Optimum Vehicle Flows in a Fully Automated Vehicle Network

Joerg Schweizer[1], Tiziano Parriani [2], Emiliano Traversi[3] and Federico Rupi[1]
[1]*Dept. of Civil, Environmental, Chemical and Materials Engineering (DICAM), University of Bologna, Bologna, Italy*
[2]*Optit, Bologna, Italy*
[3]*Dept. of Computer Science (LIPN), University of Paris 13, Paris, France*

Keywords: Self-driving Cars, Automated Vehicles, PRT Personal Rapid Transit, Unoccupied Vehicle Routing, Traffic Assignment, System Optimum, User Equilibrium.

Abstract: This paper provides a novel assignment method and a solution algorithm that allows to determine the optimum vehicle flows in a fully automated vehicle network. This assignment method incorporates the following *specific* features: (1) *optimal redistribution of occupied and unoccupied vehicles*; (2) inter-vehicle spacing is adapted to meet the *minimum safe distance criteria* on congested link, (no collision in the worst failure case); (3) trip-time minimization of all traffic participants by a centralized vehicle routing. The latter feature allows the realization of a so called *system optimum solution*, which minimizes the total time of all trips. This assignment method is applied to two, topologically different, test networks at different travel demand levels, in order to determine: the share of unoccupied vehicle, the minimum number of required vehicles, the share of congested links, the lost trip-time of occupied vehicles due to the presents of unoccupied vehicles. Furthermore, the advantage of a centralized vehicle routing is quantified by comparing the total trip-times of a scenario using a *system optimum solution* with a scenario applying the *user equilibrium solution*, without considering unoccupied vehicle flows. Regarding the investigated scenarios, the share of unoccupied vehicle flows with centralized vehicle routing in a uniform, random demand scenario is approximately $11\% - 14\%$.

1 INTRODUCTION

1.1 Motivation

According to the U.S. National Highway Traffic Safety Administration (NHTSA), a level 4 vehicle is designed to perform all safety-critical driving functions and monitor roadway conditions for an entire trip, including unoccupied vehicle movements. However, it is yet uncertain when level 4 vehicles will become available to the public and which share they may achieve in the automobile market.

In recent years, also the automation of public transport has made a significant breakthrough as the first Personal Rapid Transit (PRT) systems are successfully operating in Masdar, Abu Dhabi, UEA, at terminal 5, Heathrow airport, London, UK and Suncheon Bay, South Korea. PRT has similar service characteristics than level 4 automated vehicles, except that PRT vehicles run on dedicated guideways, which are usually elevated or underground. The operation of PRT resembles a taxi-service with taxi-stands and an automated taxi driver.

The traffic flows analysis made in this article addresses both transport technologies. For this reason we use the term *automated vehicles* for automated road vehicles as well as PRT vehicles.

With respect to the current road network used by human drivers, networks with fully automated vehicles are expected to enable major changes in terms of traffic circulation, and urban land-use: (i) *Reduction of trip times*; the vehicle routing would no longer be the choice of individual vehicles, but imposed by traffic managements, either decentralized or centralized;

(ii) *reduction of parking space*; the option to run also unoccupied vehicles, would allow shared vehicle schemes; The net effect of such a scheme would be a reduced need for urban parking space.

However, huge research efforts and/or investments are required in the forthcoming years before large scale automated vehicle networks can be realized. For this reason, it is of paramount importance to assess the potential benefits of such networks for local as well as for strategic planning. The method developed and demonstrated in this paper does exactly address these issues: the traffic assignment method allows to deter-

Schweizer, J., Parriani, T., Traversi, E. and Rupi, F.
Optimum Vehicle Flows in a Fully Automated Vehicle Network.
In *Proceedings of the International Conference on Vehicle Technology and Intelligent Transport Systems (VEHITS 2016)*, pages 195-202
ISBN: 978-989-758-185-4

mine the optimal flow distribution in automated vehicle networks, while minimizing the total travel time. With this information also the best case environmental impacts can be estimated.

1.2 Traffic Assignment Methods and Automated Vehicles

In transport planning, the traffic assignment problem for congested networks has been extensively studied since Wardrop's two optimality principles, user equilibrium (UE) and system optimal (SO), were first published (Wardrop J.G. (1952)). The limited road capacity has been modeled by incorporating link capacity constraints. But it became more common to implement capacity limits through flow deviation, using flow-dependent link cost functions, see (Nie et al. (2004)) for a comprehensive comparison. The first solution algorithm proposed by Frank-Wolfe (Frank H. and Wolfe P. (1956)) is still widely used by transport practitioners despite its drawbacks (Patriksson M. (1994)). A comparison of known solution algorithms for the general convex multi commodity flow problems can be found in (Ouorou A. et al. (2000)).

The user equilibrium (UE) assignment has received most attention, as it reflects the traffic flows in an equilibrium where all road users have minimized their own travel times, or generalized travel costs, for a comprehensive overview, see (Patriksson M. (1994)). The system optimum (SO) traffic assignment minimizes the sum of trip times over all users. The SO assignment is particularly interesting for automated vehicle networks, because such a global optimization could be performed by a centralized traffic management system, controlling the routes of all vehicles. However in literature, flows of unoccupied vehicles have not been considered.

Unoccupied vehicle routing received more attention with the emerging PRT technology. The main approach have been heuristically optimized microsimulators (Andréasson I. (1994); Koskinen K. et al. (2010)). Lees and Miller formulated as first a benchmark for optimum routing with a uniform demand (Lees-Miller J.D. et al. (2010)). A static traffic assignment method has been proposed (Schweizer J. et al. (2012)) which includes unoccupied vehicle flows: a linear programming model has been applied to a simple, uncongested network. Furthermore, a bilinear model for congested links has been formulated.

On congested networks, the vehicle flow on a link depends on link travel times, the link travel times for vehicle networks depend on the headway (which are in turn a function of the link flow). In automatic vehicle control literature, different vehicle spacing policies can be implemented. The most relevant policies are: the *constant time headway policy* and *constant safety policy*. The bulk of research deals with constant time headway spacing policy, which is usually adopted by Automated Highway Systems (AHSs) in order to form platoons of closely spaced vehicles, see for example (Horowitz R. and Varaiya P. (2000)). The constant safety policy maximizes vehicle flows at a given speed, while guaranteeing collision-free operation. However, the control system for constant safe headways are inherently non-linear and more difficult to analyze and design. Nevertheless, constant safety considerations have played a role in the design of control laws for platoon-join manœvers with AHS (Li et al. (1997)). A non-linear feedback controller which keeps vehicles at a minimum safe distance has been proposed in (Schweizer J. (2004)).

The present work focuses on: (i) the development of a Frank-Wolfe based solution algorithm (Frank H. and Wolfe P. (1956)) for the assignment model of congested, automated vehicle networks, as proposed in (Schweizer J. et al. (2012)); this assignment model assumes a constant safety policy and rerouting of occupied and unoccupied vehicles using either a decentralized or a centralized traffic management, represented by a UE assignment or a SO assignment, respectively. (ii) the application of the developed assignment method to two different real cities (with different, simplified demand scenarios), in order to show the theoretical potential of fully automated vehicle networks in terms of trip-times and vehicle requirements.

1.3 Paper Organization

The remainder of the paper is organized as follows: The next section describes the traffic assignment model and the proposed solution algorithm. In Sec. 3 the traffic assignment is applied to two different networks. Finally, in Sec. 4 some conclusions of this work and its impacts are drawn.

2 ASSESSMENT METHODOLOGY

This section focuses on the description of the traffic assignment method which will be used in successive traffic analysis. First the assignment problem is defined, which consists of the link cost function and the optimization model. Thereafter, the solution algorithm which solves the assignment problem is briefly explain. Finally the investigated traffic scenarios and performance indicators are introduced.

2.1 Assignment Problem

The link cost function $c_a(f_a)$ of link a is modeled as a non-linear function of link flow f_a and represents the effective link travel time of the vehicles running at a constant speed v. For uncongested links, this speed equals free-flow speed while for the congested links, the speed is determined by the minimum safe distance criteria. The considered link parameters are link length ℓ_a and free flow speed (or maximum speed) V_a. All automated vehicles on the network are parameterized by their brake actuation time τ, minimum guaranteed emergency brake deceleration a_E and vehicle length L. Consequently, the minimum safe nose-to-nose time-headway $T(v)$ at a constant speed v is given by:

$$T(v) = \tau + \frac{v}{2a_E} + \frac{L}{v}. \tag{1}$$

This condition is often referred to as the "brick wall stopping criteria" and guarantees collision-free operation even in case a vehicle stops instantly, preforming an infinite deceleration. For road vehicles this may be too restrictive, as it limits capacity. But the theory can be easily extended to the case where the vehicles are allowed a finite worst case deceleration. It is straight forward to show that $T(v)$ is minimal at the critical speed $v = v_{\text{crit}} = \sqrt{2a_E L}$ and the link capacity q is solely determined by the vehicle parameters:

$$q = \frac{1}{T(v_{\text{crit}})}.$$

The assumption made for the determination of the flow dependent link costs $c_a(f_a)$ is the following: the link flow f_a imposes the vehicle's time headway $\frac{1}{f_a}$. If this headway is above the minimum safe time headway $T(V_a)$, then all vehicles run at free-flow speed $v = V_a$, otherwise the vehicle's speed v must be reduced such that the minimum safe headway is maintained. We further assume that the vehicle speed on the link is always between critical speed and maximum speed ($v_{\text{crit}} < v < V_a$). This means that the network can have congestions, but no still-stands (traffic jam). In this case, one can find the following unique positive solution for the positive link flow-dependent vehicle speed

$$v(f_a) = \frac{\sqrt{-2a_E f_a^2 L + a_E^2 f_a^2 \tau^2 - 2a_E^2 f_a \tau + a_E^2}}{f_a} + \frac{-a_E f_a \tau + a_E}{f_a}. \tag{2}$$

As the link travel time equals $\frac{\ell_a}{v(f_a)}$, the link cost function $c_a(f_a)$ results in the non-linear function

$$c_a(f_a) = \begin{cases} \dfrac{\ell_a}{V_a} & \text{for} \quad f_a \leq \dfrac{1}{T(V_a)} \quad \text{(uncongested)} \\ \dfrac{\ell_a}{v(f_a)} & \text{for} \quad f_a > \dfrac{1}{T(V_a)} \quad \text{(congested)} \end{cases}. \tag{3}$$

with minimum safe headway $T(\cdot)$ from Eq. 1 and $v(\cdot)$ from Eq. 2.

The link cost function $c_a(f_a)$ is separable and piecewise continuous. But in order to apply the Frank Wolfe based solution algorithm, solving Wardrop's UE and SO principles, the link cost functions need to be continuous and monotonically increasing (Patriksson M. (1994); Cascetta E. (2001)).

For this reason, an approximated cost function $\hat{c}_a(f_a)$ is defined by introducing a small slope for the free-flow case and by interpolating with a fourth order polynomial for the congested case:

$$\hat{c}_a(f_a) = \begin{cases} \dfrac{\ell_a}{V_a} - \mu\left(\frac{1}{T(V_a)} - f_a\right) & \text{for} \quad f_a \leq \frac{1}{T(V_a)} \\ P(f_a) & \text{for} \quad f_a > \frac{1}{T(V_a)} \end{cases} \tag{4}$$

where $\mu > 0$ is a small slope and $P(f_a) = \sum_{i=0}^{4} \alpha_i \left(f_a - \frac{1}{T(V_a)}\right)^i$. The coefficients α_i, $i = 0 \ldots 4$ of polynomial $P(f_a)$ are determined such that function $\hat{c}_a(f_a)$ is continuous in $\frac{1}{T(V_a)}$, monotonically increasing and a good fit for the exact function $c_a(f_a)$. An example of the approximated and exact cost function is shown in Fig. 1.

The particularity of the traffic assignment model for shared, automated vehicles is the incorporation of a demand for *unoccupied vehicle trips*. The occupied vehicle demand is defined by an origin-to-destination demand matrix, while the unoccupied vehicle trips are generated by an additional vehicle demand that just compensates the occupied vehicle demand, as proposed in (Schweizer J. et al. (2012)), The flow-equilibrium is ensured by adding a multi-origin/multi-destination unoccupied vehicle flow to the model.

The transport problem is defined as follows: Let $\mathcal{G} = (\mathcal{V}, \mathcal{A})$ be the directed network graph where $\mathcal{V}$ and $\mathcal{A}$ are the sets of network nodes and links, respectively. Each link $a = (i, j) \in \mathcal{A}$ is associated with total link flow f_a (unoccupied plus occupied vehicle) and the travel cost function $\hat{c}_a(f_a)$ from Eq. 5.

Following Wardrop's second principle, a SO traffic assignment minimizes the objective function

$$z_{SO}(\mathbf{f}) = \sum_{a \in A} \hat{c}_a(f_a) f_a \tag{5}$$

where $\mathbf{f}$ is the link flow vector $\mathbf{f} = [f_0, \ldots]'$. In the SO case, $z_{SO}(\mathbf{f})$ is the total travel time of all trips, including unoccupied vehicle movements. The solution of the SO assignment will be compared against the solution of the UE, using the integral objective function

$$z_{UE}(\mathbf{f}) = \sum_{a\in A} \int_0^{f_a} \hat{c}_a(f)df \; . \tag{6}$$

Note that the integral objective function $z_{UE}(\mathbf{f})$ can be determined analytically because $\hat{c}_a(f_a)$ is piecewise integrable.

Let $\mathcal{C} \subset \mathcal{V}$ be the sub-set of nodes representing centroids and let $\mathcal{R} \subseteq \mathcal{V} \times \mathcal{V}$ be a set of routes $r = (s_r, t_r) \in \mathcal{R}$, where s_r and t_r denote respectively the origin- and destination centroids of route r. The total travel demand of occupied vehicles within the observation period are D trips and d_r is defined as the fractional demand representing the number of passengers traveling along route r. This means the total demand $D = \sum_{r\in\mathcal{R}} d_r$.

The residual demand D_i^{res} in centroid i is positive, in case there is a demand in node i and negative in case of a surplus. An additional variable y_a^r is introduced to represent the fractional part of the flow on route r using link a.

The occupied vehicle flow on link a is the sum of all fractional flows y_a^r multiplied by the route demand d_r. The total vehicle flow f_a on link a is the sum of the occupied vehicle flow and the unoccupied vehicle flow w_a

With the above definitions, the non-linear programming model can be stated as follows:

$$\min z(\mathbf{f}) \tag{7}$$

$$\text{s.t.} \sum_{a\in\delta^+(i)} y_a^r - \sum_{a\in\delta^-(i)} y_a^r = \begin{cases} 1 & \text{if } i = s_r \\ -1 & \text{if } i = t_r \\ 0 & \text{otherwise} \end{cases}, \tag{8}$$

$$\forall i \in V, \forall r \in \mathcal{R}$$

$$\sum_{a\in\delta^+(i)} w_a - \sum_{a\in\delta^-(i)} w_a = D_i^{res}, \forall i \in \mathcal{C} \tag{9}$$

$$\sum_{r\in\mathcal{R}} d^r y_a^r + w_a = f_a, \forall a \in \mathcal{A}, \tag{10}$$

$$y_a^r \geq 0, \forall a \in \mathcal{A}, \forall r \in \mathcal{R} \tag{11}$$

$$w_a \geq 0, \forall a \in \mathcal{A} \tag{12}$$

The objective function $z(\mathbf{f})$ can be either replaced by $z_{SO}(\mathbf{f})$ (Eq. 6) or by $z_{UE}(\mathbf{f})$ (Eq. 7) which allows to determine the flows for the system optimum model or the user equilibrium model, respectively. The constraints Eq. (9) (resp. Eq. (10)) guarantee flow conservation of occupied (resp. unoccupied) vehicles. Constraint Eq. (11) makes sure that fractional flows and unoccupied flows sum up to the total flow f_a of each link. The remaining constraints guarantee positiveness of all flows. The above solution provides not only the total flow vector $\mathbf{f}$, but also the unoccupied flow vector of all links $\mathbf{w} = [w_0, \ldots]'$. Furthermore, from the total link flows, it is possible to determine the minimum number of required vehicles $N_{\min}$ for the specific scenario:

$$N_{\min} = \sum_{a\in A} c_a f_a \; . \tag{13}$$

2.2 Solution Method based on the Frank Wolfe Algorithm

This section explains how the non-linear, uncapacitated traffic problem stated in Eq. (8)-(13) can be solved, based on the Frank Wolfe method.The iterative algorithm can be summarized in the following four steps:

Step 0: (Initialization) Set iteration counter $k = 0$ and set upper bound to $UB = +\infty$. Compute an initial feasible solution for the vehicle flow vector $\mathbf{f}^0$ and the unoccupied vehicle flow vector $\mathbf{w}^0$ by solving the linear sub-problem

$$\min \textstyle\sum_{a\in A} c_a^0 f_a \tag{14}$$
$$(10) - (13) \; .$$

with the *constant* link costs $c_a^0 = c_a(0) \,\forall a \in \mathcal{A}$.

Step 1: Calculate the direction flow vector $\mathbf{f} - \mathbf{f}^k$ by minimizing the linear programming sub-problem with the objective function $\tilde{z}(\mathbf{f}) = z(\mathbf{f}^k) + \nabla z(\mathbf{f}^k)(\mathbf{f} - \mathbf{f}^k)$ where $z(\cdot)$ represents either the objective function z_{SO} for SO assignments or z_{UE} for UE assignments. The sub-problem to be solved is

$$\min \tilde{z}(\mathbf{f}) \tag{15}$$
$$(10) - (13) \; .$$

Let $\tilde{\mathbf{f}}$ and $\tilde{\mathbf{w}}$ be the solutions to this problem. Then $LB = \tilde{z}(\tilde{\mathbf{f}})$ represents the lower bound with respect to the chosen objective function. The remaining open gap is defined by

$$GAP^k = 100 \times \frac{UB - LB}{UB}$$

where upper bound UB is updated in Step 3. The criteria $GAP^k < \varepsilon_G$ can be used as stopping criteria.

Step 2: Determine λ^k, which is the solution to the minimization problem

$$\lambda^k = \arg \min_{0\leq\lambda<1} z\left(\mathbf{f}^k + \lambda\left(\tilde{\mathbf{f}} - \mathbf{f}^k\right)\right). \tag{16}$$

This is the problem of finding the minimum of $z(\cdot)$ along the line segment joining the two points $\mathbf{f}^k$ and $\tilde{\mathbf{f}}$. Again, $z(\cdot)$ can be either of the objective functions from Eq. 6 or Eq. 7. The criteria $\lambda^k < \varepsilon_L$ can be used as stopping criteria.

Step 3: Obtain the new feasible point,

$$\mathbf{f}^{k+1} = \mathbf{f}^k + \lambda^k \left(\tilde{\mathbf{f}} - \mathbf{f}^k\right)$$

and set the new costs $c_a^{k+1} = \hat{c}_a(f_a^k)$, $\forall a \in \mathcal{A}$. Split the unoccupied vehicle flows proportionally to the total flows:

$$\mathbf{w}^{k+1} = \mathbf{w}^k + \lambda^k \left(\tilde{\mathbf{w}} - \mathbf{w}^k\right) .$$

The new upper bound is updated with $UB = z(\mathbf{f}^{k+1})$. Increase $k = k+1$ and go to Step 1.

As $k \to \infty$ both, λ^k and GAP^k tend to zero and the flow vectors $\mathbf{f}^k$ and $\mathbf{w}^k$ converge to $\mathbf{f}$ and $\mathbf{w}$.

This solution algorithm has been implemented in `C++`. The first feasible solution Eq. (16) as well as the approximation in Eq. (17) have been solved by a minimum-cost linear Multi-Commodity Flow (MCF) algorithm based on the well known Dantzig-Wolfe decomposition approach, together with column generation, see e.g. (Tomlin J.A. (1966); Lübbecke M.E. et al.(2005); Frangioni A. and Gallo G.(1999)). The implemented solver makes intensive use of the CPLEX 12.5 libraries. The Python Numpy and SciPy packages have been used to calculate λ^k and to update all cost and flow vectors.

3 RESULTS

The traffic assignment method has been applied to two network instances, central Köln and the central business district (CBD) of Portland. These networks have been chosen, because the two cities represent two completely different network topologies: The CBD of Portland has an almost regular street grid, while the historically grown street network of Köln is irregular. The Köln graph has 1007 links and 702 nodes, while the smaller Portland graph has only 506 links and 333 nodes. The transport graph of both cities has been extracted from OpenStreetMap as data source. The OSMOSIS package has been employed to extract the main streets subgraph and SUMO (Simulation of Urban Mobility) generated the directional transport graph with link attributes, such as length and speed limits.

In order to test the traffic assignment method, two demand scenarios have been considered: a *random demand scenario*, where OD-pairs have been randomly selected from the set of all graph nodes; an *asymmetric demand scenario*, where all trips are directed from the east to the west part of Portland.

The demand level has been changed, by scaling the number of trips between each O-D pair in order to match a predefined total number of trips D.

The parameters of all vehicles are $\tau = 0.5s$, $a_E = 2.5m/s^2$ and $L = 3.5m$. In this case, the critical speed $v_{\text{crit}} = 15.0km/h$ and the capacity $q = 1656veh/h$ per lane. The speed attribute of the Openstreetmap network served as information for the free-flow speeds V_a. The link cost of an example link is shown in Fig. 1. Regarding the link cost approximation $\hat{c}_a(f_a)$ from Eq. (5), the following parameters have been used: $\mu = 0.1$, $f^B = 1394vph$ and $f^C = 1525vph$.

For example, a link with length $\ell_a = 1000m$ and free flow speed $V_a = 40km/h$ shows an average error between exact and approximated costs of $\approx 13\%$ which appears to be reasonably low.

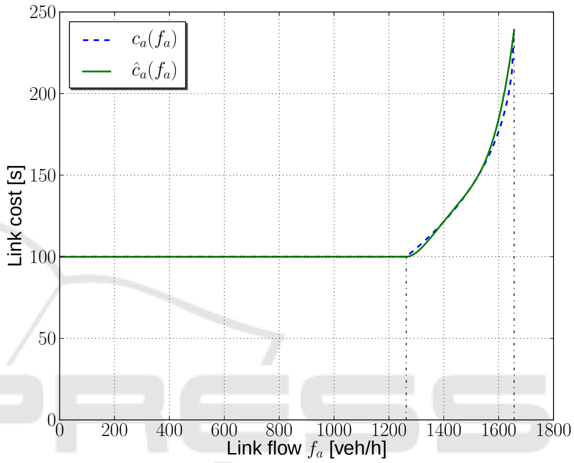


Figure 1: Exact link cost $c_a(f_a)$ and approximated link cost $\hat{c}_a(f_a)$ for the given vehicle parameters (see text) and an urban road link of length $\ell_a = 1000m$ with a free flow speed of $V_a = 40km/h$.

The simulation results from the two example networks demonstrate the theoretical potential of fully automated vehicles in terms of travel times, vehicle requirements and congestion levels. First, the average trip time $\frac{z(\mathbf{f})}{D}$ has been determined for both networks, in order to compare the results of a centralized traffic management (System Optimum assignment SO) and the optimization by each user (User equilibrium assignment UE). The result from Fig. 2 shows for the Portland network at medium demand levels slightly reduced trip-times using the SO assignment with respect to the UE assignment. Whereas for low and high demand levels, there is no significant difference in trip times between the two assignment methods. The reason is that a low demand produces no congestion effects and both assignments will choose the shortest path for all users. As demand and congestions increase, the SO assignment deviates the traffic better on alternative routes in order to avoid penalties through congestion delays, which results in lower average trip times. For high demand levels, the network is congested everywhere and there are no longer alternatives to save time by taking faster deviations, thus both assignment methods produce the same average trip times. For the Köln network (not shown), there is no difference between the SO and UE assignment method, most likely because of a lack of suitable route alternatives.

Surprisingly, the share of unoccupied vehicle flows ($100 \times \frac{\sum_a w_a}{\sum_a f_a}$) varies insignificantly with demand D: the unoccupied vehicle share ranges from 13.6% to 14.3% for the Portland network and from 11.6% to 11.95% for the Köln network.

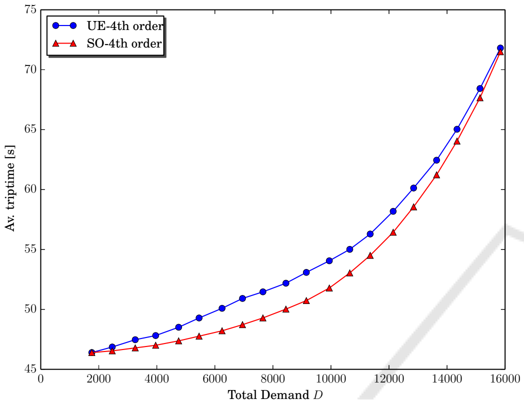


Figure 2: Portland network with random demand scenario. Average trip time $\frac{z(\mathbf{f})}{D}$ for different demand levels D and assignment methods SO ($z_{SO}(\cdot)$) (circles) and UE ($z_{SO}(\cdot)$) (triangles).

The minimum number of required vehicles $N_{\min}$ for the random and asymmetric demand scenarios is shown Fig. 3. Note the strong dependency of $N_{\min}$ on both, demand pattern and demand level. However, the required number of vehicles of the shared schemes is in both cases significantly lower than the required number of vehicles in a non-shared scheme (which corresponds to the number of trips D). Ratio of shared vehicles with respect to non-shared vehicles ($N_{\min}/D$) is 1.9% for the random demand scenario and 8.3% for the asymmetric demand scenario.

In an attempt to quantify the impact of the unoccupied vehicle flows on trip-times and congestion level, the following assignment-scenarios have been compared:

1. the SO assignment *with* unoccupied vehicles, identical to the scenario shown in Fig. 2.
2. the SO assignment *without* unoccupied vehicles.
3. the UE assignment *without* unoccupied vehicles.

Scenarios 2 and 3 correspond to SO and UE assignments without unoccupied vehicles, this means vehicles are parked at the destination. In particular, scenario 3 represents the user equilibrium of present, non shared car-traffic, without centralized traffic management. The total trip-times $z(\mathbf{f})$ (the sum of all, occupied and unoccupied vehicle trips) are shown in Fig. 4 (a). One can observe that scenario 1 (the assignment with unoccupied vehicle flows) has an approximately 14% higher total trip time compared with the other scenarios (without unoccupied vehicle flows). As the 14% correspond to the share of unoccupied vehicles, one can conclude that the presence of unoccupied vehicle flows does not prolongate the occupied vehicle trips.

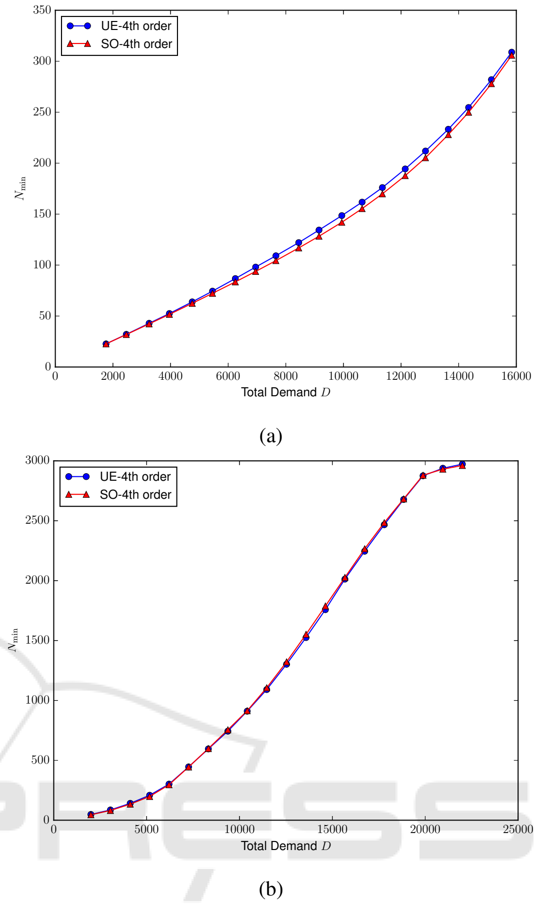
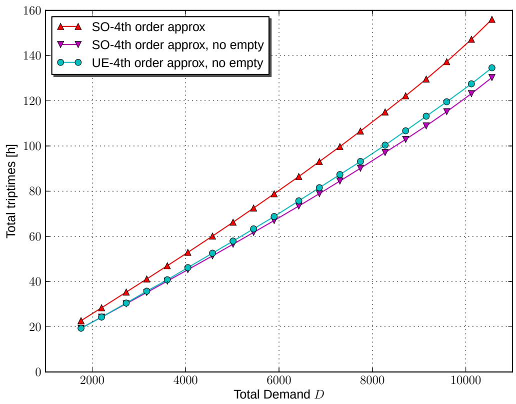
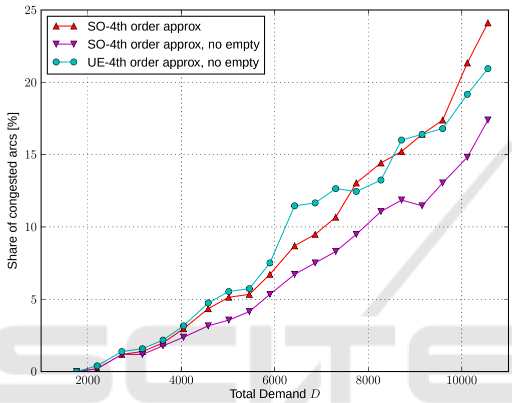


Figure 3: Minimum number of vehicles $N-\min$ for different demand scenarios and demand levels D. Portland network with (a) random demand scenario and (b) asymmetric demand scenario.

A clearer picture can be gained by looking at congested links — a link a is considered congested if the travel speed is below free-flow speed, which means $f_a > \frac{1}{T(V_a)}$. Figure 4 (b) shows that the numbers of congested links in scenario 1 is almost equal to the one of scenario 3, which is the assignment without unoccupied vehicles and UE assignment. In case of the investigated Portland network, the results suggest that the additional unoccupied vehicle flows do obvi-



(a)



(b)

Figure 4: Three assignment with Portland network with random demand scenario: Scenario 1 (SO-4th order approx), Scenario 2 (SO-4th order approx, no empty) and Scenario 3 (SO-4th order approx, no empty). (a) Total trip times $z(\mathbf{f})$ and (b) Share of congested links (=arcs).

ously add to the total trip times, but they do not add to congestion levels. In other words, the unoccupied vehicles do not cause delays for the occupied vehicles in case of a central traffic management. However, this result cannot be generalized as it may be due to the particular one-way grid of the Portland network.

4 CONCLUSIONS

The traffic assignment method developed in this work has allowed to determine the optimal flows of a network with automated vehicle traffic, considering also unoccupied vehicle trips, a decentralized or centralized traffic management and a minimum safe headway vehicle-spacing. The assignment problem has been solved with a method based on the well known Frank-Wolfe algorithm.

The assignment models have been applied to two example networks: the centers of Portland and Köln. As expected, for moderately congested networks the SO assignment has resulted in lower average trip times with respect to the UE assignment. The difference between SO and UE has been notably larger for the Portland network, most likely due to its grid-network, offering more route alternatives for the SO assignment algorithm to distribute flows.

The number of required shared, automated vehicles depends strongly on the network and demand patterns. The ratio of shared vehicles with respect to non-shared vehicles to satisfy the same demand has been found to be in the range of $1.9 - 8.3\%$ for Portland center.

In order to study the impact of unoccupied vehicle flows, the SO and UE traffic assignments have been applied to the Portland network with and without the generation of unoccupied vehicle trips (comparison of shared and non shared scenario). The results suggest that a centrally optimized traffic management could prevent unoccupied vehicles from delaying occupied vehicles. However, this is not generally true and depends on the network topology and demand patterns.

The proposed assignment models do have limitations. The derived link cost-function is an over-simplification of the real network, neglecting junctions and multiple lanes. The share of unoccupied vehicles depends predominantly on the location of origins and destinations (11% - 14% in case of randomly chosen origins/destinations and 39% - 48% in case of a strongly asymmetric demand in Portland center).

The presented assignment algorithm is useful for strategic planning, network design, technology assessments and benchmarking of real-time traffic managements of automated vehicle networks. An interesting future research topic is to use this fast, static traffic assignments as short term predictors within a vehicle scheduling process.

REFERENCES

NHTSA'S Office of Behavioral Safety Research 2012 Some theoretical aspects of road traffic research. facts DOT HS 811 611. http://www-nrd.nhtsa.dot.gov/CATS/index.aspx

Wardrop J.G. (1952) Some theoretical aspects of road traffic research. Proc. Inst. Civ. Eng. 2: pp. 325-378.

Frank H. and Wolfe P. (1956) An Algorithm for quadratic programming, Naval Research Logistics Quarterly, 3, pp. 95-110.

Beckmann M.J. (1965) On optimal tolls for highways, tunnels and bridges Vehicular Traffic Science, American Elsevier, New York (1965), pp. 331341.

Sheffi Y. (1985) Urban Transportation Networks: Equilibrium Analysis with Mathematical Programming Methods Prentice-Hall, Englewood Cliffs, NJ (1985)

Ouorou A., Mahey P., Vial J.-Ph. (2000) A Survey of Algorithms for Convex Multicommodity Flow Problems Management Science INFORMS Vol. 46, No. 1, pp. 126-147

Yang H., Huang H. J. (1998), Principle of marginal-cost pricing: How does it work in a general network? Transportation Research, 32A (1998), pp. 45-54

Patriksson M. (1994), The Traffic Assignment Problem: Models and Methods, VSP, Utrecht, the Netherlands.

Nie Y., Zhang H.M., Lee D. (2004) Models and algorithms for the traffic assignment problem with link capacity constraints Transportation Research Part B 38, pp. 285312

Xu M., Qu Y., Gao Z. (2008) Implementing Frank-Wolfe Algorithm under Different Flow Update Strategies and Line Search Technologies, J Transpn Sys Eng & IT, 8(3), pp. 14-22

Cascetta E. (2001) Transportation systems engineering: theory and methods. Kluwer Academic Publisher

Inouea S., Maruyama T. (2012) Computational Experience on Advanced Algorithms for User Equilibrium Traffic Assignment Problem and Its Convergence Error, Procedia - Social and Behavioral Sciences 43, pp. 445-456

Tomlin J.A. (1966) Minimum-Cost Multicommodity Network Flows. Operations Research, pp. 45-51.

Lübbecke M.E., Desrosiers J. (2005) Selected topics in column generation. Operations Research, 53.6: pp. 1007-1023.

Frangioni A., Gallo G. (1999) A bundle type dual-ascent approach to linear multicommodity min-cost flow problems. INFORMS Journal on Computing, 11.4: pp. 370-393.

Andréasson, I (1994). Vehicle Distribution in Large Personal Rapid Transit Systems. Transportation Research Record, No. 1451, pp 95-99, Transportation Research Board, Washington, D.C.

Schweizer J., Danesi A., Rupi F.,Traversi E. (2012) Comparison of static vehicle flow assignment methods and microsimulations for a personal rapid transit network, J. Adv. Transp.; 46: pp. 340-350

Lees-Miller J.D, Hammersley J.S, Wilson R.E (2010) Theoretical Maximum Capacity as a Benchmark for Empty Vehicle Redistribution in Personal Rapid Transit, Journal of the Transportation Research Board, No. 2146, pp 76-83, Transportation Research Board, ISSN: 0361-1981

Koskinen K., Luttinen T., Kosonen I. (2007) Developing a microscopic simulator for personal rapid transit (PRT) systems, Transportation Research Board 86th Annual Meeting, Washington, D.C.

Horowitz R. and Varaiya P. (2000) Control design of automated highway system, *IEEE Proc.*, vol. 88, no. 7.

Li P., Alvarez L., and Horowitz R. (1997) “AHS safe control laws for platoon leaders,” *IEEE Trans. Contr. Syst. Technol.*, vol. 5, no. 6, pp. 615–628.

J. Schweizer (2004) Non-linear feedback control for short time headways based on constant- safety vehicle-spacing. In IEEE Intelligent Vehicles Symposium, pp. 167 - 172.