

Individual Mobility Patterns in Urban Environment

Pierpaolo Mastroianni¹, Bernardo Monechi², Vito D. P. Servedio^{3,4}, Carlo Liberto¹, Gaetano Valenti¹
and Vittorio Loreto^{3,2}

¹*ENEA, Casaccia Research Center, Via Anguillarese 301, 00123, Rome, Italy*

²*Institute for Scientific Interchange Foundation, Via Alassio 11/c, 10126, Turin, Italy*

³*Sapienza University of Rome, Physics Dept., P.le Aldo Moro 2, 00185 Roma, Italy*

⁴*Institute for Complex Systems (ISC-CNR), Via dei Taurini 19, 00185 Roma, Italy*

Keywords: Urban Mobility, Daily Patterns, Optimization, Circadian Rhythm.

Abstract: The understanding and the characterization of individual mobility patterns in urban environments is important in order to improve liveability and planning of big cities. In relatively recent times, the availability of data regarding human movements have fostered the emergence of a new branch of social studies, with the aim to unveil and study those patterns thanks to data collected by means of geolocalization technologies. In this paper we analyze a large dataset of GPS tracks of cars collected in Rome (Italy). Dividing the drivers in classes according to the number of trips they perform in a day, we show that the sequence of the traveled space connecting two consecutive stops shows a precise behavior so that the shortest trips are performed at the middle of the sequence, when the longest occur at the beginning and at the end when drivers head back home. We show that this behavior is consistent with the idea of an optimization process in which the total travel time is minimized, under the effect of spatial constraints so that the starting points is on the border of the space in which the dynamics takes place.

1 INTRODUCTION

The spreading of ICT (Information and Communication Technology) devices across the population has led to the unprecedented possibility to monitor the daily activity of citizen's in almost real time (Mayer-Schonberger and Cukier, 2013). Despite privacy issues (Tene and Polonetsky, 2012; Rubinstein, 2013), these new technology are of utmost importance in social science studies, since they are opening the possibility for a better understanding of large-scale collective phenomena (Gonzalez-Bailon, 2013; Lazer et al., 2009; Eluru et al., 2009). Among the possibility offered by the large availability of human activity data, the study of how people move and interact within a urban environment could give an important contribute to the future social challenges in terms of reducing pollution and increase the livability of big cities (United Nations Secretariat, 2014). Moreover, the study of other phenomena like epidemic spreading (Eubank et al., 2004; Colizza et al., 2007) cannot abstract from the understanding of human mobility. For these reasons, many relatively recent works have focused on the derivation of universal statistical

laws characterizing the patterns of human movements (Brockmann et al., 2006; Gonzalez et al., 2008; Song et al., 2010; Simini et al., 2012; Wang et al., 2014). In this paper we address the problem of the characterization of the daily patterns of car drivers, in order to understand how they move between different areas of the city during each day. Similarly to other works (Bazzani et al., 2010; Gallotti et al., 2012; Rambaldi et al., 2007; Gallotti, 2013; Gallotti et al., 2015), we analyze a large database of GPS tracks of private cars collected in the Rome (Italy) district during the whole month of May 2011. The study of the dynamics of car travel has a long tradition in Complex Systems and Physics framework, modelling traffic flows from both an Eulerian and Lagrangian perspectives (Treiber and Kesting, 2013; Rambaldi et al., 2007). Our findings suggest that there is a universal pattern in the way drivers choose the sequence of places they have to visit, so that independently of their number the sequence of the length of the trips connecting them has a "parabolic" shape. Note that a similar approach has been used also considering mobile phone data (Calabrese et al., 2013), which is a proxy for multi-modal mobility, i.e. all the possible kind of means of trans-

portation are considered. By means of a model introduced in (Mastroianni et al., 2015), we show how this pattern can emerge from the interplay between the geometric constraints of the space in which the stops are placed and the need to optimize the overall travel time in order to get back to the starting point at the end of the day.

2 GPS TRACKS DATA AND DAILY DYNAMICS PATTENRS

2.1 Data

Approximately 4% and 8% of the whole vehicle population inside the Rome (Italy) district during the months of May 2011 and May 2013, respectively, was monitored on behalf of an insurance company (Oct, 2014). For that, time, position, velocity and covered distance of single vehicles were recorded by sampling each trajectory at a time scale of 30 seconds (on fast speed roads, e.g., on highways) or at a spatial scale of 2 km (elsewhere). This sampling strategy was chosen by that company to ensure a better sampling rate on arterial roads. A further signal was also recorded each time the engine was switched on or off so that a travel is defined as the temporal ordered sequence of points between the engine start and stop. Due to privacy issues, it is not possible to know any information about the owner of the vehicle performing the trips. Hence, in the following we do not distinguish between private or professional drivers, nor the reasons why a trip has been performed. In total, we are able to study the spatial pattern of 13,527 vehicles during 20 working days with an average number of trips per day equal to 11,524. Errors due to GPS signal quality have been already considered in (Mastroianni et al., 2015), we refer to this work for details. We consider as the same trip, two consecutive journeys performed by the same car if the stop time between them, i.e. the difference between the switch off time of the engine at the end of the first trip and the switch on time at the beginning of the second, is smaller than 5 minutes. The distribution $P(n)$ of the number n of trips performed by a driver during a day can be approximated for $n \geq 5$ by an exponential law with a typical length $n_0 = 2.51(2)$ as displayed in Fig. 1. The trips made up by a small number of stops clearly dominate overall our sample.

2.2 Parabolic Pattern of Trips

Considering the sequence of the n trips performed in a day by the same driver it is possible to define the

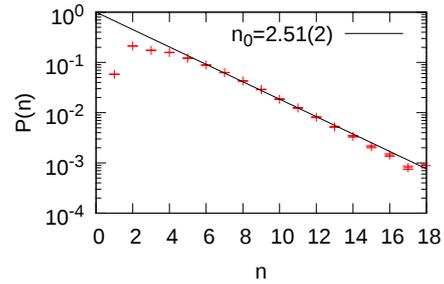


Figure 1: Distribution of the number of trips n performed by a driver in each day. Black line represents the best fit with an exponential function $\propto \exp(-\frac{n}{n_0})$.

quantity l_k^n , i.e., the distance traveled during the k^{th} trips. We can then compute \bar{l}_k^n by averaging over all the k^{th} of the drivers with n movements. This operation of average might hide the differences inside the sample it makes regular patterns emerge from the data. If the way in which each driver chooses the sequence of stops performed during a day would be random, the sequence of \bar{l}_k^n would not depend on k . Instead they follow the behavior in Fig. 2 panel a. For every value of n , the sequence of \bar{l}_k^n strongly depends on k . Moreover, the sequence of \bar{l}_k^n seems to be particularly regular being symmetric around $k = \frac{(n+1)}{2}$, where it reaches its minimum value. Thus, \bar{l}_k^n decreases starting from $k = 0$ until $k = \frac{(n+1)}{2}$ is reached, then starts growing again until $k = n$. Note that \bar{l}_0^n and \bar{l}_n^n are the highest values of the sequence and their values become more similar as n grows. This suggests that for each n , \bar{l}_k^n can be approximated by a parabola, whose coefficients depend on n :

$$p_n(k) = a_n k^2 + b_n k + c_n. \quad (1)$$

For every value of n , $p_n(k)$ is a parabola with symmetry axis orthogonal to the k axis. Fitting $p_n(k)$ by means of the sequence \bar{l}_k^n with the corresponding value of n , the coefficients a_n , b_n and c_n grow with n as a power-law (Fig. 2 panel b).

Thus it is possible to write:

$$\begin{aligned} a_n &= A n^{-\eta_a} \\ b_n &= B n^{-\eta_b} \\ c_n &= C n^{-\eta_c}. \end{aligned} \quad (2)$$

where $A = 8.8(7) \text{ Km}$, $B = 21(1) \text{ Km}$, $C = 24.5(8) \text{ Km}$, $\eta_a = 2.00(4)$, $\eta_b = 1.33(4)$ and $\eta_c = 0.55(2)$. Thus, combining equations (1) and (2),

$$p_n(k) = A n^{-\eta_a} k^2 + B n^{-\eta_b} k + C n^{-\eta_c}. \quad (3)$$

Dividing each members of (3) by $C n^{-\eta_c}$, we get

$$\frac{p_n(k)}{c_n} = \frac{A}{C} n^{-(\eta_a - \eta_c)} k^2 + \frac{B}{C} n^{-(\eta_b - \eta_c)} k + 1. \quad (4)$$

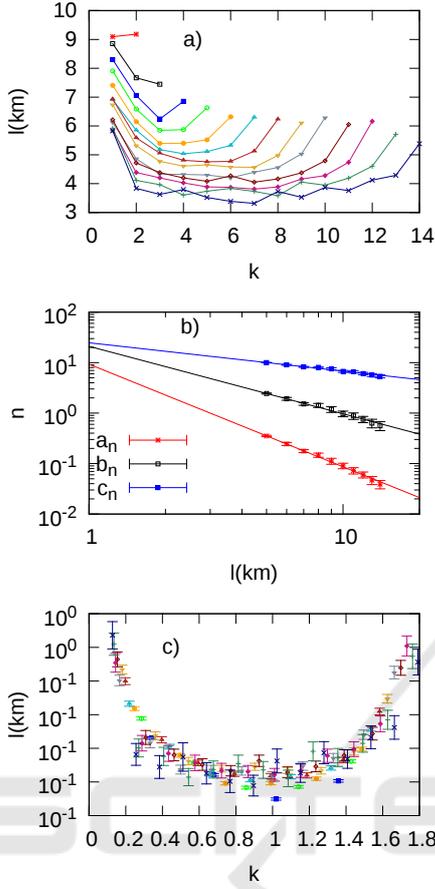


Figure 2: (a) sequences \bar{l}_k^n for some values of n . Different colors correspond to different values of the number of trips n . (b) sequences of the coefficients of the parabolic fit $p_n(k)$ for $n \geq 4$. Continuous lines are power law fit of the sequences. (c) sequences \bar{l}_k^n divided by c_n as functions of \hat{k} .

Note that with the estimated values of the parameters,

$$\begin{aligned} \eta_a - \eta_c &= 1.45(4) \\ \eta_b - \eta_c &= 0.78(4). \end{aligned} \quad (5)$$

This indicates that, within the estimated errors, the following relation holds

$$\eta_a - \eta_c = 2(\eta_b - \eta_c). \quad (6)$$

The relation (6) allows to derive a scaling law for equation (3). In fact

$$\begin{aligned} \frac{p_n(k)}{c_n} &= \frac{A}{C} n^{-2(\eta_b - \eta_c)} k^2 + \frac{B}{C} n^{-(\eta_b - \eta_c)} k + 1 \\ &= \frac{A}{C} (n^{-(\eta_b - \eta_c)} k)^2 + \frac{B}{C} n^{-(\eta_b - \eta_c)} k + 1. \end{aligned} \quad (7)$$

Then it is possible to define a new variable $\hat{k} = n^{-(\eta_b - \eta_c)} k$, obtaining a universal expression for the parabola independent of n

$$p(\hat{k}) = \frac{A}{C} \hat{k}^2 + \frac{B}{C} \hat{k} + 1. \quad (8)$$

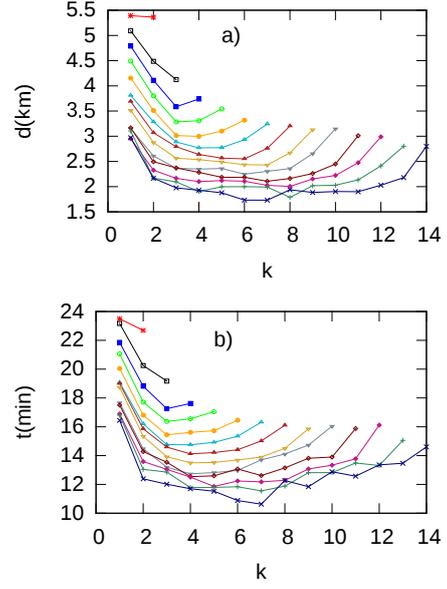


Figure 3: sequences of euclidean distances \bar{d}_k^n (panel a) and travel times \bar{t}_k^n (panel b) for some values of n .

This indicates that rescaling all the values of \bar{l}_k^n by c_n , all the data would collapse over the curve defined by Eq. (8). Fig. 2 in panel c shows the collapse of the data for all the \bar{l}_k^n sequences. The variability of the data, combined with the parabolic approximation used, does not provide a very precise collapse but the indication of a universal scaling law still holds. Similar properties can also be found for the sequence of the euclidean distances between subsequent stops d_k^n and for the sequence of the travel times t_k^n . Their corresponding parabolas are shown in Fig. 3. Despite the approximation of the parabolic fit, some general features can be identified for all the previously presented sequences. Independently of the value of n , the first value of the sequence with $k = 1$ is the largest one. Apart from $n = 2$ and $n = 3$, all the sequences are initially decreasing until a certain value of k , say k_n^* , is reached. Around $k = k_n^*$ the sequences reach a minimum, then for $k > k_n^*$ they start growing with k . As n grows, the value of k_n^* gets closer to the central value of the sequence $k = \frac{n+1}{2}$. Since the dynamics of the individual is certainly related to the circadian rhythm, the emergence of these parabolic pattern are probably related to the constraint of going back to the origin of the trip.

2.3 Spatial Constraints on Daily Stops

Indicating with \vec{x}_k the coordinates of the k -th stops of a driver, we can consider the point \vec{x}_0 as the starting point of the daily dynamics and study the spatial rela-

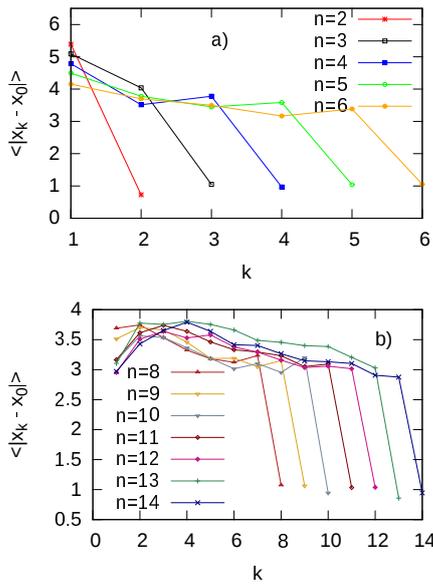


Figure 4: sequences $d_n(\vec{x}_k, \vec{x}_0)$ of the average distances from the starting point.

tions between it and the other points $k \in [1, n]$. Considering all the sequences with the same number of trips n , we can define the sequence of average distances between the k -th stop and the starting point $d_n(\vec{x}_k, \vec{x}_0) = \langle |\vec{x}_k - \vec{x}_0| \rangle$, where $|\cdot|$ indicates the euclidean distance. Fig. 4 shows these values for some value of the number of trips per day n . It is evident how the distance between the last point and the first one is usually smaller with respect to the others, indicating that usually at the end of the dynamics drivers tend to go back to the starting point. However this final distance is between 0.5 km and 1 km, so that the final point does not coincide exactly with the first one. The fact that the distance between the initial and the other stops was usually constant suggested two kinds of possible dynamics for the choice of the sequence of the stops: an *orbital dynamics* and a *bipolar dynamics*, depicted in Fig. 5. In the orbital dynamics the stops are disposed at a constant radius around the origin. In the other case the driver performs a long movement and then a series of shorter movements. These movements are so that the distances between the points they connect and the origin are quite constant so that they are displaced on average over a circular arc centered in the origin. The last movement is again a long one in order to get back to the origin. In order to discriminate between these two dynamics, we consider the barycenter of the intermediate stops (i.e. with $k \neq 0$ and $k \neq n$)

$$\vec{r}_c = \frac{1}{n-1} \sum_{k=1}^{n-1} \vec{x}_k. \quad (9)$$

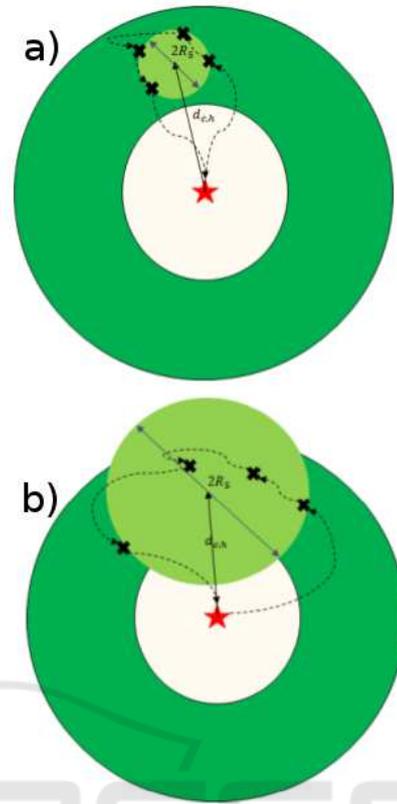


Figure 5: Representations of an orbital (a) and bipolar (b) dynamics for a driver performing 4 stops. In (a) the radius of gyration R_5 is comparable to the distance between the origin and the barycenter $d_{c,h}$ since the points are rather scattered, thus leading to $\rho_5 > 0$. On the contrary in (b) the points are quite clustered around the barycenter and thus $\rho_5 < 0$.

The gyration radius around this point is

$$R_n^2 = \frac{1}{n-1} \sum_{k=1}^{n-1} |\vec{x}_k - \vec{r}_c|^2, \quad (10)$$

measuring how much the dispersion of the intermediate stops around their barycenter. We can also define the quantity $d_{c,h} = |\vec{r}_c - \vec{x}_0|$, i.e. the distance between the starting point and the barycenter. If $R_n < d_{c,h}$, then the dispersion of the intermediate points will be smaller than the distance between the starting point and their barycenter, resulting in a bipolar dynamics. The opposite condition will indicate instead an orbital dynamics. To summarize these conditions with an adimensional metrics, we define

$$\rho_n = \frac{R_n - d_{c,h}}{R_n + d_{c,h}}, \quad (11)$$

which is bounded between -1 and 1 , so that $\rho_n < 0$ and $\rho_n > 0$ correspond respectively to the bipolar and orbital dynamics. Fig. 6 shows the average values of

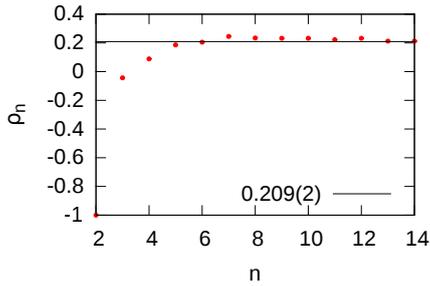


Figure 6: sequences ρ_n . The horizontal black line is the best fit of a constant function for points with $n > 4$.

R_n as function of n . For $n \geq 5$, R_n has a constant value of $0.209(2)$, while for $n < 5$ its values are smaller but still larger than 0. The only exception is represented by the point $n = 3$ which is slightly smaller than 0, and the point $n = 2$ which is equal to -1 by definition since $R_2 = 0$. Hence, the orbital dynamics seems to be the dominant one across the whole sample of tracks.

3 DAILY DYNAMICS AS CONSTRAINED OPTIMIZATION

3.1 Modeling Road Network

In (Mastroianni et al., 2015) we have introduced a simple model of road network, where the regular roads of the city were represented as an $N \times N$ grid whose links were of unit length and that could be traveled at constant unit speed. We introduced a certain number $N_{shortcut}$ of additional links connecting random nodes of the grid, that could be traveled at a higher speed $v > 1$ and their length was the euclidean distance between the nodes. These longer links represented arterial roads exploited by car driver in order to cross rapidly the urban environment. We showed how the interplay between the arterial roads and the ability of the car drivers to optimally choose the path connecting the origin and the destination of a trip (i.e., minimizing the travel time), was able to explain the sub-linear growth of the average speed during a trip with the total trip length. However, in order to reproduce the correct paths the optimization has to be sub-optimal: we supposed that the drivers are not able to choose the globally optimal path due to their limited knowledge of the urban environment, but can optimize it piece-wise between known locations until the car reach their destination. Thus, we defined a Navigation Algorithm (NA) depicted in Fig. 7. Assuming that a driver must go from the node A to the node B

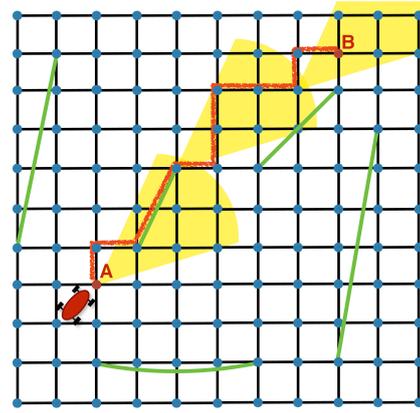


Figure 7: Representation of the Grid Network with shortcuts (green links) and of the Navigation Algorithm used to build synthetic paths connecting the nodes A and B .

of the grid, the navigation algorithm proceeds as follows:

- Assuming that the current node visited by the algorithm at its i^{th} step is n_i , we choose an optimization distance l_{optim} from a uniform distribution in $[3, l(n_i, B)]$, where $l(n_i, B)$ is the euclidean distance between n_i and B .
- The next visited node n_{i+1} is chosen randomly between all the nodes whose distance from n_i is less than l_{optim} . Moreover, the angle between the lines connecting n_i and n_{i+1} and A and B is smaller than an assigned value α ($\alpha = 30^\circ$ in the following).
- If B satisfies the conditions in the previous point, it is automatically chosen as n_{i+1} .
- If $n_{i+1} = B$ the process ends.

Once a sequence of nodes $\{A, n_1, n_2, \dots, B\}$ is made with the algorithm, the path connecting A and B is built by concatenating all the shortest paths connecting each n_i with its successive node n_{i+1} . In the next paragraph we used the NA and our urban network model to build some synthetic paths and studied how the parabolic patterns might emerge from a sequence of trips.

3.2 The Traveling Salesman

We have seen how the sequence of trips performed by a driver is linked to his circadian rhythm, since the last one is usually performed to go back to his initial location. Moreover, the sequence of stops is chosen according to an orbital dynamics, i.e., they are deployed at a constant radius around the origin. Thus, we can use this information to build a model that can reproduce these patterns in a simple way. By using our grid model and the NA, we assume that a driver

starts his daily dynamics from node i_{start} and has to perform other $n - 1$ stops before going back home. He may choose this node randomly from all the nodes in the grid, but we this choice is fundamental in order to obtain the desired pattern. Practically, the algorithm defining the sequence proceeds as follows:

- We choose $n - 1$ distinct nodes as intermediate stops. These nodes are chosen randomly between all the nodes in the grid.
- We build a path connecting each pair of stops i and j (including i_{start}) using the navigation algorithm. Thus, we construct the matrix $t_{i,j}$ of the travel times of the paths connecting every pair of stops.
- The sequence of stops $\{i_{start}, i_1, i_2, \dots, i_{n-1}, i_{start}\}$ is then the one that minimizes the total travel time, i.e.,

$$T = \sum_{k=0}^{n-1} t_{i_k, i_{k+1}}, \quad (12)$$

where $i_0 = i_n = i_{start}$.

- At the end of the procedure we have the optimal sequences d_k^n , t_k^n and l_k^n .

Note that this procedure would be equivalent to the *Traveling Salesman Problem* (Flood, 1956) on the grid with shortcuts if the shortest path between two successive nodes would have been chosen instead of the one build with the navigation algorithm. In the following, we sampled more than 2,000 paths for each value of n and $N_{shortcuts}$ in the grid model, and computed \overline{d}_k^n , \overline{t}_k^n and \overline{l}_k^n . For each path, the shortcuts are reassigned over the grid. The speed on the links in the grid and in the shortcuts has been chosen as constant, with values of 1 and 2 respectively. Different choices lead to qualitatively similar results. The first check that can be made is that the choice of i_{start} must not be random.

Fig. 8 shows the sequences of \overline{d}_k^n for some values of n in grids with $L = 100$ and $N_{shortcuts} = 0$ and $N_{shortcuts} = 100$. The values of the sequence as n increases are quite constant or follow an irregular pattern, rather different from the one observed in the data. Fig. 9 shows the same sequences with i_{start} chosen randomly at the border of the grid. In this case it is evident that the sequences show the desired behavior: an initial decrease towards a minimum and then an increase until a value similar to the initial one is reached. This indicates that the interplay between the optimization process and the confined geometry of the urban environment is crucial in order to understand how the observed patterns emerge. The ‘‘parabola-like’’ sequence seems to be the results of an optimization process performed within a limited area, starting

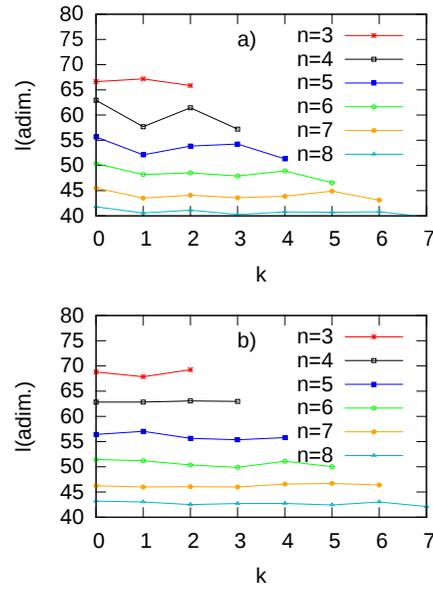


Figure 8: Sequence of trip lengths \overline{l}_k^n on the grid model with $N_{shortcuts} = 0$ (a) and $N_{shortcuts} = 100$ (b) when the origin of the trips is randomly chosen on the grid.

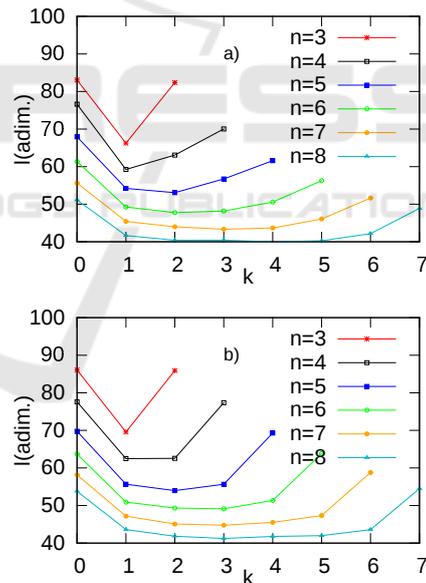


Figure 9: Sequence of trip lengths \overline{l}_k^n on the grid model with $N_{shortcuts} = 0$ (a) and $N_{shortcuts} = 100$ (b) when the origin of the trips is randomly chosen at the border of the grid.

from a point close to the boundaries of such area. Note that, as we have seen in the data, also \overline{t}_k^n and \overline{d}_k^n possess this kind of structure (Fig. 3).

In Section 2.2 it has been shown that approximating the sequence of \overline{d}_k^n with a parabola $p_n(k)$ it is possible to derive a scaling law for the sequence. Despite the fact that in our case the parabolic fit does

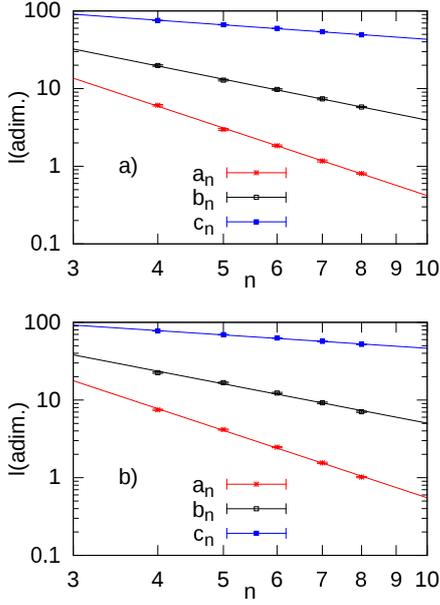


Figure 10: Coefficients a_n , b_n and c_n of the parabolic fit for the sequence \overline{l}_k^n for some values of n on the grid with $N_{shortcuts} = 0$ (a) and $N_{shortcuts} = 100$ (b). Continuous lines are power-law fits of the points.

not describe well the sequence, we can still use it in order to see if there is a similar scaling also for the sequences on the grid. Therefore, we fit the curves in Fig. 9 with Eq. (1). In both cases we check that the coefficients $p_n(k)$ are power-law decreasing with n , so that equations (2) hold (see Fig. 10). Moreover, also the relations between the coefficients in Eq. (6) is still valid even though the approximation is worse for $N_{shortcuts} = 100$ (Table 1 displays the values of the exponents of Eq. 2 for some values of $N_{shortcuts}$).

Fig. 10 shows the collapsed sequences together with the universal parabola independent from n . It is evident that, despite a scaling law seems to exist, the parabola does not describe well the curves since the found behavior shows a smaller curvature.

Table 1: Values of the exponents of Eq. (2) inferred from the grid model.

$N_{shortcuts}$	0	50	100
η_a	2.89(8)	2.91(7)	2.95(5)
η_b	1.76(8)	1.72(8)	1.74(5)
η_c	0.62(1)	0.58(2)	0.58(2)
$\eta_c - \eta_a$	-2.26(1)	-2.34(9)	-2.30(7)
$2(d\eta_c - \eta_b)$	-2.27(1)	-2.3(2)	-2.32(7)

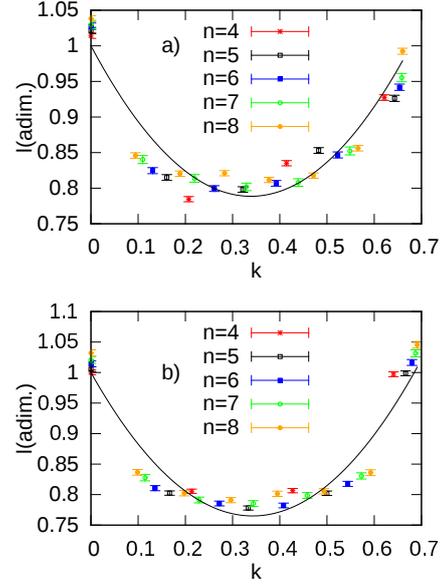


Figure 11: Collapsed sequence of trip lengths \overline{l}_k^n on the grid model with $N_{shortcuts} = 0$ (a) and $N_{shortcuts} = 100$ (b) when the origin of the trips is randomly chosen on the border of the grid.

4 CONCLUSIONS

In this paper we analyzed a large dataset of GPS tracks about vehicles collected within the city of Rome. After dividing the drivers according to the number of trips performed within a day, we showed that the sequence of travel length, as well as the sequences of travel times and geodesic distances between stops, exhibit a “parabolic pattern” so that each sequence is initially decreasing until a minimum is reached and then grows again to another maximum. By fitting these sequences with a parabolic law we showed that each one can be rescaled in a universal form independent from the number of trips, suggesting the existence of a universal mechanism responsible of this observed pattern. By using a model introduced in (Mastroianni et al., 2015) to produce synthetic paths in a simplified urban environment, we showed that these findings are consistent with the idea of drivers trying to minimize the total travel time. The geometry of the problem seems to be crucial in order to reproduce the correct behavior, so that the starting point must be at the border of a space that constrains the dynamics. Despite the fact that the simulated sequences of trip lengths are not well described by parabolic laws, still the model exhibit a scaling law similar to the one found empirically.

Despite the simplicity of the modeling scheme, the empirical patterns found in the data are qualitatively

reproduced. We argue that a more realistic model, taking into account different traffic conditions could lead to a better agreement with the data and a better understanding on the behavioral changes of car drivers. Finally, the universality of our findings have still to be completely proven by performing similar measurements in different urban environments and time-frames. The observed patterns might be useful in order to develop improved info-mobility systems taking into account the possible behavior of a driver on his next trip. The understanding of the behavior of individual car movements could in fact help at improving traffic forecast systems.

ACKNOWLEDGEMENTS

The authors acknowledge support from the KREYON project funded by the Templeton Foundation under contract n. 51663. VDPS acknowledges the EU FP7 Grant 611272 (project GROWTHCOM), the CNR PNR Project “CRISIS Lab” for financial support. We acknowledge interesting discussions with P. Gravino. We thank M. Mancini for his valuable work on OctoTelematics data pre-processing.

REFERENCES

- Octotelematics. <https://www.octotelematics.com/en/>. Accessed: 2014-10-03.
- Bazzani, A., Giorgini, B., Rambaldi, S., Gallotti, R., and Giovannini, L. (2010). Statistical laws in urban mobility from microscopic gps data in the area of florence. *Journal of Statistical Mechanics: Theory and Experiment*, 2010(05):P05001.
- Brockmann, D., Hufnagel, L., and Geisel, T. (2006). The scaling laws of human travel. *Nature*, 439(7075):462–465.
- Calabrese, F., Diao, M., Di Lorenzo, G., Ferreira, J., and Ratti, C. (2013). Understanding individual mobility patterns from urban sensing data: A mobile phone trace example. *Transportation research part C: emerging technologies*, 26:301–313.
- Colizza, V., Barrat, A., Barthélemy, M., Valleron, A. J., and Vespignani, A. (2007). Modeling the worldwide spread of pandemic influenza: Baseline case and containment interventions. *PLOS MEDICINE*, 4(1):95–110.
- Eluru, N., Sener, I., Bhat, C., Pendyala, R., and Axhausen, K. (2009). Understanding residential mobility: joint model of the reason for residential relocation and stay duration. *Transportation Research Record: Journal of the Transportation Research Board*, (2133):64–74.
- Eubank, H., Guclu, S., Kumar, V. S. A., Marathe, M., Srinivasan, A., Toroczkai, Z., and Wang, N. (2004). Controlling Epidemics in Realistic Urban Social Networks. *Nature*, 429.
- Flood, M. M. (1956). The traveling-salesman problem. *Operations Research*, 4(1):61–75.
- Gallotti, R. (2013). *Statistical Physics and Modelling of Human Mobility*. PhD thesis, Physics Department, University of Bologna.
- Gallotti, R., Bazzani, A., and Rambaldi, S. (2012). Towards a statistical physics of human mobility. *International Journal of Modern Physics C*, 23(09):1250061.
- Gallotti, R., Bazzani, A., and Rambaldi, S. (2015). Understanding the variability of daily travel-time expenditures using gps trajectory data. *EPJ Data Science*, 4(1):1–14.
- Gonzalez, M. C., Hidalgo, C. A., and Barabasi, A.-L. (2008). Understanding individual human mobility patterns. *Nature*, 453(7196):779–782.
- Gonzalez-Bailon, S. (2013). Social Science in the Era of Big Data. *Social Science Research Network Working Paper Series*.
- Lazer, D., Pentland, A., Adamic, L., Aral, S., Barabási, A.-L., Brewer, D., Christakis, N., Contractor, N., Fowler, J., Gutmann, M., Jebara, T., King, G., Macy, M., Roy, D., and Van Alstyne, M. (2009). Computational social science. *Science*, 323(5915):721–723.
- Mastroianni, P., Monechi, B., Liberto, C., Valenti, G., Servedio, V. D. P., and Loreto, V. (2015). Local optimization strategies in urban vehicular mobility. *PLoS ONE*, 10(12):e0143799.
- Mayer-Schonberger, V. and Cukier, K. (2013). *Big Data: A Revolution That Will Transform How We Live, Work, and Think*. Houghton Mifflin Harcourt, Boston.
- Rambaldi, S., Bazzani, A., Giorgini, B., and Giovannini, L. (2007). Mobility in modern cities: Looking for physical laws. In *Proceedings of the European Conference on Complex Systems*, Dresden. ECCS07.
- Rubinstein, I. S. (2013). Big Data: The End of Privacy or a New Beginning? *International Data Privacy Law*. im Erscheinen, NYU School of Law, Public Law Research Paper Nr. 12-56.
- Simini, F., González, M. C., Maritan, A., and Barabási, A.-L. (2012). A universal model for mobility and migration patterns. *Nature*, 484(7392):96–100.
- Song, C., Koren, T., Wang, P., and Barabasi, A.-L. (2010). Modelling the scaling properties of human mobility. *Nature Physics*, 6(10):818–823.
- Tene, O. and Polonetsky, J. (2012). Big Data for All: Privacy and User Control in the Age of Analytics. *Northwestern Journal of Technology and Intellectual Property*.
- Treiber, M. and Kesting, A. (2013). Traffic flow dynamics. *Traffic Flow Dynamics: Data, Models and Simulation*, Springer-Verlag Berlin Heidelberg.
- United Nations Secretariat (2014). World urbanization prospects: The 2014 revision. Technical report, Department of Economic and Social Affairs.
- Wang, X.-W., Han, X.-P., and Wang, B.-H. (2014). Correlations and scaling laws in human mobility. *PLoS ONE*, 9(1):e84954.