Computer Supported Evolution Inside Van Hiele Levels 1 and 2

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Abstract: The goal of the study is to clarify the communicative abilities of 5th grade students related to the computer supported geometry education. Theoretical frame is the Van Hieles' model. The experimental teaching gives an idea of how the language characteristics of Level 1 and 2 could be improved by a short-term game-like math education. Some coding-decoding activities make students to be more accurate in written communications.

1 A MODERN VECTOR IN MATH EDUCATION

In the last 30 years ICT enhanced math education became routine practice in secondary school. It was based mainly on the development of computer algebra systems (CAS) and dynamic geometry software (DGS). Key role in this trend plays the so-called inquiry-based approach (Rocard et al., 2007) which is a kind of a modern Socratic style (Lazarov, 2014). But in 2007 there appeared a paper where the role of the mathematics education in secondary school was reconsidered (Haapasalo, 2007). It is a matter of fact that the advanced usage of ICT refers to some specific communication skills and applications of mathematics methods. No other school subject than mathematics can face better these demands of life, so the vector of the secondary school math education should contain components that meet the ICT needs of a modern individual.

Our recent research shows that high school students who use dynamic geometry software (DGS) in studying mathematics developed intuitively specific communication skills (Lazarov, 2015). Their written math slang includes synthetic symbols and icons (like dynamic pictures and graphs) parallel to the traditional formulas and shorthands. This slang evolves along with the educational process and reflects the level of student's geometrical reasoning. Our practice clearly shows that the constructing of DGS applets requires a student to have reached at least Van Hiele Level 3[1]. In fact any dynamically stable construction (i.e. such that preserves the geometrical properties of the objects after some transformations) is made following an algorithm reflecting the properties of the figures. The design of such algorithm requires student to apply at least short deductive chains using the DGS syntax. Our experience confirms that the proper usage of DGS syntax needs a long training 'within the following categories concerning what modern technology can maintain and promote:
(1) Links between conceptual and procedural knowledge,
(2) Metacognitions and problem-solving skills,
(3) Sustainable components of mathematics making,
(4) Interplay between systematic approaches and minimalist instruction,
(5) Learning by design' (Haapasalo, 2007).

But following the Van Hieles' theory, in order to be at Level 3 a student should pass consecutively Levels 1 and 2. So the question is what kind of math activities will contribute to the development of ICT skills in pre-deductive phase. We started exploring the mathematics and informatics curriculum to find the most adequate starting point for introducing integrated mathematics and IT approaches.

2 THE STATUS QUO IN BULGARIA

The change in the teaching style that happens on the borderline between primary and secondary school

[1]We are going to make a lot of references to the first three Van Hiele levels so we give a brief description of them in an Appendix.

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in Bulgaria (4th-5th grade) has several dimensions. First of all, it considers the subject-oriented approach which comes to substitute the (more or less) topic-oriented mode of teaching in primary school. A second important moment is the change of the teachers who are engaged with a particular class. The related subjects like mathematics and information technology are already separate disciplines in the school plan, sometimes covered by one teacher, but more often by different persons. The math and IT syllabus are made by different commissions at the Ministry of Education which yields to relatively poor interrelated connections. The Bulgarian math syllabus up to 2015/2016 scholastic year does not provide additional space for activities of mixed (conventional and IT based) type in which the necessary skills for inquiry-based education to be built. Such status quo makes it hard to take advantages of IT in traditional math education and vice versa. So the teachers and educators are searching their own way to achieve integration of IT in math education in the mode they think is most appropriate for a particular target group.

Our way started with the target group of high ability senior secondary school students (Lazarov, 2014). At this stage all students have reached the Van Hiele Level 3, and some of them proceeded on Level 4. Students' knowledge, skills and attitude (KSA) are transferable from the conventional context of math education to the DGS environment, therefore we can speak about students' competence of synthetic type (synthetic competence). Further, we tried to apply similar approach to intermediate secondary school students with average abilities, i.e. incomplete Van Hiele Level 3. The results were far from satisfactory (Shabanova & Lazarov, 2014). Only a small fraction of the target group managed to transfer the math KSA into a new context of DGS exploration. It became clear that the foundation of the transferability should be established somewhere in the early secondary school.

3 APPLICATION OF A CLASSICAL MODEL

The Van Hiele model of learning geometry provides a convenient base for interpreting and analyzing the students' levels of understanding. In parallel, this model clarifies the way they form geometrical reasoning. Special role in the model plays the development of the language in which students express their knowledge, as far as each level has its own linguistic symbols and own network of relationships connecting those symbols (Usiskin, 1982). For instance, student's progress from Level 1 to Level 2 yields a significant structuring of relationships and a refinement of concepts. Teacher should feel how such transition occurs and should tune his/her language for adequate verbalization of the intuitive knowledge, because the verbalization goes together with a restructuring of concepts. As we mentioned above, the necessary level for meaningful use of DGS is Level 3, so the concept restructuring must first occur at Level 2 before students can start exploring the logical relationships needed for creating DGS applets.

The geometry education in Bulgarian 5th grade is characterized with a significant intensification. As evidence we point that the number of new geometrical concepts in 5th grade is about 4 times bigger than all geometrical concepts introduced in the previous 4 years. Moreover, many problems require a construction to be done and a solution to be written which needs more developed language for communications in both directions - understanding the statements and composing statements. For instance, let us consider the following problem:

Draw $\angle POQ = 30^\circ$ *and a point A on the ray OQ such that* $OA = 10$ *cm. Find the distance from A to the ray OP.* (Lozanov et al., 2011)

Here students are expected to turn a description into a picture before starting the solution. They should draw an arbitrary ray, then measure the angle (using protractor), then find a specific point and erect a perpendicular from it to a line (to do this, students should know that the distance equals the length of the perpendicular). The solution of the problem is based on both conceptual and procedural knowledge and skills. This two knowledge types seem to be developed iteratively (Rittle-Johnson & Koedinger, 2004). In our opinion such activities are accessible for Level 2 and up and could be built by integrating IT in math education.

What follows refers to our research in lower secondary school (Bulgarian 5th grade), which supposes the Van Hiele Level 2 (sometimes incomplete) has been reached. We tried to manipulate students' attitude towards elaborating more precise and reliable communication style in learning mathematics by applying IT. We hope this approach will guarantee the foundation of KSA which is necessary for the next steps in building synthetic competence.

4 FRAMEWORK OF THE STUDY

The goal of our study was to clarify the communicative abilities of 5th grade students related to the computer supported geometry education. Designing our experimental teaching we took into account the fol-

lowing two requirements (Kadijevich, 2006):
(1) when utilize mathematics, don't forget available tool(s); when make use of tool, don't forget the underlying mathematics;
(2) to solve the assigned task, use, whenever possible, a process approach as well as an object approach, working with different representations .
According to the above requirements, we chose the MS Paint application with *Basic shapes* for the IT activities. This application provides relevant resources for Levels 1 and 2: ready-made shapes as isosceles triangle, right-angled triangle, as well as three types of transformations: rotation, stretching, dragging.

Another moment in the research design was the choice of the topic. We recognize the unavoidable usage of familiar everyday concepts on the first two levels, so we expected students' descriptions to be in the form of a meta-language (mixture of geometrical and everyday life language, expanded with pictures and shorthands). Students were assigned to sketch a monster using some geometrical shapes. Among the other educational reasons, this topic was selected because of the anthropomorphic and zoomorphic terms that potentially could help the composite figures to be properly depicted. Let us point the anthropomorphic origin of some concepts in geometry like *legs* of a triangle and trapezium in Bulgarian math language. The idea of the topic came from a creative writing project, proposed by Linda Yollis in her blog (Yollis, 2014). The accent in her project work was on developing creative thinking and writing skills.

5 PARAMETERS OF THE STUDY

The target group was composed of 5th grade students (no indication for additional interest in mathematics among them). The experimental teaching lasted 6 academic hours distributed in the following manner:
– Diagnostic test.
– Math class. Revision of geometrical students' knowledge.
– ICT class. Students created on computer their monsters, constructed by different geometrical figures, using a simple computer graphic program - MS Paint.
– Language class. Writing a description of a picture. Each student wrote a short description of his/her own monster.
– ICT class. Students exchanged their descriptions and based on the written texts they created a copy of the original monsters on computer.
– Control test.

Students worked individually but grouped by pairs on the next assignments:

(A1) A monster to be drawn using the following basic shapes: rectangle (including square), triangle (isosceles, right), and circle.
(A2) The own monster to be described by words and to be sent to a classmate for depicting.
(A3) The descriptions to be interchanged in the classmate pair, the other monster to be depicted following only the description and to be compared with the original.

During the lessons some comments and remarks on the assignment were done. Students were pointed out that the figure type is invariant when applying any of the three transformations.

6 INDICATORS AND DATA COLLECTION

We observed several indicators but some of them were covered by all students (like classifying the general type of a polygon or reconstructing an abstract figure following a verbal description), so we took them away from the analysis. The following indicators were used to determine the initial Van Hiele level including the degree of completeness inside the level:
(i1) recognizing the square and the rectangle no matter how it is oriented;
(i2) the square is considered as rectangle;
(i3) recognizing the type of a triangle (isosceles, right);
(i4) combining two properties of triangle (isosceles and right);
(i5) usage of geometrical properties of the figure in description;
(i6) usage of elements of the figure like *side* and *vertex* in depiction;
(i7) applying elements of the figure like *side* and *vertex* in reconstruction;
(i8) reconstruction of abstract figure following verbal description.

Indicators for determining the individual Van Hiele level are based on the Burger-Shaughnessy operationalization (Burger & Shaughnessy, 1986), given in the Appendix. Most of them refer to Level 2, but some indicate Level 1, like (i1). Burger-Shaughnessy features suppose a direct communications with students to analyze the geometrical reasoning. Our approach is oriented mostly to analyze the features of the student's written language at Levels 1 and 2. So we collected data from the tests, students' computer pictures and written descriptions of their pictures. There were 22 pairs of students who took part in the experimental teaching. Below we are going to present the details about 4 pairs that are representative for the most typical cases. The observed students are coded as PgA, PpA, RsA, RkA, KoG, KrG, RaG, SaG.

7 STATISTICS

Table 1 shows the coverage of the indicators: 0 means that an indicator is not covered and 1 stands for a covered indicator. Some indicators were partially covered, e.g. the corresponding test item is correct but in the written material there were mistakes or gaps – we scored these cases with 1.

Table 1: Coverage of the indicators.

	i1	i2	i3	i4	i5	i6	i7	i8	Σ
PgA	1	1	1	1	1	0	0	1	6
PpA	1	1	1	0	0	0	0	0	3
RsA	1	1	1	1	1	1	1	1	8
RkA	1	0	1	1	1	0	0	0	4
KoG	1	0	1	1	1	0	0	1	5
KrG	0	0	1	0	0	0	0	0	1
RaG	0	0	0	0	0	0	0	0	0
SaG	1	0	1	1	1	0	0	1	5

We consider a total score $\Sigma \geq 4$ as reaching Level 2, and $\Sigma \leq 3$ as reaching Level 1. Initially, we introduced more indicators, but these indicators were either covered by all students or there was no student who covered them. For instance:
usage of non geometrical concepts in description – all;
application of elements of the figure like side and vertex in description – none.

8 EXAMPLES AND COMMENTS

In this section we are going to consider some particular cases which are emblematic for the different stages of language forming inside Levels 1 and 2. Let us highlight that we did not register any usage of pure geometrical concepts. All students' descriptions of their monsters were based on anthropomorphic features; all geometrical shapes were colored and usually the color stands before the shape in description.

Case 1

The monster created by PgA and its replica reconstructed by KrG are shown in Figure 1.
PgA (Level 2, $\Sigma = 6$) uses simple sentences to describe his monster, e.g. *My body is a gray rectangle as a traffic light. My neck is an orange rectangle.* There is no detailed information about sizes, directions, triangle types and so on. Just shapes, colors and relations *over and bellow*.

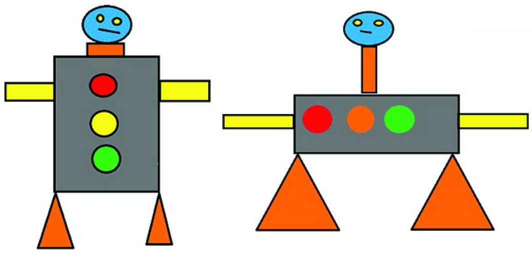

Figure 1: Monster created by PgA and its decoded replica by KrG.

Nevertheless, even with this simple and insufficient information, the replica created by KrG (Level 1, $\Sigma = 1$) and based on the text description, is quite accurate. We could explain the poor description with limited students' knowledge of geometry at this level. But this limited geometrical resource was sufficient enough for the students to communicate with each other.

Case 2

The monster created by PpA (Level 1, $\Sigma = 3$) is shown in Figure 2 (left). The monster created by PpA is shown in Figure 2. The student, from the position of his monster, wrote: *My head is a blue square with rounded edges*.

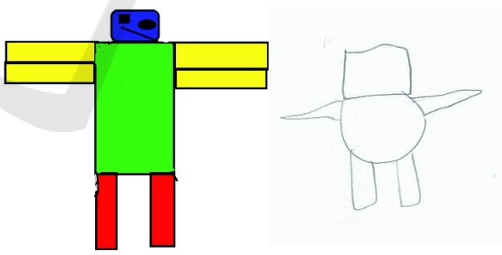

Figure 2: Monster created by PpA and a picture from his test.

From mathematical point of view, this description is completely wrong, because a polygon cannot have rounded edges. But in computer graphic software applications (including MS Paint) we could see an icon called *rounded rectangle*. Some students expand their language including icon-labels on an equal level with geometrical concepts. The teacher could use such contradictions to clarify the terms and to upgrade students' knowledge.

The right image on Figure 2 is taken from the PpA's control test. The test task was to *draw a figure* *where the arms are triangles and legs are rectangles; the arms and the legs should be connected with the circle body at only one vertex.* PpA follows mainly the anthropomorphic context, without paying enough attention to geometrical details.

Case 3

The monster created by KoG (Level 2, $\Sigma = 5$) and its replica reconstructed by PpA are shown in Figure 3.

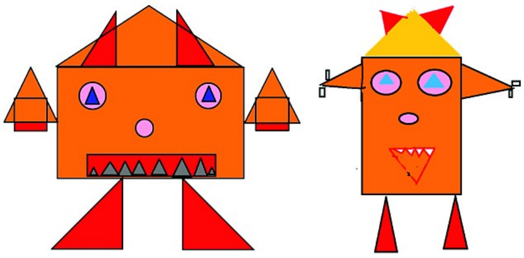

Figure 3: Monster created by KoG and its replica decoded by PpA.

KoG's description is the richest one in geometrical concepts. He described the arms as *two orange equilateral triangles*, the horns as right triangles. But no details about the position of these triangles was given. So the reconstructor PpA has put the arms connected to the body by side, not by vertex. Such problem solving examples, connected with coding and decoding processes, could be used to develop students' critical thinking. Here one can see how anthropomorphic context dominates over geometrical knowledge.

Case 4

The monster, created by KrG and its replica by PgA (Level 2, $\Sigma = 6$) are shown in Figure 4.

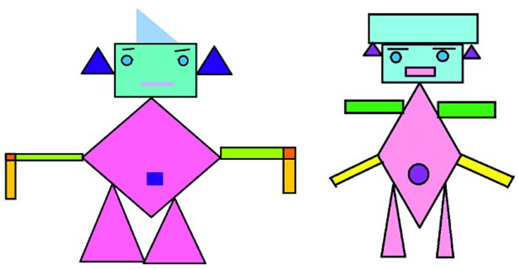

Figure 4: Monster created by KrG and the reconstruction by PgA.

In her description KrG wrote that *the arms are formed by four rectangles: the straight are green and the down are yellow*. Such multimodality of the spoken language (Ginsberg, 2015) applied to geometrical purposes is an evidence for incomplete Level 2 of formation of mathematical concepts and terminology. However, KrG showed significant progress during the experimental teaching. Her starting point was recognized as Level 1, but further in her description language appeared properly used geometrical concepts as right triangle and rhombus. KrG covered (i6) and (i7) and approached Level 2.

9 CONCLUSIONS

Ginsberg (ibid., pp 4-5) gives very interesting example of misunderstanding in communication between teacher and students. Similar misunderstanding appears every time when the teacher's expectations about the geometry reasoning of the students are not coherent with their actual Van Hiele level. There were also quite different standing points between the authors of this article before the analysis of the experimental data was done. One of us was quite sure that students operate at least at Level 2, but the other was more skeptic. Our experimental teaching was held in the beginning of 5th grade before the new geometry topics from the school plan started. Based on the outcomes of our study, we recommend Bulgarian teachers to be very careful when using professional math slang in their instructions.

Indeed, our observed students used mainly everyday life expressions and images instead of geometrical concepts. Some of them recognized the importance of clarity in communications after getting some coding-decoding experience during the experimental teaching. However, we consider problems like the one quoted in section 3 to be still beyond the average 5th graders' zone of proximal development.

De Villiars stated the following open question: *could hierarchical thinking be developed earlier at Van Hiele Levels 1 and 2 through various strategies and using tools such as dynamic geometry software?* (De Villiers, 2010). We claim that the ability to use DGS is equally related to mathematics and ICT, but also it needs specific communication skills to express the KSA. Evolution along the Van Hiele levels causes development of a meta-language that reflects the degree of geometrical reasoning but also accelerates the evolution itself. However, DGS is not a relevant educational tool for construction activities at Levels 1 and 2 – there is not enough mathematical KSA accumulated for proper use of DGS. Even the basic understanding of a DGS interface requires significant math knowledge (compare with the Level 3 features in Appendix). Thus some preparatory training should be done for some connections between elements of the figures at Level 1 and 2. Short deductive chains appear naturally during such training and computer applications of lower class than DGS allow to achieve clarity and precision of the expression of these chains.

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REFERENCES

Burger, W. F. & Shaughnessy, J. M. (1986). Characterizing the Van Hiele levels of development in geometry. *Journal for Research in Mathematics Education*, 17(1), pp 31-48.

De Villiers, M. (2010) Some Reflections on the Van Hiele theory. Invited plenary presented at the *4th Congress of teachers of mathematics of the Croatian Mathematical Society, Zagreb*, 30 June – 2 July 2010.

Ginsberg, D. (2015) Multimodal semiotics of mathematics teaching and learning. Dissertation, *Georgetown University. Washington*, DC, 2015, pp 9-10

Haapasalo, Lenni (2007) Adapting Mathematics Education to the Needs of ICT. eJMT, Vol. 1, No 1.

Kadijevich, Dj. (2006). Coordinating the Process and object features of mathematical knowledge by CAS. *Conference Proceedings of DES-TIME*-2006.

Lazarov, B. (2014) Socratic Style Support for Transferability of Math Knowledge and Skills. The Korean Society of Mathematical Education. Proceedings of the 2014 *Int'l Conference on Mathematical Education, Cheongju National University of Education*, October 17-18, 2014, pp 75-85.

Lazarov, B. (2015) Project-oriented Education as a Platform for Transfer of Math KSA. Proceedings of the *7th International Conference on Computer Supported Education Volume 2*, Lisbon, Portugal, 23-25 May, 2015, pp 325-330

Lozanov, C., Vitanov, T. & Kalcheva, A. (2011) Mathematics for 5th grade. Sofia, Anubis, p 77, problem 8 (in Bulgarian)

Rittle-Johnson, B. & Koedinger, K. (2004). Comparing instructional strategies for integrating conceptual and procedural knowledge. In D. Mewborn, P. Sztajn, D. White, H. Hiegel, R. Bryant and K. Nooney (Eds.), Proceedings of the *Twenty-fourth Annual Meeting of the North American Chapters of the International Group for the Psychology of Mathematics Education*. ERIC Clearinghouse for Science, Mathematics, and Environmental Education. OH: Columbus, pp. 969-978.

Rocard, M. et al. (2007) Science Education Now: A Renewed pedagogy for the Future of Europe. High Level Group on Science Education. *EUROPEAN COMMISSION, Directorate-General for Research*.

Shabanova, M. & Lazarov, B. (2014) Detecting Math-and-ICT Competence. Proceedings of the *6th International Conference on Computer Supported Education, Volume 2*, Barcelona, Spain, 1 - 3 April, 2014, pp 153-158

Van Hiele, P. M. (1984) A child's thought and geometry. In D.Fuys, D.Geddes and R.Tischler (Eds), English translation of selected writings of Dina van Hiele-Geldof and P.M. van Hiele, pp. 237-241. *Brooklyn College, N.Y.* 1984.

Usiskin, Z. (1982) Van Hiele levels and Achievement in Secondary School Geometry. Final report of the *CDASSG Project. Chicago: Univ. of Chicago*.

Yollis, L. (2014) http://yollisclassblog.blogspot.bg/2014/10/monstrous-fun.html (active in Dec 2015)

APPENDIX

Operationalization of Van Hiele Levels 1-3

Burger & Shaughnessy characterized pupils' geometrical reasoning at the first three Van Hiele levels as follows:

Level 1 (Recognition)
(1) Often use irrelevant visual properties to identify figures, to compare, to classify and to describe.
(2) Usually refer to visual prototypes of figures, and is easily misled by the orientation of figures.
(3) An inability to think of an infinite variation of a particular type of figure (e,g. in terms of orientation and shape).
(4) Inconsistent classifications of figures; for example, using non-common or irrelevant properties to sort figures.
(5) Incomplete descriptions (definitions) of figures by viewing necessary (often visual) conditions as sufficient conditions.

Level 2 (Analysis)
(1) An explicit comparison of figures in terms of their underlying properties.
(2) Avoidance of class inclusions between different classes of figures, eg. squares and rectangles are considered to be disjoint.
(3) Sorting of figures only in terms of one property, for example, properties of sides,
while other properties like symmetries, angles and diagonals are ignored.
(4) Exhibit an uneconomical use of the properties of figures to describe (define) them, instead of just using sufficient properties.
(5) An explicit rejection of definitions supplied by other people, e.g. a teacher or textbook, in favour of their own personal definitions.

(6) An empirical approach to the establishment of the truth of a statement; e.g. the use of observation and measurement on the basis of several sketches.

Level 3 (Ordering)
(1) The formulation of economically, correct definitions for figures.
(2) An ability to transform incomplete definitions into complete definitions and a more spontaneous acceptance and use of definitions for new concepts.
(3) The acceptance of different equivalent definitions for the same concept.
(4) The hierarchical classification of figures, e.g. quadrilaterals.
(5) The explicit use of the logical form "if ... then' in the formulation and handling of conjectures, as well as the implicit use of logical rules such as modus ponens.
(6) An uncertainty and lack of clarity regarding the respective functions of axioms, definitions and proof.