Total Information Transmission between Autonomous Agents

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Abstract: This note explores the current framework of information theory to quantify the amount of semantic content of a given message is sent in a given communicative exchange. Meaning issues have been out of the mainstream of information theory since its foundation. However, in spite of the enormous success of the theory, recent advances on the study of the emergence of shared codes in communities of autonomous agents revealed that the issue of meaningful transmission cannot be easily avoided and needs a general framework. This is due to the absence of designer/engineer and the presence of functional/semantic pressures within the process of shaping new codes or languages. To overcome this issue, we demonstrate that the classical Shannon framework can be expanded to accommodate a minimal explicit incorporation of meaning within the communicative exchange.

1 INTRODUCTION

The exploration of the emergence of communication has been a hot topic of research recent years (Hurford, 1989; Nowak, 1999; Cangelosi, 2002; Komarova, 2004; Niyogi, 2006; Steels, 2003). In particular, the emergence of shared, non-designed codes between autonomous agents pushed by selective pressures has been a source of interesting results (Nowak, 1999; Steels, 2001). In most of these studies, codes emerge among agents by the need to communicate things about the world they are immersed in. These agents are autonomous, therefore, no designer or engineer is explicitly behind the communicative exchanges ensuring the correct transmission and interpretation of the message. The role of code designer is taken by evolution and its associated selective pressures. These selective pressures apply at different levels: the standard information-theoretic level –the physical coding and transmission of the events of the world shared by the autonomous agents– and the semantic/functional level. By this semantic/functional level we refer to the *content* of the message, which, in turn, can be split in two parts: 1) The *relevance* of the event to be transmitted, a crucial issue in a selective framework and 2) the proper referentiation of such event during a communicative exchange (Corominas-Murtra, 2013) -see figure (1). Both dimensions of the communicative phenomenon will lead the functional response of the agent and its potential success through the selective process. It is worth observing that these semantic/functional issues are totally absent in standard information theory (Hopfield, 1994).

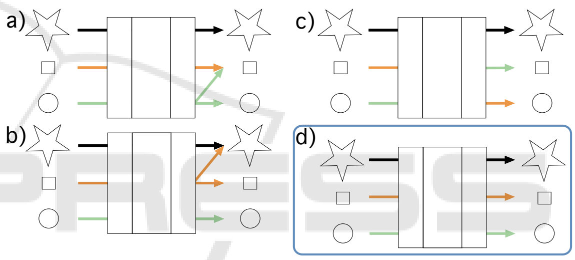

Figure 1: The two problems of Shannon's Information concerning the simplest meaning transmission: Blindness of it to i) internal hierarchy of relevance of events and ii) Referentiality conservation. We have a communication system consisting on three objects $\Omega = \{x_1, x_2, x_3\}$ appearing, for the sake of simplicity, with equal probabilities, namely $p(x_1) = p(x_2) = p(x_3) = \frac{1}{3}$. The appearance of an object is coded in some way by the coder agent $\mathcal{A}$, passes through the channel Λ and is decoded by the decoder agent $\mathcal{B}$, which gives a referential value to the signal received. Standard mutual information does not distinguish between a) and b). However, if we assume that the information contained in x_1 is larger than the others, the same communication mistake involving this object should be penalised higher in the overall information transfer. In c) and d) we depict the referentiality problem. Mutual information is blind to referentiality mistakes. Only d) represents a perfect communication schema where all semantic aspects are respected. In this paper we derive the information-theoretic functional that accounts for the internal hierarchy of relevance of events. By accounting the semantic value or weight of each event to be coded and sent through the channel, we derive the proper functional able to distinguish between a) and b).

In this article we present a minimal incorporation of the semantics of elements in a given information/theoretic functional, quantifying the amount statistical bits properly transmitted plus the average

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semantic content of them in a given communicative exchange between two autonomous agents. This provides a solution to problem 1) pointed out above concerning meaning/functional issues in standard information theory: The *relevance* of the event to be transmitted -see figure (1). In this new framework information becomes a 2-dimensional entity: on the one hand, one has the classical Shannon information and, on the other hand, one has the semantic information which is carried by those *statistical* bits. The sum of both is the *total information transmitted*, the amount of bits which will be taken into account for the selective pressures, presented in equation (11). Point 2), concerning the conservation of the referential value, stems from the conception of the dual nature of the communicative sign, a primitive kind of *Saussurean duality* (Saussure, 1916; Hopfield, 1994; Corominas-Murtra, 2013) which considers the signal and the reference as the fundamental unit to be conserved in a communicative exchange. The incorporation of such duality in a consistent information-theoretic framework has been already addressed in (Corominas-Murtra, 2013). Crucially, this approach did not take into account any meaning quantification, and this is the target of this work.

The inclusion of meaning can be performed through any justified quantification to the elements of the *world*. In selective scenarios, meaning/functional quantification may be understood as emerging from a kind of *language game*, in Wittgenstein's interpretation (Wittgenstein, 1953; Kripke, 1982; Steels, 2001). This game would combine the interaction between the environment and the autonomous agents with the interaction among the autonomous agents themselves. In a primary approach, one can attribute the meaning quantification out of this language game to be tied to the relevance of the functional response to the events to be coded and transmitted. In general, and following Wittgenstein's footsteps, no *absolute* measure of meaning quantification is assumed to be achievable, and such a quantification, if possible, must be the outcome of an agreement/consensus among the agents under the conditions imposed by evolutionary pressures. We point out that, in formal systems, standard approaches developed to quantify the amount of information of abstract objects beyond the statistical framework can provide solid formal background for meaning quantification. As paradigmatic examples, the approach to semantic information proposed by Yehoshua Bar-Hillel and Rudolf Carnap over logical systems (Carnap, 1953) or the theory of Algorithmic Complexity to quantify the amount of information required to describe a formal object (Kolmogorov, 1965; Chaitin, 1966; Li, 1997; Cover, 2001). To better grasp the intuition behind this paper, let us stop a while with an example of Bar-Hillel and Carnap's semantic information theory (Carnap, 1953). Roughly speaking, the main idea underlying their approach stems from the following observation: let us imagine that we have a *world* made of two variables $\mathbf{p}, \mathbf{q}$ and that the 'state' of the world is given by formulas:

$$\mathbf{p},\ \mathbf{q},\ \mathbf{p} \vee \mathbf{q},\ \mathbf{p} \wedge \mathbf{q}.$$

Now assume that the sender transmits events from the world, as long as the formulas are satisfied, i.e., their truth values, at a given point in time are 1. Undoubtedly, $\mathbf{p} \wedge \mathbf{q}$ will contain much more *information* about the state of the world than $\mathbf{p} \vee \mathbf{q}$. This happens because the conditions under which $\mathbf{p} \wedge \mathbf{q}$ is satisfied are much more restrictive than the ones than satisfy $\mathbf{p} \vee \mathbf{q}$. This is classic work from (Carnap, 1953). Now imagine that what we have a receiver which decodes the message of the sender after a presumably noisy channel. How much of the semantic information has been transmitted? In other words, even $\mathbf{p} \vee \mathbf{q}$ and $\mathbf{p} \wedge \mathbf{q}$ may have the same probability of appearing – and therefore, the same weight in the computation of the statistical information– is there a way to take into account that $\mathbf{p} \wedge \mathbf{q}$ contains more information, in terms of message content? As pointed out above, if the sender/receiver system is autonomous, being able to perform such a distinction can make a difference in terms, for example, survival chances in selective scenarios.

Nevertheless, it is worth to emphasise that the choice of the semantic framework is incidental, and it remains deliberately open. A final word of caution is required: the presented results are tied to the range of applicability of standard information theory, and therefore concern closed systems, and as such, extremely simple abstractions of real scenarios. Nonetheless, it demonstrates that in such controlled cases, transmission of meaning is affordable from a consistent, information-theoretic viewpoint.

2 TOTAL INFORMATION TRANSMISSION

2.1 Quantification of Meaning

In general, we have a finite set of objects $\Omega = \{x_1, ..., x_N\}$ and a function Q,

$$Q : \Omega \longrightarrow \mathbb{R}^+, \tag{1}$$

such that $Q(x_k)$ is the *information content* of x_k. We will have in mind the following schema: Two autonomous agents $\mathcal{A}$ and $\mathcal{B}$ immersed in a shared world whose events/objects are members of the set $\Omega = \{x_1, ..., x_N\}$. Agents exchange information about Ω. Objects in $\Omega = \{x_1, ..., x_N\}$, appear following a given random variable $X_\Omega \sim p$. Every object $x_k \in \Omega$ will thus appear with probability $p(x_k)$. The entropy of such an ensemble of objects will give us the minimal amount of information to describe the statistical behaviour of X_Ω, namely:

$$h(X_\Omega) = -\sum_{x_k \in \Omega} p(x_k) \log p(x_k).$$

As it is standard in information theory, X_Ω is an information source sending $h(X_\Omega)$ bits to the information channel -we follow the communication schema provided in figure (1).

2.2 Total Information of an Ensemble of Objects

Total information of an ensemble of objects.- How much of *semantic* information is sent, in average, if we consider X_Ω as an information source? To answer this question, we first define the vector ϕ, whose elements $\phi(x_k)$ are defined as:

$$\phi(x_k) \equiv -\frac{p(x_k) \log p(x_k)}{h(X_\Omega)}. \quad (2)$$

Then, the amount of semantic information sent by X_Ω as an information source is:

$$\langle Q \rangle_\phi = \sum_{x_k \in \Omega} \phi(x_k) Q(x_k), \quad (3)$$

namely, the *average semantic information* $\times$ *per bit of the information source defined by* X_Ω. In other words: The amount of *semantic* bits carried by *statistical* bits; or how much meaning can you send, in average, having the ensemble of objects Ω which is sampled using X_Ω.

The total information we get, in average, from the ensemble of objects Ω, to be named $H(\Omega, X_\Omega)$, will thus be:

$$H(\Omega, X_\Omega) = h(X_\Omega) + \langle Q \rangle_\phi, \quad (4)$$

i.e., the statistical information per event plus the average amount of semantic information carried by such statistical information. Notice that the derivation we provided for the total information differs from the one given by Gell-mann and Lloyd and Ay et al (Gell-mann, 1996; Ay, 2010). Although inspired in Gell-mann and Lloyd's definition, the definition here provided better fits in a broad information/theoretic frame where transmission is taken into account. To have a consistent framework accounting for transmission, it is necessary to weight the semantic content with the informative contribution of a given signal/object to the overall information content. As we shall see in the following lines, this is crucial get consistent results when, for example, the channel is totally noisy and all information is destroyed. The framework here presented will be able to cope with the natural assumption that, in these cases, both the semantic and statistic information transmitted must be zero. The ontological discussion between these two approaches, even interesting from the epistemological and formal viewpoint, exceeds the scope of this paper.

Now we proceed our construction by defining

$$q(x_k) \equiv \frac{Q(x_k)}{h(X_\Omega)},$$

from which we can rewrite $H(\Omega, X_\Omega)$ as:

$$H(\Omega, X_\Omega) \equiv h(X_\Omega) \sum_{x_k \in \Omega} \phi(x_k)\left(1 + q(x_k)\right). \quad (5)$$

We observe that another approach would be to consider $H(\Omega, X_\Omega)$ a two-dimensional vector $H(\Omega, X_\Omega) \equiv \langle h(X_\Omega), \langle Q \rangle_\phi \rangle$, thereby highlighting the two-dimensional character of the approach. We take the definition provided in equation (4) for the sake of simplicity.

2.3 Transmission of Semantic and Statistic Information

Formally, we have an ensemble of objects Ω whose behaviour is described by X_Ω. This is the information *source*. Agents $\mathcal{A}$ and $\mathcal{B}$ sharing the world made by the objects of Ω transmit messages about it among them. Information provided by the source X_Ω is coded in some way by agent $\mathcal{A}$, and agent $\mathcal{B}$ assign the received message to a given object $x_i \in \Omega$, being this assignment process depicted by the random variable X'_Ω. We say that X'_Ω is the *reconstruction* of X_Ω made by agent $\mathcal{B}$. The communication channel between agents $\mathcal{A}$ and $\mathcal{B}$ is described by the matrix Λ

$$\Lambda(x_k, x_j) \equiv p(X'_\Omega = x_j | X_\Omega = x_k).$$

For simplicity, we will simply write $p(x_j|x_k)$. Likewise, we will refer to the join probability $p(X_\Omega = x_k, X'_\Omega = x_j)$ simply as $p(x_k, x_j)$ and, to the conditional probability $p(X_\Omega = x_j | X'_\Omega = x_k)$ simply as $p'(x_j|x_k)$. We finally note that X'_Ω follows the probability distribution p' defined as:

$$p'(x_k) = \sum_{x_i \in \Omega} p(x_k|x_i) p(x_i).$$

Having all the ingredients properly defined, we first put Shannon information in a suitable way to work with, namely:

$$I(X_\Omega : X'_\Omega) = \; = \sum_{x_k, x_\ell \in \Omega} p(x_k, x_\ell) \log \frac{p(x_k, x_\ell)}{p(x_k) p'(x_\ell)} = \sum_{x_k \in \Omega} p(x_k) D(p(X'_\Omega | x_k) || p'),$$

where

$$D(p(X'_\Omega | x_k) || p') \equiv \sum_{x_\ell \in \Omega} p(x_\ell | x_k) \log \frac{p(x_\ell | x_k)}{p'(x_\ell)},$$

is the *Kullback-Leibler* (KL) divergence between distributions $p(X'_\Omega | x_k)$ and p' (Cover, 2001). The KL divergence $D(p(X'_\Omega | x_k) || p')$ can be interpreted as the *information gain* agent $\mathcal{B}$ has from observing x_k, in statistical terms. The KL divergence is non-negative and, in this particular problem, is bounded as follows:

$$\begin{aligned} 0 &\leq D(p(X'_\Omega | x_k) || p') \\ &= \sum_{x_i \in \Omega} p(x_i | x_k) \log \frac{p(x_k, x_i)}{p'(x_i)} - \log p(x_k) \quad (6) \\ &\leq -\log p(x_k). \end{aligned}$$

In this framework, $p(x_k) D(p(X'_\Omega | x_k) || p')$ is the average contribution of x_k to the mutual information. Thus, the ratio $\phi'(x_k)$, defined as:

$$\phi'(x_k) \equiv p(x_k) \frac{D(p(X'_\Omega | x_k) || p')}{h(X_\Omega)}, \quad (7)$$

depicts the average fraction of bits from the source X_Ω coded by agent $\mathcal{A}$ due to x_k that are properly transmitted. By defining the vectors $\phi \equiv (\phi(x_1), \ldots, \phi(x_N))$ and $\phi' \equiv (\phi'(x_1), \ldots, \phi'(x_N))$, one can completely describe the effect of the channel Λ as a transformation of the vector ϕ:

$$\phi \to \phi'. \quad (8)$$

Understanding the effect of the channel as the transformation depicted in equation (8) creates a compact and intuitive way to express the effect of the channel over all the information-theoretic functionals that we will derive. From equations (6,7) we observe that, for any $x_k \in \Omega$,

$$0 \leq \phi'(x_k) \leq \phi(x_k). \quad (9)$$

Notice that, in general, ϕ' is not a probability. Now, we observe that we can rewrite the usual mutual information among X_Ω and X'_Ω, $I(X_\Omega : X'_\Omega)$, in terms of ϕ and ϕ':

$$I(X_\Omega : X'_\Omega) = h(X_\Omega) \sum_{x_k \in \Omega} \phi'(x_k),$$

and consistently, the standard noise term $h(X_\Omega | X'_\Omega)$ (Cover, 2001), can be expressed as:

$$h(X_\Omega | X'_\Omega) = h(X_\Omega) \sum_{x_k \in \Omega} (\phi(x_k) - \phi'(x_k)).$$

The transmitted semantic information, to be referred to as $\mathbf{Q}_\Omega(\Lambda)$ can be now easily derived in terms of ϕ and ϕ':

$$\mathbf{Q}_\Omega(\Lambda) = \sum_{x_k \in \Omega} \phi'(x_k) Q(x_k). \quad (10)$$

Equation (10) quantifies the *average semantic content of the information received by agent $\mathcal{B}$ after being decoded in some way by agent $\mathcal{A}$ and sent through the channel* Λ. Consistently to what we have done above with the standard mutual information, the semantic noise or the loss of semantic information, $\eta(X_\Omega | X'_\Omega)$ can be expressed as:

$$\eta(X_\Omega | X'_\Omega) = \sum_{x_k \in \Omega} (\phi(x_k) - \phi'(x_k)) Q(x_k).$$

2.4 Transmission

We are now in the position to compute the *total* information transmission from agent $\mathcal{A}$ to agent $\mathcal{B}$. This involves the mutual information between X_Ω and its reconstruction made by agent $\mathcal{B}$, X'_Ω plus the semantic content that can be properly conveyed. According to this, one has that total information transmission, $I_T(X_\Omega, X'_\Omega, \Omega)$ is defined as:

$$I_T(X_\Omega, X'_\Omega, \Omega) = I(X_\Omega : X'_\Omega) + \mathbf{Q}_\Omega(\Lambda). \quad (11)$$

which can be rewritten, as we did in equation (4), as:

$$I_T(X_\Omega, X'_\Omega, \Omega) = h(X_\Omega) \sum_{x_k \in \Omega} \phi'(x_k) (1 + q(x_k)).$$

We observe that the above expression is identical to the one we derived in equation (5) describing total information of an ensemble of objects, but with changing $\phi \to \phi'$. Finally, if one wants to highlight the role of the source and the different noise contributions, mimicking the standard formulation of information theory, $I_T(X_\Omega, X'_\Omega, \Omega)$ can be rewritten as:

$$I_T(X_\Omega, X'_\Omega, \Omega) = H(X_\Omega, \Omega) - h(X_\Omega | X'_\Omega) - \eta(X_\Omega | X'_\Omega).$$

2.5 Properties

We finally point out some of the some of the properties of I_T as defined in equation (11). We first observe that, if the channel is totally noisy, $(\forall x_k \in \Omega)(\phi'(x_k) = 0)$. Indeed, in a totally noisy channel Λ, $(\forall x_k, x_j \in \Omega)(\Lambda(x_k, x_j) = 1/N)$, leading to

$$(\forall x_k \in \Omega) \;\; \phi'(x_k) = 0,$$

which results, consistently, into

$$I_T(X_\Omega, X'_\Omega, \Omega) = 0.$$

On the contrary, if the channel is noiseless, Λ is a $N \times N$ *permutation matrix*, namely, a $N \times N$ matrix which has exactly one entry equal to 1 in each row and each column and 0's elsewhere It is straightforward to check that this leads to $\phi' = \phi$, so that $I(X_\Omega, X'_\Omega) = h(X_\Omega)$ and $\mathbf{Q}_\Omega(\Lambda) = \langle Q \rangle_\phi$, and, accordingly:

$$I_T(X_\Omega, X'_\Omega, \Omega) = H(X_\Omega, \Omega).$$

We observe that there are $\binom{N}{2}$ permutation matrices, so there are $\binom{N}{2}$ different configurations that lead to the above result. Finally, using equation (9) we can properly bound I_T:

$$0 \leq I_T(X_\Omega, X'_\Omega, \Omega) \leq H(\Omega, X_\Omega).$$

The above chain of inequalities ensures that I_T has a good behaviour concerning the intuitive constraints one has to assume for an information measure: 1) a totally noisy channel implies that no information is transmitted and 2) no information is created in the process of sending and processing.

3 DISCUSSION

We presented an information-theoretic framework to evaluate the communication between two autonomous agents which includes its semantic relevance. Specifically, we derived, within the Shannon's paradigm, the amount of *total information* transmitted: The standard mutual information plus a term accounting for the amount of *semantic* bits carried by each statistical bit. The main result of the paper, provided in equation (11) shows that it is possible to evaluate the transmission of message content using the standard framework. Crucially, the specific quantification of the content of the message is left from the theory and remains deliberately open, giving a total generality to the derived results. Beyond the generality of the result, it paves the way towards a rigorous information-theoretic exploration of code emergence in scenarios where autonomous agents develop and evolve under evolutionary constraints. This is the case, among others, of artificial intelligence studies based on autonomous robots or simplified biological problems concerning the resilience and emergence of shared codes. Further works should explore how to match the obtained results accounting for meaning transmission –without referentiality conservation assumed– with the results provided in (Corominas-Murtra, 2013), where the problem of referentiality conservation –without meaning quantification– is addressed.

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