

Efficient Proxy Signature Scheme from Pairings

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Abstract: A proxy signature enables an entity to transfer its signing rights to another entity, called the proxy signer, without actually sharing its signing key. Most of the proxy signatures in literature have been designed using bilinear pairing on the elliptic curve group with the aim of providing either the property of being identity-based or efficiency or security. But almost all of these schemes do not provide all these three desirable properties together and most of the identity-based proxy signature (IBPS) schemes are either too inefficient or their security is based on non-standard assumptions to have practical significance. In this paper, we propose an efficient and provably secure identity-based proxy signature scheme from bilinear pairing based on a standard assumption, the hardness of the computational Diffie-Hellman problem. The proposed scheme is secure against existential forgery on adaptive chosen-message and adaptive chosen-ID attack in the random oracle model. Moreover, we do an efficiency analysis and show that our scheme is significantly more efficient in the view of computation and operation time than the existing similar schemes.

1 INTRODUCTION

We live in a digital era in which communications and transactions are mostly online and it is crucial to have efficient and usable tools to verify the authenticity and integrity of exchanged information. Digital signature is a cryptographic primitive which facilitates the above desired properties and guarantees the sender's non-repudiation. In many fields, including e-government and e-commerce, there may be many situations in which the signatory entity itself is unable to apply the signature and needs to delegate its rights to another entity. For example, the CEO of a company may wish to authorize the chief financial officer (CFO) or the chief operating officer (COO) to sign certain messages on his behalf during a certain period of his absence. A proxy signature scheme enables a signer (entity) O , called the *original signer*, also called the *designator* or *delegator*, to delegate its signing rights (without transferring the private key) to another entity P , called the *proxy signer*, to produce, on the delegator's behalf, signatures that can be verified by the receiver R under the delegator O 's public key. In other words, whenever a receiver entity verifies a proxy signature, it verifies the signature itself along with original signer delegation.

1.1 Related Work

The idea of proxy signature was introduced in (Gasser et al., 1989), but the first formal construction was presented in (Mambo et al., 1996) in 1996. The formal model of security for the proxy signature was structured by (Boldyreva et al., 2012). (Schuldt et al., 2008) strengthened the security model for proxy signatures in more formal way and have also extended it on the identity-based setting.

Since then, many proxy signature schemes have been proposed in the identity-based setting using bilinear pairings. (Xu et al., 2005) formalized the notion of security for identity-based proxy signature schemes based on the work of (Boldyreva et al., 2012) and (Malkin et al., 2004). However, (Wu et al., 2007) showed that the model defined in (Xu et al., 2005) does not capture the definitions of adaptive chosen-message and chosen-identity attack in identity based system and proposed a new scheme. (Wang, 2008) showed that the schemes given in (Xu et al., 2005) are vulnerable to proxy key exposure attack.

1.2 Our Contribution

We propose an efficient and provable secure identity-

based proxy signature (IBPS) scheme suitable for real-life applications. The construction of our scheme is motivated by the identity based signature of (Choon and Cheon, 2003) which is secure against existential forgery on adaptive chosen-message and given-ID attack in the random oracle model. We do a non-trivial transformation to their scheme to construct a proxy signature which is secure against existential forgery on adaptive chosen-message and adaptive chosen-ID attack in the random oracle model. Our scheme follows a delegation by warrant approach and is secure under the computational Diffie-Hellman assumption.

Since the last decade most of the works on proxy signature have been carried out with some extensions viz. multi-proxy signature, proxy multi-signature, multi proxy multi- signature (Cao and Cao, 2009a; Cao and Cao, 2009b; Sahu and Padhye, 2015). Considering them for one-to-one delegation, these schemes may be seen as simple proxy signature schemes. Some recent basic proxy signature schemes (Singh and Verma, 2012; Asaar et al., 2014) have been proposed with an additional property of message recovery, in which the message need not to be sent with signature and can be recovered from the signature by one more additional step after verification. In this paper, we show that even these schemes are less efficient than our proposed scheme. Due to the page limitation, though we do not compare our scheme with the schemes with multi-delegation here, it can be easily seen that our scheme is more efficient than these schemes when those are used for single delegation. Our scheme beats even one of the most recent one-to-one delegation scheme (Sarde and Banerjee, 2015) by more than 25% in computation time. Moreover, the scheme is up to 39% more efficient in the view of computation and operation time than the existing proxy signature schemes (Xu et al., 2005; Chow et al., 2005; Wu et al., 2007; Wang, 2008; Singh and Verma, 2012; Asaar et al., 2014; Sarde and Banerjee, 2015).

Most of the new primitives of proxy signature as multi-proxy signature, proxy multi-signature, multi-proxy multi signature, proxy blind signature, threshold proxy signature etc. use a simple proxy signature as a building block. Hence, the construction of an efficient and provable secure proxy signature scheme is desired. Almost all the proxy signature schemes from bilinear pairings defined on the elliptic curve group are either inefficient or they have some security issues. As a consequence, the new primitives using these schemes as building blocks also have the same issues and, in fact, these issues are magnified depending on the nature of the construction. Thus our efficient and provably secure identity-based proxy

signature scheme would be very useful as a concrete scheme to be further used to construct efficient extensions of proxy signature scheme.

1.3 Outline of the Paper

The rest of this paper is organized as follows. In Section 2, we introduce some related mathematical definitions, problems and assumptions. In Section 3, we present the formal definition of an identity-based proxy signature scheme and formal security model for it. Our proposed identity-based proxy signature scheme is presented in Section 4. In Section 6 we analyze the security of our scheme and in Section 7 we do an efficiency comparison with the state-of-art. Finally, in Section 8 we draw a conclusion of our work.

2 PRELIMINARIES

In this section, we introduce some relevant definitions, mathematical problems and assumptions.

2.1 Notations

We denote by $y \leftarrow A(x)$ the operation of running a randomized or deterministic algorithm $A(x)$ and storing the output to the variable y . If X is a set, then $v \xleftarrow{\$} X$ denotes the operation of choosing an element v of X according to the uniform random distribution on X . We say that a given function $f : N \rightarrow [0, 1]$ is *negligible in n* if $f(n) < 1/p(n)$ for any polynomial p for sufficiently large n . For a group G and $g \in G$, we write $G = \langle g \rangle$ if g is a generator of G .

Definition 1 (Bilinear Map). Let G_1 be an additive cyclic group with generator P and G_2 be a multiplicative cyclic group with generator g . Let both the groups are of the same prime order q . Then, a map $e : G_1 \times G_1 \rightarrow G_2$ satisfying the following properties, is called a *cryptographic bilinear map*:

1. *Bilinearity*: For all $a, b \in \mathbb{Z}_q^*$, $e(aP, bP) = e(P, P)^{ab}$, or equivalently, for all $Q, R, S \in G_1$, $e(Q + R, S) = e(Q, S)e(R, S)$ and $e(Q, R + S) = e(Q, R)e(Q, S)$.
2. *Non-Degeneracy*: There exists $Q, R \in G_1$ such that $e(Q, R) \neq 1$. Note that, since G_1 and G_2 are groups of prime order, this condition is equivalent to the condition $e(P, P) \neq 1$, which again is equivalent to the condition that $e(P, P)$ is a generator of G_2 .
3. *Computability*: There exists an efficient algorithm to compute $e(Q, R) \in G_2$, for any $Q, R \in G_1$.

Definition 2 (Computational Diffie-Hellman Problem). Let G_1 be a cyclic group with generator P .

1. Let $a, b \in \mathbb{Z}_q^*$ be randomly chosen and kept secret. Given $P, aP, bP \in G_1$, the *computational Diffie-Hellman problem* (CDHP) is to compute $abP \in G_1$.
2. The (t, ε) -CDH assumption holds in G_1 if there is no algorithm which takes at most t running time and can solve CDHP with at least a non-negligible advantage ε .

3 IBPS SCHEME AND ITS SECURITY

In this section, we give the formal definition and the security model for an identity-based proxy signature (IBPS) scheme. Here onwards we mean by IBPS an identity-based proxy signature.

3.1 Definition of IBPS Scheme

In an IBPS scheme, an original signer delegates its signing rights to a proxy agent to make a signature on its behalf, where the public keys of the original and proxy signers can be computed from their identities by anyone and their private keys are generated using their corresponding identities by a trusted authority, the private key generator (PKG). Let O be the original signer with identity ID_O and \mathcal{P} be the proxy signer with identity $ID_{\mathcal{P}}$. Precisely, an IBPS scheme consists of the following algorithms:

Params \leftarrow **Setup**(1^λ): For a security parameter 1^λ as input, the PKG runs this algorithm and generates the public parameters *params* of the system and a master secret.

$(Q_{ID}, S_{ID}) \leftarrow$ **KeyGen**(ID): With this private key generation algorithm, the PKG outputs the private key S_{ID} for a given identity ID .

$(d_{\mathcal{P}}) \leftarrow$ **ProxyKeyGen**($w, S_{ID_O}, S_{ID_{\mathcal{P}}}$): This is a protocol between the original signer and the proxy signer during which they agree on the warrant w , which includes some specific information regarding the message as restrictions on the message, time of delegation, identity of original and proxy signer, period of validity, etc. After the successful interaction, the proxy signer \mathcal{P} outputs its proxy signing key, $d_{\mathcal{P}}$.

$(\sigma_{\mathcal{P}}) \leftarrow$ **ProxySignature**($m, w, d_{\mathcal{P}}$): This randomized algorithm outputs an IBPS $\sigma_{\mathcal{P}}$ on a message m satisfying a warrant w .

$(0/1) \leftarrow$ **Verification**($m, w, \sigma_{\mathcal{P}}, Q_{ID_O}, Q_{ID_{\mathcal{P}}}$): This deterministic algorithm outputs 1 if the signature $\sigma_{\mathcal{P}}$ is a valid IBPS on the message m by the proxy signer on behalf of the original signer and outputs 0 otherwise.

3.2 Security Model for IBPS Scheme

In this model, an adversary \mathcal{A} tries to forge the proxy signature working against a user, either against the original signer say O or against the proxy signer say \mathcal{P} . The adversary \mathcal{A} can access polynomial number of hash queries, extraction queries, delegation queries, proxy key generation queries and proxy signature queries. Consider that the response to each query is provided to \mathcal{A} by using the random oracle. The goal of adversary \mathcal{A} is to produce one of the following forgeries:

1. An IBPS $\sigma_{\mathcal{P}}$ for a message m on behalf of the original signer, where user \mathcal{P}' is the proxy signer, such that either the original signer never designate user \mathcal{P}' , or m was not submitted to the proxy signing oracle.
2. An IBPS $\sigma_{\mathcal{P}}$ for a message m by the proxy signer on behalf of the user O' , where user O' plays the role of original signer, and the proxy signer was never designated by the user O' .

Definition 3. An IBPS scheme is said to be existential unforgeable against adaptive chosen-message and adaptive chosen-ID attack if no probabilistic polynomial time adversary \mathcal{A} has a non-negligible advantage against the challenger \mathcal{C} in the following game:

1. **Setup:** The challenger \mathcal{C} runs the setup algorithm and provides the public parameters *params* to the adversary \mathcal{A} .
2. **Hash queries:** On hash query of adversary \mathcal{A} , challenger \mathcal{C} responds through random oracle and maintains lists say L_{H_1} and L_{H_2} for the hash queries.
3. **Extraction queries:** On key extraction query by \mathcal{A} for an identity ID , \mathcal{C} provides the corresponding private key S_{ID} to \mathcal{A} .
4. **Delegation queries:** \mathcal{A} produces a warrant w' and receives its corresponding delegation value T'_O from \mathcal{C} .
5. **Proxy key generation queries:** \mathcal{A} produces a valid warrant w' with respect to an adaptively chosen identity ID and receives its corresponding proxy signing key d_{ID} from \mathcal{C} .
6. **Proxy signature queries:** \mathcal{A} produces a message m' , a valid warrant w' corresponding to the message m' and identity ID and receives from \mathcal{C} an IBPS σ'_{ID} on the adaptively chosen message.

After the series of queries, \mathcal{A} outputs a new IBPS $\sigma_{\mathcal{P}}^*$ on the message m^* under a warrant w^* for identities ID_O and $ID_{\mathcal{P}}$, where $-$ \mathcal{A} has not requested the private key for at least one of the identities ID_O and $ID_{\mathcal{P}}$, in extraction queries; \mathcal{A} did not request a delegation query on warrant w^* and identity ID_O ; \mathcal{A} did not request a proxy key generation query including warrant

w^* and identity $ID_{\mathcal{P}}$; \mathcal{A} never requests a proxy signature query on message m^* with warrant w^* and identities $ID_{\mathcal{P}}$. The adversary \mathcal{A} wins the above game if it is able to provide a validity proof of the new IBPS $\sigma_{\mathcal{P}}^*$ on message m^* under the warrant w^* .

Definition 4. An adversary \mathcal{A} $(t, q_{H_1}, q_{H_2}, q_E, q_d, q_{pk}, q_{ps}, 2, \epsilon)$ -breaks an 2-user IBPS scheme by adaptive chosen-message and adaptive chosen-ID attack, if \mathcal{A} wins the above game with probability ϵ within time t and makes at most q_{H_1} H_1 queries, q_{H_2} H_2 queries, q_E extraction queries, q_d delegation queries, q_{pk} proxy key generation queries and q_{ps} proxy signature queries.

Definition 5. An IBPS scheme is $(t, q_{H_1}, q_{H_2}, q_E, q_d, q_{pk}, q_{ps}, 2, \epsilon)$ -secure against adaptive chosen-message and adaptive chosen-ID attack, if no probabilistic polynomial time adversary can $(t, q_{H_1}, q_{H_2}, q_E, q_d, q_{pk}, q_{ps}, 2, \epsilon)$ -break it.

4 PROPOSED SCHEME

In this section, we present our IBPS scheme. As defined in Section 3, our scheme consists of the following phases: *Setup*, *KeyGen*, *ProxyKeyGen*, *ProxySignature*, and *Verification*.

Setup. In the setup phase, the private key generator (PKG), on input security parameter 1^λ , generates the system's master secret key s and the system's public parameters

$$params = (\lambda, G_1, G_2, q, e, H_1, H_2, P, Pub),$$

where G_1 is an additive cyclic group of prime order q with generator P ; G_2 is a multiplicative cyclic group of prime order q with generator g ; $e : G_1 \times G_1 \rightarrow G_2$ is a bilinear map as defined in Section 2; $H_1 : \{0, 1\}^* \rightarrow G_1$ and $H_2 : \{0, 1\}^* \times G_1 \rightarrow \mathbb{Z}_q^*$ are two cryptographically secure hash functions, and $Pub = sP \in G_1$ is system's public key.

KeyGen. Given an identity ID , the PKG returns the public and private keys for ID as follows:

- public key: $Q_{ID} := H_1(ID) \in G_1$; and
- private Key: $S_{ID} := sQ_{ID} \in G_1$.

Thus, the original signer O has its private key S_{ID_O} while anyone can compute the corresponding public key Q_{ID_O} . Similarly, for the proxy signer \mathcal{P} , the public key is $Q_{ID_{\mathcal{P}}}$ and corresponding private key is $S_{ID_{\mathcal{P}}}$.

ProxyKeyGen.

Make Warrant: In this phase, the original signer O delegates its signing capability to the proxy signer through a signed warrant w . The warrant w includes the identity of original signer O , the identity of the proxy signer \mathcal{P} , the time of delegation, the period of validity, the nature of messages that can be signed, etc.

Sub Proxy Generation: The original signer O randomly chooses $x_O \in \mathbb{Z}_q^*$ and computes

- $S_O = x_O Q_{ID_O} \in G_1$,
- $h_2 = H_2(w, S_O) \in \mathbb{Z}_q^*$, and
- $T_O = (x_O + h_2)S_{ID_O} \in G_1$ and appends to the warrant w ,

Finally, the original signer O sends $D = (w, S_O, T_O)$ to the proxy signer through a secure channel, with T_O as a delegation value.

Sub Proxy Verification: The proxy signer \mathcal{P} , accepts the delegation value T_O on warrant w , if the equality

$$e(T_O, P) = e(S_O + h_2 Q_{ID_O}, Pub)$$

holds. Otherwise, it asks for a new delegation value or terminate the protocol.

Proxy Key Generation: After receiving the (correct) delegation value, the proxy signer \mathcal{P} generates its proxy signing key as

$$d_{\mathcal{P}} = T_O + h_2 S_{ID_{\mathcal{P}}}.$$

ProxySignature. To generate the proxy signature on a message m , on behalf of the original signer O , the proxy signer \mathcal{P} does the following:

- randomly selects $y \in \mathbb{Z}_q^*$
- computes $U_{\mathcal{P}} = y \cdot Pub \in G_1$
- $h_3 = H_2(m, U_{\mathcal{P}}) \in \mathbb{Z}_q^*$
- and $V_{\mathcal{P}} = (y + h_3)d_{\mathcal{P}}$.

The proxy signature generated by the proxy signer \mathcal{P} on the message m , on behalf of the original signer O is $\sigma_{\mathcal{P}} = (S_O, U_{\mathcal{P}}, V_{\mathcal{P}})$.

Verification. Receiving a proxy signature $\sigma_{\mathcal{P}} = (w, S_O, U_{\mathcal{P}}, V_{\mathcal{P}})$ and message m , the verifier proceeds as follows:

1. Checks the validity of message m with respect to the warrant w . Continue, if it is a valid one. Rejects otherwise.
2. Checks the authorization of the proxy signer by the original signer. Stop the verification, if the proxy signer is not authorized by the warrant. Continue otherwise.
3. Finally, accepts the proxy signature if the equality

$$e(V_{\mathcal{P}}, P) = e(S_O + h_2(Q_{ID_O} + Q_{ID_{\mathcal{P}}}), U_{\mathcal{P}} + h_3 Pub)$$

holds.

5 PROOF OF CORRECTNESS

The correctness of verification of our scheme holds as follows:

$$\begin{aligned}
& e(S_O + h_2(Q_{ID_O} + Q_{ID_P}), U_P + h_3Pub) \\
&= e(x_O Q_{ID_O} + h_2 Q_{ID_O} + h_2 Q_{ID_P}, yPub + h_3Pub) \\
&= e((x_O + h_2)Q_{ID_O} + h_2 Q_{ID_P}, (y + h_3)Pub) \\
&= e((x_O + h_2)S_{ID_O} + h_2 S_{ID_P}, (y + h_3)P) \\
&= e(T_O + h_2 S_{ID_P}, (y + h_3)P) \\
&= e(d_P, (y + h_3)P) \\
&= e((y + h_3)d_P, P) \\
&= e(V_O, P)
\end{aligned}$$

6 PROOF OF SECURITY

In this section, we give the theorem which proves of the security of our scheme against existential forgery on adaptive chosen-message and adaptive chosen-identity attack in the random oracle model. In the full paper, we will use the rewinding technique and the forking lemma (Pointcheval and Stern, 2000) to prove this theorem.

Theorem 6. If there exists an adversary $\mathcal{A}(t, q_{H_1}, q_{H_2}, q_E, q_d, q_{pk}, q_{ps}, 2, \epsilon)$ which breaks the proposed IBPS scheme in time t with success probability ϵ , then there exists an adversary $\mathcal{B}(t', \epsilon')$ which solves CDHP with success probability at least $\epsilon' \geq \frac{(1-1/q)\mathbb{M}}{q_E + q_d + 2q_{pk} + 3q_{ps} + 3}\epsilon$ in time at most $t' \geq t + (q_{H_1} + q_E + 2q_d + q_{pk} + 2q_{ps} + 4)C_{G_1}$ where C_{G_1} denotes the maximum time taken for scalar multiplication in G_1 and \mathbb{M} is a maximum value as defined in the proof.

Let, for a security parameter 1^λ , the adversary \mathcal{B} is challenged to solve the CDHP for $\langle q, G_1, P, sP, bP \rangle$ where G_1 is an additive cyclic group of prime order q with generator P and $s, b \in \mathbb{Z}_q^*$. The goal of \mathcal{B} is to solve CDHP by computing $sbP \in G_1$ using \mathcal{A} , the adversary who claims to forge our proposed IBPS scheme. \mathcal{B} simulates the security game with \mathcal{A} as described in section 3, and solves an instance of CDHP, using the values \mathcal{A} requires to forge the proposed signature. The probability of success and time have been calculated as given in the above theorem, in the full version we will describe the complete analysis we have done, with all the required steps.

7 EFFICIENCY COMPARISON

In this section, we compare the efficiency of our IBPS

scheme with the existing popular IBPS schemes (Xu et al., 2005), (Chow et al., 2005), (Wu et al., 2007) and (Wang, 2008), and with the recently proposed IBPS schemes (Singh and Verma, 2012; Asaar et al., 2014; Sarde and Banerjee, 2015) and show that our scheme is more efficient in the sense of computation and operation time than these existing schemes. As few of the recently proposed proxy signature schemes (Singh and Verma, 2012; Asaar et al., 2014) provide the message recovery property, we do not count the computation for the message recovery during the comparison. To evaluate the total operation time in the efficiency comparison tables, we use the method from (Cao et al., 2010; He et al., 2011). We note that the OT for one pairing computation is 20.04ms, for one map-to-point hash function it is 3.04ms, for one modular exponentiation it is 5.31ms, for one scalar multiplication it is 6.38ms and for one general hash function it is $< 0.001ms$. For example, during the proxy key generation phase of our scheme, each proxy signer computes 2 pairings (P), 0 map-to-point hash (H), 0 modular-exponentiation (E), and 4 scalar multiplications (S), hence the total operation time can be calculated as: $2 \times 20.04 + 0 \times 3.04 + 0 \times 5.31 + 4 \times 6.38 = 65.60ms$. The OT for each phase of all the schemes has been computed similarly.

From the efficiency comparison tables, it is clear that our scheme is computationally more efficient than the existing popular and recent identity-based proxy signature schemes. In particular, our scheme is 39%, 19%, 33%, 22%, 26%, 20%, 28% and 25% more efficient than the schemes given in (Xu et al., 2005), (Chow et al., 2005), (Wu et al., 2007), (Wang, 2008), (Singh and Verma, 2012), (Asaar et al., 2014) and (Sarde and Banerjee, 2015) respectively.

8 CONCLUSION AND FUTURE WORK

We have proposed an efficient and provably secure identity-based proxy signature scheme from bilinear pairing based on the hardness of the computational Diffie-Hellman problem. In the full version of this paper, we will provide a full security analysis of our scheme. Our scheme is useful for the application in various real world scenarios including those discussed in Section 1. In the full version we will envisage a few more suitable applications for practical implementation of our scheme.

Proxy key generation

Scheme	P	H	E	SM	OT (ms)
(Xu et al., 2005)	3	3	0	3	88.38
(Chow et al., 2005)	3	1	0	3	82.30
(Wu et al., 2007)	0	1	0	2	15.80
(Wang, 2008)	3	1	0	2	77.92
(Singh and Verma, 2012)	3	1	2	3	92.92
(Asaar et al., 2014)	3	2	2	0	76.82
(Sarde and Banerjee, 2015)	4	1	2	4	119.26
Our scheme	2	0	0	4	65.60

Proxy signature generation

Scheme	P	H	E	SM	OT (ms)
(Xu et al., 2005)	0	1	0	2	15.80
(Chow et al., 2005)	0	1	0	2	15.80
(Wu et al., 2007)	0	3	0	6	47.40
(Wang, 2008)	0	1	0	2	15.80
(Singh and Verma, 2012)	1	0	1	1	31.73
(Asaar et al., 2014)	1	0	1	0	35.71
(Sarde and Banerjee, 2015)	0	0	0	2	12.76
Our scheme	0	0	0	2	12.76

Verification

Scheme	P	H	E	SM	OT (ms)
(Xu et al., 2005)	5	4	1	0	117.66
(Chow et al., 2005)	3	1	0	1	69.54
(Wu et al., 2007)	5	4	0	0	112.36
(Wang, 2008)	4	4	0	0	92.32
(Singh and Verma, 2012)	2	0	1	0	45.39
(Asaar et al., 2014)	3	2	2	0	76.82
(Sarde and Banerjee, 2015)	2	0	1	1	51.77
Our scheme	2	2	0	2	58.92

Overall operation time

Scheme	P	H	E	SM	OT (ms)
(Xu et al., 2005)	8	8	1	5	221.84
(Chow et al., 2005)	6	3	0	6	167.64
(Wu et al., 2007)	5	8	0	8	175.56
(Wang, 2008)	7	6	0	4	186.04
(Singh and Verma, 2012)	6	1	4	4	170.04
(Asaar et al., 2014)	7	4	5	0	189.35
(Sarde and Banerjee, 2015)	6	1	3	7	183.79
Our scheme	4	2	0	8	137.28

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