

$$V_1(t, \xi_t) = \int_{t-\tau_m}^{t} \xi^T(s) Q_1 \xi(s) ds$$

$$V_2(t, \xi_t) = \int_{t-\tau_M}^{t-\tau_m} \xi^T(s) Q_2 \xi(s) ds$$

$$V_3(t,\xi_t) = \tau_m \int_{-\tau_m}^{0}\int_{t+s}^{t} \dot{\xi}^T(\theta)R_1\dot{\xi}(\theta)d\theta ds$$

$$V_4(t,\xi_t) = \int_{-\tau_M}^{-\tau_m}\int_{t+s}^{t} \dot{\xi}^T(\theta)R_2\dot{\xi}(\theta)d\theta ds$$

$$V_5(t,\xi_t) = (\tau_M - \tau(t))\int_{t-\bar{\tau}(t)}^{t} \dot{\xi}^T(\theta)R_3\dot{\xi}(\theta)d\theta$$

$$V_6(t,\xi_t) = (\tau_M - \tau(t))\left[(\xi(t)-\xi(t-\bar{\tau}(t)))^T R_4 \right. \\ \left. (\xi(t)-\xi(t-\bar{\tau}(t)))\right]$$

$$V_7(t,\xi_t) = \int_{-\tau}^{0}\int_{t+s}^{t}\int_{t-\lambda}^{t} \dot{\xi}^T(\theta)G_1\dot{\xi}(\theta) \\ \int_{-\tau}^{0}\int_{t+s}^{t}\int_{t-\lambda}^{t} dsd\theta d\lambda$$

where $\xi_t = \xi(t+\theta)$, $\forall\theta \in [-\tau_M, 0]$, $P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22}\end{bmatrix}$ with $P_{ii} = P_{ii}^T$, $Q_i = Q_i^T, (i = 1,2)$, $R_i = R_i^T, (i = 1,2,3,4)$, $G_1 = G_1^T, (i = 1,2)$.

The main difference between this functional and the one proposed in Zhang et al. (2015) is the introduction of the triple integral term in V_7. Notice that alternative 02 makes use of the same Lyapunov functional as Zhang et al. (2015). Given all of this, we are now able to present the conditions in the result below.

Theorem 1: Given positive scalars $\gamma, \tau_m, \tau_M, \varepsilon, \delta_i (i = 1,2,\ldots,r), \sigma$ and a weighting matrix $U > 0$, under the event-triggered communication scheme, the system 8 is asymptotically stable with the L_2-gain tracking performance, if there exist symmetric matrices $W > 0, P_{ii} > 0, Q_i > 0, R_i > 0, G_i > 0$ and matrices $P_{12}, X, S, Z, M_i, N_i (i = 1,2,\ldots,r), T_{ij} = T_{ji}^T (i, j = 1,2,\ldots,2r)$ such that the following LMIs hold:

$$\begin{bmatrix} P_{11} & P_{12} \\ * & P_{22}\end{bmatrix} > 0, \tag{13}$$

$$\begin{bmatrix} R_2 & S \\ * & R_2\end{bmatrix} > 0, \tag{14}$$

$$\begin{bmatrix} \Xi_{ij}^{(1)} & \Gamma_i & \tau Z^T & e_1^T X^T C_i^T \\ * & -\gamma^2 I & 0 & 0 \\ * & * & -\tau R_3 & 0 \\ * & * & * & -U^{-1} \end{bmatrix} \leq 0, \tag{15}$$

$$\begin{bmatrix} \Xi_{ij}^{(2)} & \Gamma_i & e_1^T X^T C_i^T \\ * & -\gamma^2 I & 0 \\ * & * & -U^{-1} \end{bmatrix} \leq 0, \tag{16}$$

$$\begin{bmatrix} T_{1,1} & T_{1,2} & \ldots & T_{1,2r} \\ T_{2,1} & T_{2,2} & \ldots & T_{2,2r} \\ \vdots & \vdots & \ddots & \vdots \\ T_{2r,1} & T_{2r,2} & \ldots & T_{2r,2r} \end{bmatrix} < 0, \tag{17}$$

$$T_{ij} + T_{ji} - 2M_i \leq 0, \tag{18}$$

$$-2N_j - T_{(j+r),(i+r)} - T_{(i+r),(j+r)} \leq 0. \tag{19}$$

where $e_i = I(i,:) \otimes I$ $(i = 1,2,\ldots,9)$ are the $p \times 9p$ matrices, I denotes a matrix identity of order p, $I(i,:)$ denotes the i-*th* row of an 9×9 identify matrix, p is the dimension of $\xi(t)$,

$$\begin{aligned}
\Xi_{ij}^{(1)} =& \Omega_{ij}^{(0)} + e_3\varepsilon W e_3 + sym\left\{e_3^T \varepsilon W e_4\right\} \\
& -(1-\varepsilon)We_4 - \kappa_0 \Upsilon \kappa_0^T, \\
\Xi_{ij}^{(2)} =& \Omega_{ij}^{(0)} + \tau\Omega_{ij}^{(1)} + e_3\varepsilon W e_3 + sym\left\{e_3^T \varepsilon W e_4\right\} \\
& -(1-\varepsilon)We_4 - \kappa_0 \Upsilon \kappa_0^T, \\
\Omega_{ij}^{(0)} =& e_1^T(A_iX + X^TA_i^T + Q_1 + P_{12} + P_{12}^T - 9R_1 - \\
& R_4)e_1 + e_2^T(\tau_m^2 R_1 + \tau^2 R_2 - \sigma X + \sigma X^T)e_2 + \\
& e_3^T(S^T + S - 2R_2)e_3 - e_5^T R_4 e_5 + \\
& e_6^T(Q_2 - Q_1 - 9R_1 - R_2)e_6 + e_7^T(Q_2 + R_2)e_7 \\
& - e_8^T 180 R_1 e_8 - e_9^T 720 R_1 e_9 + sym\left\{e_1^T\left(\sigma X^T A_i^T \right.\right. \\
& \left.\left. -X + P_{11}\right)e_2 + e_1^T B_i Y_j e_3 - e_1^T B_i Y_j e_4\right\} \\
& + sym\left\{e_1^T R_4 e_5 - e_1(2R_1 + P_{12})e_6 \right. \\
& \left. + e_1^T(\tau_m P_{22} + 6R_1)e_8\right\} \\
& sym\left\{e_2^T \sigma B_i Y_j e_3 - e_2^T \sigma B_i Y_j e_4 + e_2^T \tau_m P_{12} e_8\right\} \\
& + sym\left\{e_3^T(R_2 - S^T)e_6 + e_3^T(R_2 - S)e_7 + \right. \\
& \left. e_6^T S e_7\right\} + sym\left\{e_6^T(6R_1 - \tau_m P_{22})e_8 + \right. \\
& \left. Z^T(e_1 - e_5)\right\}, \\
\Omega_{ij}^{(1)} =& sym\left\{e_1^T R_4 e_2 - e_2^T R_4 e_5\right\} + e_2^T R_3 e_2, \\
\Gamma_i =& e_1^T E_i + e_2^T \sigma E_i, \\
\Upsilon_{ij} =& \frac{T_{ij} + T ji + T_{(i+r),(j+r)} + T_{(j+r),(i+r)}}{2} \\
& + T_{i,(j+r)} + T_{(j+r),i} \\
& - \sum_{k=1}^{r} \delta_k \left\{ M_i + \frac{T_{ik} + T_{ki}}{2} + N_j + \right. \\
& \left. \frac{T_{(j+r),(k+r)} + T_{(k+r),(j+r)}}{2} \right\}.
\end{aligned}$$

The fuzzy control gains and the matrix in the event-triggered communication scheme are given by $\bar{F}_j = Y_j X^{-1} (j = 1,2,\ldots,r)$ and $W = X^{-T}\bar{W}X^{-1}$.

Proof. The proof is similar to that of Proposition 1 in (Zhang et al., 2015, Appendix A and B) with the exception of using Lemmas (Park et al., 2015b, Lemma 5.1) instead of (Zhang et al., 2015, Lemma 2), and thus is omitted.

Remark: The other proofs are also similar. Alternative 1 uses the Lyapunov functional 12 and (Zhang et al., 2015, Lemmas 1, 2, 3 and 4) to the integral terms. Alternative 2 uses the same Lyapunov functional of Zhang et al. (2015) and the (Park et al., 2015b, Lemma 5.1) instead of (Zhang et al., 2015, Lemma 2).

3 EXAMPLE

We consider the example in (Zhang et al., 2015, Section 5, p. 38). The purpose is to compare the maximum allowable upper bounds that guarantee the asymptotic stability of each system. We consider an initial state $x(0) = [1 \;\; 0]^T$ and $x_r(0) = -2$, disturbance input $\omega(t) = 12\cos(t)e^{-t}$ and reference input $r(t) = 6\sin(1.2t)e^{-0.11t}$. Consider the following T-S fuzzy system.

$$\bar{A}_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -0.1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \bar{A}_2 = \begin{bmatrix} 0 & 1 & 0 \\ 25 & -0.1 & 0 \\ 0 & 0 & -1 \end{bmatrix},$$
$$\bar{E}_1 = \bar{E}_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^T,$$
$$\bar{B}_1 = \bar{B}_2 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T,$$
$$\bar{C}_1 = \bar{C}_2 = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix},$$
$$\mathbb{D} = \{x(t) : |x_1(t)| \le 5, |x_2(t)| \le 4\}$$
$$\mu_1(\theta(t)) = 1 - x_1^2(t)/25,\ \mu_2(\theta(t)) = x_1^2(t)/25,$$
$$\theta(t) = x_1(t).$$

This system is described in the augmented form as in (8). Table 1 presents the maximum allowable upper bound to the time-varying delay, τ_2, achieved for a given lower bound, τ_1.

Table 1: System.

Method	$\tau_1 = 0$	$\tau_1 = 0.5$	$\tau_1 = 1.0$
Zhang et al. (2015)	0.12	0.72	1.41
Alternative 1	1.45	2.23	2.69
Alternative 2	2.13	2.73	3.27
Alternative 3	2.47	2.79	3.58

The results obtained show that the alternatives proposed can provide higher upper bounds than existing results. For comparision purpose, the maximum time-varying delay is set to 1.0 s. The gains of the state feedback controller were obtained through Alternative 3, which was the best alternative among the alternatives proposed, that is: $\bar{F}_1 = [-5.9879 \;\; -7.2412 \;\; 22.7661]$ and $\bar{F}_2 = [-3.2195 \;\; -0.93731 \;\; 11.763]$. Figure 2 depicts the output tracking errors. In a tracking problem, it is expected that the tracking error tends to zero as time increases. Notice that the error obtained by Alternative 3 is smaller than the error obtained in Zhang et al. (2015).

Generally, considering small values of delay, the result obtained by Zhang et al. (2015) is very similar to the results obtained through the alternatives proposed in this article. The great advantage of the methods proposed in this article is that they maintain a better output tracking dynamic for a greater range of delay values.

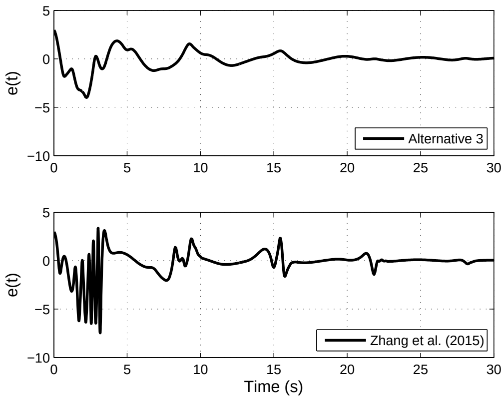

Figure 2: Output Tracking Error.

Figure 3 presents the output tracking responses. Applying an input signal with limited energy, it is expected that the output signal of the system has limited energy. As can be seen, for the maximum delay value equal to 1.0 s, both systems present output signal with limited energy. According to the figure, the controller designed through Alternative 3 presents a better disturbance rejection in relation to the controller designed by Zhang et al. (2015).

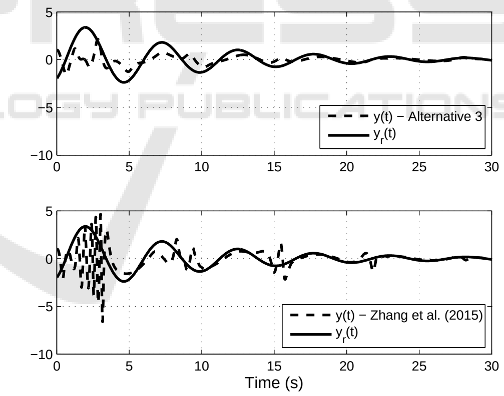

Figure 3: Output Tracking Responses.

Figure 4 depicts the performance of the system states. What is expected in this problem is to minimize the effect of disturbances in the first state variable, acting only in the second state variable of the model as denoted by the input matrices B_i. As we can note, the controller obtained using Alternative 3 rejects the disturbance in the second state variable better than the controller designed by Zhang et al. (2015). Note that the axes of this figure have different amplitudes, in order to obtain the best resolution possible.

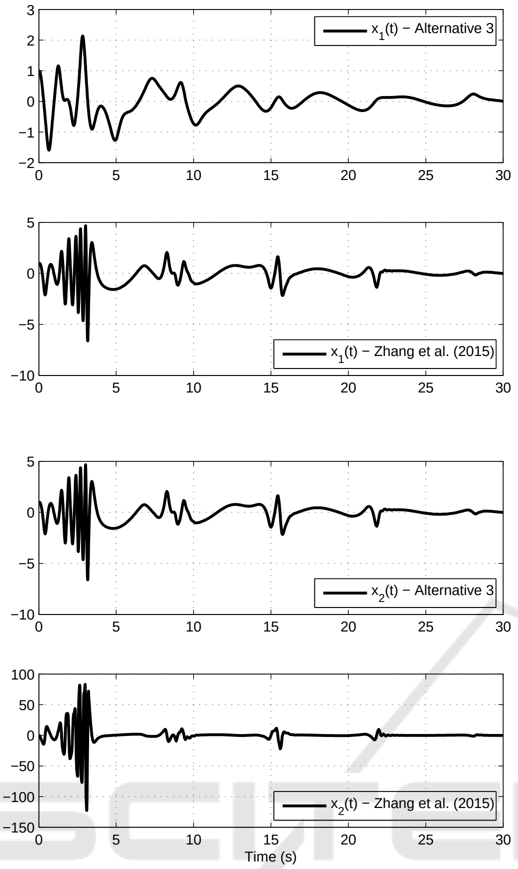

Figure 4: States of the Systems.

4 CONCLUSION

In this paper the output tracking problem for networked control has been handled considering different choices for integral inequalities relaxations as well as proper selection of Lyapunov functionals. In general, the results obtained suggest that less conservative conditions can be obtained just considering slight alterations in very recent results.

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REFERENCES

Albert, A. (2004). Comparison of event–triggered and time–triggered concepts with regard to distributed control systems. *Embedded World*, 2004:235–252.

Feng, Z. and Zheng, W. X. (2016). Improved stability condition for Takagi–Sugeno fuzzy systems with time–varying delay. *IEEE Transactions on Cybernetics*.

Hu, S., Zhang, Y., and Du, Z. (2012). Network–based $\mathcal{H}_\infty$ tracking control with event–triggering sampling scheme. *IET Control Theory & Applications*, 6(4):533–544.

Jia, X., Zhang, D., Hao, X., and Zheng, N. (2009). Fuzzy tracking control for nonlinear networked control systems in Takagi–Sugeno fuzzy model. *IEEE Transactions on Cybernetics*, 39(4):1073–1079.

Park, M., Kwon, O., Park, J. H., Lee, S., and Cha, E. (2015a). Stability of time-delay systems via Wirtinger–based double integral inequality. *Automatica*, 55:204–208.

Park, P., Lee, W. I., and Lee, S. Y. (2015b). Auxiliary function–based integral inequalities for quadratic functions and their applications to time-delay systems. *Journal of the Franklin Institute*, 352(4):1378–1396.

Peng, C., Han, Q., and Yue, D. (2013). To transmit or not to transmit: a discrete event–triggered communication scheme for networked Takagi–Sugeno fuzzy systems. *IEEE Transactions on Fuzzy Systems*, 21(1):164–170.

Seuret, A. and Gouaisbaut, F. (2013). Wirtinger–based integral inequality: application to time–delay systems. *Automatica*, 49(9):2860–2866.

Souza, F. O., Campos, V. C. S., and Palhares, R. M. (2014). On delay–dependent stability conditions for Takagi–Sugeno fuzzy systems. *Journal of the Franklin Institute*, 351(7):3707–3718.

Sun, J., Liu, G., and Chen, J. (2009). Delay–dependent stability and stabilization of neutral time–delay systems. *International Journal of Robust and Nonlinear Control*, 19(12):1364–1375.

Tseng, C., Chen, B., and Uang, H. (2001). Fuzzy tracking control design for nonlinear dynamic systems via Takagi–Sugeno fuzzy model. *IEEE Transactions on Fuzzy Systems*, 9(3):381–392.

Yue, D., Tian, E., and Han, Q. (2013). A delay system method for designing event–triggered controllers of networked control systems. *IEEE Transactions on Automatic Control*, 58(2):475–481.

Zhang, D., Han, Q., and Jia, X. (2015). Network–based output tracking control for Takagi–Sugeno fuzzy systems using an event-triggered communication scheme. *Fuzzy Sets and Systems*, 273:26–48.

Zhang, X. and QL.Han (2013). Novel delay–derivative–dependent stability criteria using new bounding techniques. *International Journal of Robust and Nonlinear Control*, 23(13):1419–1432.