

# Gap Processes for Analysing Buyers' Burstiness in E-Business Process

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**Keywords:** E-Business Process, Buyers' Burstiness, Gap Processes, Binary Customer Behaviour.

**Abstract:** Success of e-business process requires analysis of buyers' burstiness. The research question is as follows: How to describe buyers' burstiness in e-business by a mathematical model? The aim of the research is to provide a mathematical model for evaluation of buyers' burstiness in e-business process. In this work the buyers' burstiness in e-business process is described by a mathematical concept based on gap processes. Therefore, the meaning of the key concepts of buyers' burstiness, binary customer behavior and gap processes is studied. Moreover, the analysis demonstrates how the key concepts are related to the idea of e-business process and shows a potential model for development, indicating how the steps of the process are related following a logical chain: conceptual framework → model development → empirical study → conclusions. The results of the present research show the model for evaluation of buyers' burstiness in e-business process. The model itself is based on the assumption that the gaps between two buyers are statistically independent from each other. The novel contribution of the paper is revealed in the newly created mathematical model for evaluation of buyers' burstiness in e-business process via gap processes.

## 1 INTRODUCTION

The phenomenon of burstiness attracts more and more research efforts as it influences the flow of a number of processes including e-business. It should be noted that the flow of e-business process is permanently optimized in order to increase the profit (Ahrens et al., 2015a). Optimization of e-business process implies choices about quantity of goods to be delivered, number of the staff to be employed as highlighted in (Ahrens et al., 2015a), goods' pricing, goods discounts, computer software to be installed, networking between a business company and its customers to be established, etc. Additionally, such a result of business process as purchase and/or sale of a good or service indicates the output of this process. By phenomenon's burstiness, intervals of high-activity alternating with long low-activity periods are meant. Tab. 1 demonstrates the phenomenon of burstiness in a range of scientific fields.

Beginning in 1960 Gilbert presented the first model in telecommunications which emphasized that bit errors occurred in bundles or, in other words, bursts (Gilbert, 1960; Elliott, 1963). Since then, the issues of a general procedure to evaluate the performance or, in other words, e-business process in the present research, as well as a basic set of parameters

or, in other words, criteria, are still relevant today.

In business including e-business, burstiness of workload is traditionally analyzed (Heinrich, 2014). However, the paradigm has changed from an input based business process or, in other words, burstiness of workload to an outcome based process or, in other words, burstiness of buyers (Ahrens et al., 2015a). The shift from analysis of burstiness of workload to evaluation of burstiness of buyers allows increasing the efficiency of e-business process and, consequently, e-business profit. It should be noted that the concept burstiness of buyers is developed by the international group of researchers such as Ahrens, Purvinis, Zašcerinska, Andreeva (Ahrens et al., 2015a). The previous work in the field of burstiness of buyers includes elaboration of a conceptual framework on criteria for qualitative decisions in business (Ahrens et al., 2015b), mathematical analysis of gap processes underpinning elaboration of a simulation model of binary customer behaviour within business related processes (Ahrens et al., 2015a), design of a simulation model of binary customer behaviour in a bursty business process based on gap processes (Ahrens et al., 2016).

The research question is as follows: How to describe buyers' burstiness in e-business by a mathematical model?

Table 1: Burstiness in different scientific fields

Scientific field	Phenomenon of burstiness
Telecommunications	Burstiness of bit-errors in data transmission
Economics	Burstiness of crises
Natural sciences	Burstiness of disasters or earthquakes
Logistics	Burstiness of traffic
Social media	Burstiness of hot topic, keyword or event
Business	Burstiness of workload
E-Business	Burstiness of buyers

The aim of the research is to provide a mathematical model for evaluation of buyers' burstiness in e-business process. The novel contribution of the paper is revealed in the newly created mathematical model for evaluation of buyers' burstiness in e-business process via gap processes. For the model elaboration, the meaning of the key concepts of buyers' burstiness, binary customer behavior and gap processes is studied. Moreover, the analysis demonstrates how the key concepts are related to the idea of e-business process and shows a potential model for development, indicating how the steps of the process are related following a logical chain: conceptual framework → model development → empirical study → conclusions.

The present contribution employs interdisciplinary research as it assists in synthesizing, connecting and blending ideas, data and information, methods, tools, concepts, and/or theories from two or more disciplines in order *to make whole* (Repko, 2012). For the design of a mathematical model for evaluation of buyers' burstiness in e-business process, the synergy between e-business and telecommunications is promoted as the phenomenon of customers in the e-business process as well as bit-errors in data transmission appear to be of a similar nature, namely, the bursty nature. Such mathematical models that consider the bursty nature of bit-errors in data transmission have been successfully implemented in telecommunications for optimizing data communication protocols and will be adopted in this work to the buyers' burstiness in e-business process. It should be noted that the present research is not limited to only two scientific disciplines, namely e-business and telecommunication, but is based on a number of scientific disciplines such as business, social media, logistics, literature, etc.

The remaining part of this paper is organized as follows: Section 2 introduces buyers' burstiness in e-business process. A mathematical model for evaluation of buyers' burstiness in e-business process via gap processes is presented in Section 3. The associated results of an empirical study will be discussed

in Section 4. Finally, some concluding remarks are provided in Section 5.

## 2 BUYERS' BURSTINESS IN E-BUSINESS PROCESS

By e-business process, the process of buying and/or selling of goods and/or services through Information and Communication Technologies (ICT) is meant. In the present work, e-business process is built within the paradigm of binary customer behaviour. For defining binary customer behaviour, such an everyday e-business situation is considered as potential customers have to solve an issue formulated already in 1603 by William Shakespeare in his play Hamlet such as *to be, or not to be* (Shakespeare, 1825). Regarding a modern interpretation of potential customers' contemporary problems, Shakespeare's words may sound as *to buy, or not to buy*. It should be noted that *to buy, or not to buy* is considered as binary customer behavior depicted in Fig. 1. Fig. 2 shows a typical scenario in which a buyer who



Figure 1: Elements of customers' binary option

made a purchase as the output of e-business process is highlighted (represented by "x") within a sequence of people (represented by "-") who visited an e-shop. It should be noted that by e-shop visitor any customer who seeks and examines a product without buying it is understood. E-business process which ends without a purchase or sale creates a gap between two buyers (Ahrens et al., 2015a). These gaps are assumed to be statistically independent from each other (Ahrens et al., 2015a).

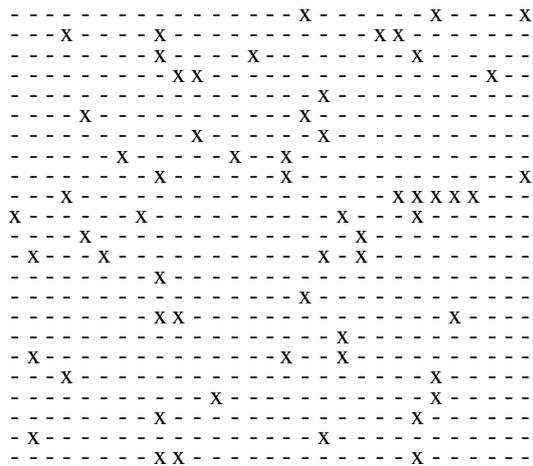


Figure 2: A buyer (represented by "x") within a sequence of e-shop visitors (represented by "-")

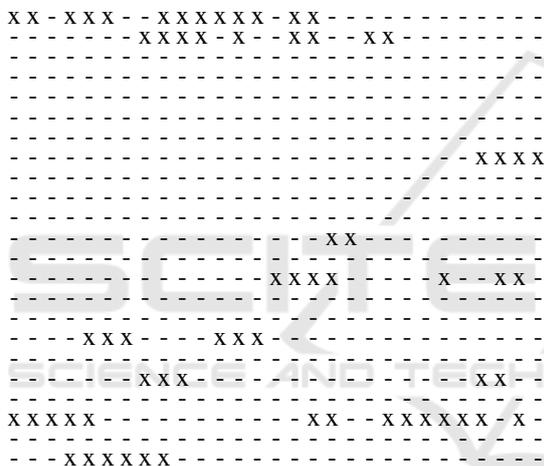


Figure 3: Buyers' burstiness (represented by "x") within a sequence of e-shop visitors (represented by "-")

It should be noted that the terms "gap", "gap process" and "gap distribution function" are used synonymously in the present contribution. Gaps are rooted in the Hidden Markov Models (HMM) (Gilbert, 1960; Elliott, 1963). What has however interested communication protocol developers and coding theorists, are the probabilities of error structures in any finite time interval such as the block length or the cycle length of a transmission procedure. These probabilities are typically difficult to present analytically. Some solutions were presented by Wilhelm in 1976 resulting in gap models such as the L-model or the A-model (Wilhelm, 1976; Ahrens, 2000). With these models the bursty nature of transmission errors in ICT could be simulated. This approach based on gap processes is now considered as a possible solution of evaluation of buyers' burstiness in e-business process. Fig. 4 il-

lustrates the e-business process between two buyers described by gaps. However, the buyers can be more

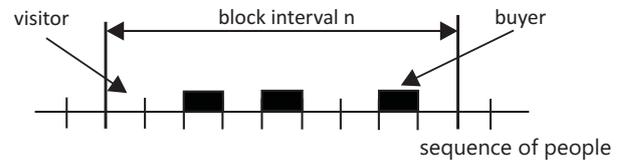


Figure 4: Buyer's gap for describing binary customer behavior

independently distributed over e.g. a day or they can appear really concentrated as highlighted in Fig. 3.

In situations where binary decisions in e-business processes such as selling or buying are made, not only purchases and sales are of any interest but also how concentrated goods are sold or bought. That is why models which focus only on the purchases and sales with a given probability are not exact enough to describe e-business process. In general, the buyers' probability can serve as a clear indicator of how often people decide to buy e.g. a product. However, the buyers' probability does not deliver any information about how concentrated the purchases and/or sales are. Thus, buyers' burstiness is a criterion in e-business process. The criterion of buyers' burstiness includes such indicators as buyers' probability and buyers' concentration (Ahrens et al., 2015a) as summarized in Tab. 2.

Table 2: Criterion and indicators of burstiness in e-business process

Criterion	Indicator
Buyers' burstiness	Buyers' probability and Buyers' concentration

For comparison purposes of the present interdisciplinary research, Tab. 3 demonstrates the criterion and indicator of evaluation of burstiness of hot topic, keyword, event, etc. in a sequence of batched georeferenced documents in social media. This model is developed by a group of Japanese researchers as geannotated user-generated data on social media sites is becoming one of the most influential sources of information (Kotozaki et al., 2015).

Table 3: Criterion and indicator of burstiness in social media

Criterion	Indicator
Burstiness of hot topic or keyword in a sequence of batched georeferenced documents	Locality

This group of Japanese researchers built their model of evaluation of burstiness of hot topic, keyword, etc in a sequence of batched georeferenced documents on Kleinberg's burst detection algorithm, which is based on a queuing theory for detecting bursty network traffic (Kotozaki et al., 2015). It should be noted that Kleinberg's solution does not provide clear distinction between within-burst and out-of-burst records (Mai et al., 2015).

A comparison of the model of evaluation of burstiness of hot topic, keyword, etc. in social media shown by the group of Japanese researchers (Kotozaki et al., 2015) with the model for evaluation of buyers' burstiness in e-business process is reflected in Tab. 4. The comparative analysis of Tab. 4 reveals that Kleinberg's burst detection algorithm, which is based on a queuing theory, is built on a sequence of phenomena while gap distribution function is featured by sequential independence of gaps between two buyers.

The comparative analysis assists in concluding that e-business process is characterized by sequential independence of gaps between two buyers. Consequently, the methodological background for evaluation of buyers' burstiness in e-business process should take it into account while developing a mathematical model for evaluation of buyers' burstiness in e-business process.

### 3 MODEL FOR EVALUATION OF BUYERS' BURSTINESS IN E-BUSINESS PROCESS

The term "model" is of great research interest. In pedagogy, by model a pattern is meant. In mathematics, a model is an interpretation of a theory. In engineering, business and computer sciences, a model describes a system. Interdisciplinary (pedagogy, mathematics, engineering, business and computer sciences) analysis of the term model leads to such a newly defined notion of the term model as a pattern of individual's or individuals' interpretation of a phenomenon. Models can be presented in a variety of forms such as verbal, graphic, computer, etc.

For model creation, we can define a block interval  $n$  (identified as the probability  $p_B(n)$  where at least one buyer appears. The parameter  $n$  refers e.g. to the number of people entering a shop in a given time e. g. a day. Choosing the parameter  $n = 1$  the probability  $p_B(n)$  equals the buyers probability  $p_e$ .

The block buyer probability  $p_B(n)$  can be described as a function of the buyers' probability  $p_e$  and the block interval length  $n$  as investigated by real data

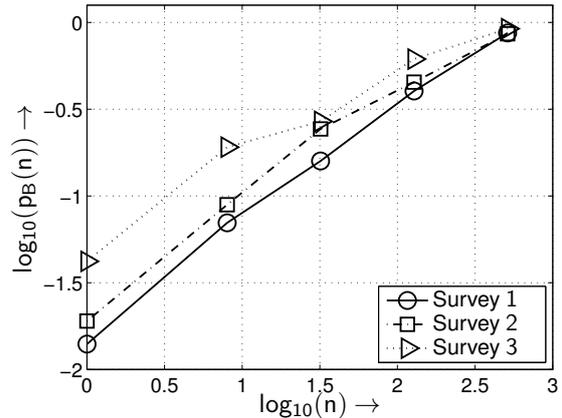


Figure 5: Measured block buyer's probabilities  $p_B(n)$  as a function of the interval length  $n$

analysis (see Fig. 5). Here, the following approximation is used

$$p_B(n) = \begin{cases} p_e \cdot n^\alpha & 1 \leq n \leq n_0 \\ 1 & n > n_0 \end{cases} \quad (1)$$

Re-writing (1) yields

$$\log_{10}(p_B(n)) = \alpha \log_{10}(n) + \log_{10}(p_e) \quad (2)$$

Therein, the value  $\alpha$  denotes the linear dependence between  $\log_{10}(p_B(n))$  and  $\log_{10}(n)$  and is a measure for the buyers' concentration (also referred to the concentration of buying). The value of  $n_0$  indicates the maximum interval length to which the linear-dependence can be maintained (see also Fig. 6).

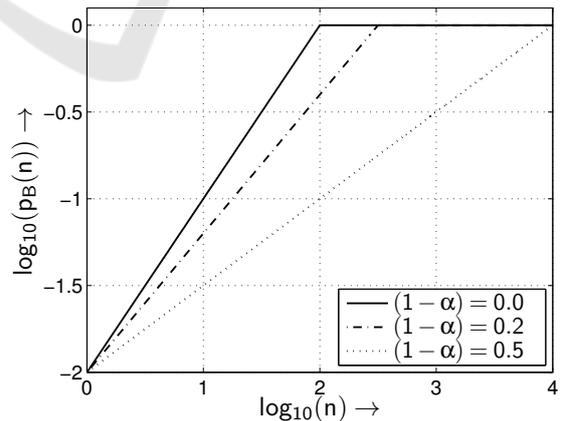


Figure 6: Approximated block buyer's probability  $p_B(n)$  as a function of the interval length  $n$  for different parameters of the  $(1 - \alpha)$  at a buyer's probability of  $p_e = 10^{-2}$

The analysis of concentration parameters  $(1 - \alpha)$  (referred to the concentration of buying) has shown that parameters in the range of 0.0 until 0.5 describe

Table 4: Comparison of models for evaluation of burstiness in social media and e-business process

Model's Element	Social Media	E-Business Process
Feature	Sequence of batched georeferenced documents	Sequential independence of gaps between two buyers
Methodological background	Kleinberg's burst detection algorithm based on queuing theory	Gap distribution function

realistic scenarios. Thereby, a parameter  $(1 - \alpha) = 0$  describes the situation where the potential buyers appear independently distributed from each other. With increasing parameter  $(1 - \alpha)$  the buyers appear more and more concentrated and the probability  $p_B(n)$  decreased for a given  $n$ .

Describing the buying process by gaps, the block buyer probability  $p_B(n)$  can be defined by the gap-distribution function  $u(k) = P(X \geq k)$ , which describes the probability of a gap larger than  $k$ , as highlighted in Fig. 7. With the assumption that the distances (gaps  $k$ ) between neighboring buyers are statistically independent from each other, the buyers' characteristic, namely the occurrence of bursty buyers, is defined by the buyers' gap-distribution function  $u(k) = P(X \geq k)$  completely. The approach

$$p_B(n) = \begin{cases} p_e \cdot \sum_{k=0}^{n-1} u(k) & 1 \leq n \leq n_0 \\ 1 & n > n_0 \end{cases} \quad (3)$$

can be used to develop the buyer's gap distribution function  $u(k)$  for the buyers' gaps step by step. Comparing (1) and (3), one gets:

$$\sum_{k=0}^{n-1} u(k) = n^\alpha \quad 1 \leq n \leq n_0 \quad (4)$$

and for the searched error-gap distribution  $u(k)$  we yield step by step:

$$\begin{aligned} n = 1 & : u(0) & = 1 \\ n = 2 & : u(0) + u(1) & = 2^\alpha \\ n = 3 & : u(0) + u(1) + u(2) & = 3^\alpha \\ \dots & : \dots & = \dots \\ n \leq n_0 & : u(0) + u(1) + \dots + u(n-1) & = n^\alpha \end{aligned}$$

The buyer's-gap distribution function  $u(k)$  results finally in

$$u(k) = \begin{cases} (k+1)^\alpha - k^\alpha & 0 \leq k < n_0 \\ 0 & k \geq n_0 \end{cases} \quad (5)$$

Re-writing of  $u(k)$  leads to the buyers-gap density function  $v(k) = P(X = k)$ , which describes the probability of a gap  $X$  equal to  $k$ :

$$\begin{aligned} u(k) & = v(k) + v(k+1) + v(k+2) + \dots \\ u(k+1) & = v(k+1) + v(k+2) + \dots \end{aligned}$$

and by calculating the difference between  $u(k)$  and  $u(k+1)$  the buyers-gap density function  $v(k) = P(X = k)$  can be obtained

$$v(k) = u(k) - u(k+1) \quad (6)$$

Assuming that the buyers are independently distributed, i.e.  $(1 - \alpha) = 0$ , and using equation (5) and (6) one gets the following result for the buyers-gap density function  $v(k)$ :

$$v(k) = \begin{cases} 1 & k = (n_0 - 1) \\ 0 & k \neq (n_0 - 1) \end{cases} \quad (7)$$

With this result, the disadvantage of the model setup becomes evident. The model setup defined in (1) leads to a deterministic buyers-gap process. In situations, where the buyers appear concentrated, i.e.  $(1 - \alpha) > 0$ , one can also find an enlarged value at  $v(n_0 - 1)$ . This error leads to engraving inaccuracies in the simulation process. The reason is the discontinuity at  $n = n_0$  in equation (1). A modification of this model setup is necessary.

The following solution can be assumed: The linear increases of  $\log_{10}(p_B(n))$  can only be accepted for small parameters of  $n$ . The value of  $\log_{10}(p_B(n))$  has to change steadily into the value  $\log_{10}(p_B(n)) = 0$  for larger  $n$ . To the minimization of the model inaccuracy at  $v(n_0 - 1)$  equation (5) has to be multiplied by the value  $e^{-\beta k}$ . The modification of the model approach is highlighted in Fig. 8 with respect to the block buyer's probability  $p_B(n)$ .

For the buyers-gap distribution function  $u(k)$  the following expression arises:

$$u(k) = ((k+1)^\alpha - k^\alpha) \cdot e^{-\beta k} \quad 0 \leq k \leq \infty \quad (8)$$

with

$$\lim_{k \rightarrow \infty} e^{-\beta k} = 0 \quad \beta > 0 \quad (9)$$

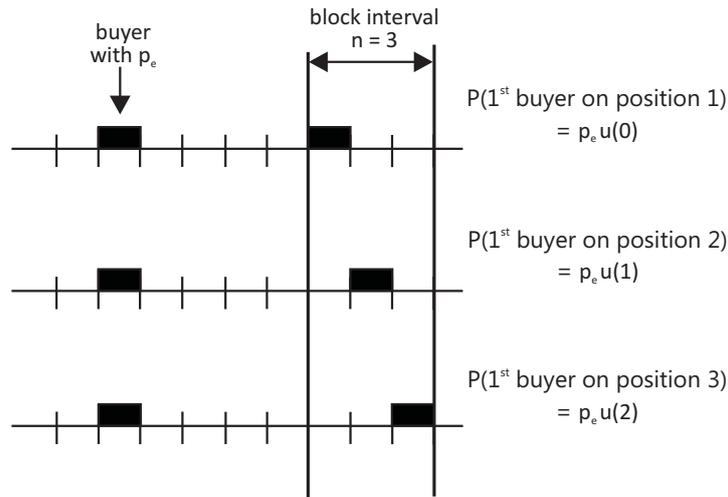


Figure 7: Calculating the block-error probability  $p_B(n)$  using the error-gap distribution function  $u(k)$

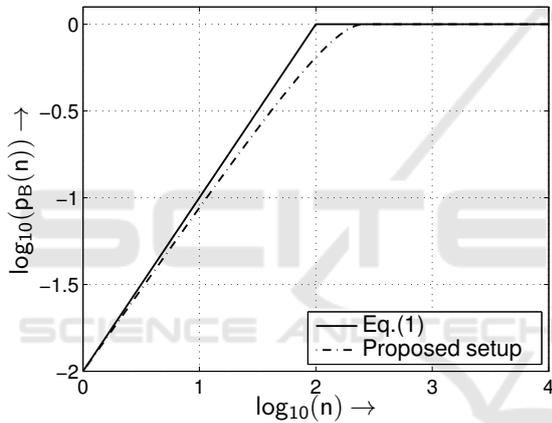


Figure 8: Approximated relationship between the probability  $p_B(n)$  and the block interval  $n$  for different parameters of the  $(1 - \alpha)$

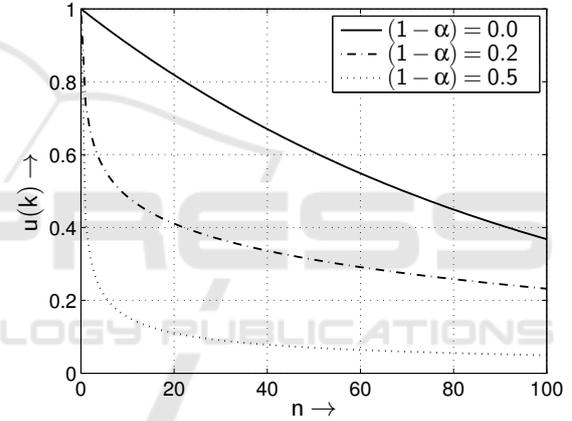


Figure 9: Buyers-gap distribution function  $u(k)$  for different parameters of the  $(1 - \alpha)$  at a buyer's probability of  $p_e = 10^{-2}$

and

$$\beta \approx p_e^{1/\alpha} \quad (10)$$

Fig. 9 illustrates the buyers-gap distribution function  $u(k)$  for different parameters  $(1 - \alpha)$  assuming a buyer's probability of  $p_e = 10^{-2}$ . The resultant buyers-gap density function  $v(k)$  is depicted in Fig. 10. With these modifications, the buyers characteristic can be modeled by two parameters (the buyer's probability  $p_e$  and the buyer's concentration value  $(1 - \alpha)$ ).

#### 4 EMPIRICAL ANALYSIS

The present part of the contribution demonstrates the design, results and findings of the empirical study.

The design of the study comprises the purpose and question, materials and methodology.

The empirical study was aimed at evaluating the buyers' burstiness in a e-business process. It should be noted that for the empirical study's purposes, by e-business process, the process of buying a scientific paper is determined. The empirical study's question was as follows: Is the e-business process, namely the online process of selling and buying a scientific paper, characterized by buyers' burstiness? The present empirical study was carried out in January 2016. Analysis of statistical documents of e-shop which sells scientific papers was carried out.

Interpretive research paradigm was used in the present empirical study. The interpretive paradigm aims to understand other cultures, from the inside

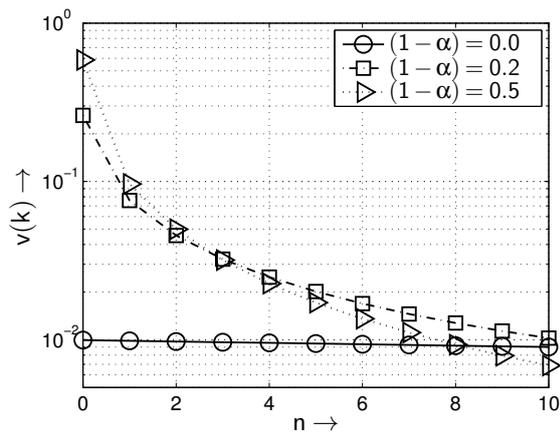


Figure 10: Buyers-gap density function  $v(k)$  for different parameters of the  $(1 - \alpha)$  at a buyer's probability of  $p_e = 10^{-2}$

through the use of ethnographic methods such as informal interviewing, participant observation and establishment of ethically sound relationships (Taylor and Medina, 2013).

Exploratory research aimed at generating new questions and hypothesis was employed in the empirical study (Phillips, 2006). The exploratory methodology proceeds from exploration in Phase 1 through analysis in Phase 2 to hypothesis development in Phase 3.

The qualitatively oriented empirical study allows the construction of only few cases (Mayring, 2004). The cases themselves are not of interest, only the conclusions and transfers we can draw from these documents (Flyvbjerg, 2006). Selecting the cases for the case study comprises use of information-oriented documents, as opposed to random documents (Flyvbjerg, 2006). This is because an average case is often not the richest in information. In addition, it is often more important to clarify the deeper causes behind a given problem and its consequences than to describe the symptoms of the problem and how frequently they occur (Flyvbjerg, 2006).

Analysis of statistical documents of e-shop which sells scientific papers revealed that one author's website was visited 1161 times in 2015. From around 100 scientific papers of this particular author, one scientific paper was bought 109 times in 2015. Fig. 11 demonstrates the monthly distribution of the purchases of this particular scientific paper. Summarizing content analysis (Mayring, 2004) of the data reveals the buyers' burstiness in buying an e-product.

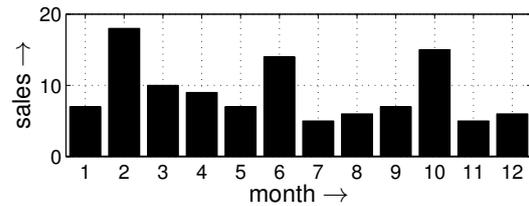


Figure 11: Monthly distribution of the purchases of this particular scientific paper.

## 5 CONCLUSIONS

The theoretical analysis of the binary customer behaviour assists in outlining gap processes in e-business process. The theoretical findings on the inter-relationship between e-business process, binary customer behaviour, the buyers' burstiness and gap processes allow determining the model for evaluation of buyers' burstiness in e-business process. The model for evaluation of buyers' burstiness in e-business process is based on the chosen gap distribution function. The proposed model could be used for both, namely simulation and detection of burstiness of buyers. The presented mathematical model could be used when making business decisions about the resources needed to provide a service.

The empirical findings of the research allow drawing the conclusions on the buyers' burstiness in e-business process. The following new research question has been formulated: How to optimize e-business process based on gap processes?

The present research has such limitations: The inter-connections between e-business process, binary customer behaviour, the buyers' burstiness and gap processes have been set. Another limitation is the empirical study based on one case only. Therein, the results of the study cannot be representative for the whole area. Nevertheless, the results of the research, namely the elaborated mathematical model for evaluation of buyers' burstiness in e-business process that is based on gap processes, may be used as a basis of analysis of buyers' burstiness in e-business process. If the results of other cases had been available for analysis, different results could have been attained. There is a possibility to continue the study.

Further research tends to facilitate the advancement of interconnections between buyers' burstiness and e-business process. The search for relevant methods, tools and techniques for evaluation of buyers' burstiness in e-business process is proposed. Future research tends to analyse the implementation of the elaborated mathematical model based on gap processes for evaluation of buyers' burstiness in e-

business process. A comparative research of models for evaluation of burstiness in other scientific fields could be carried out, too.

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