Earth Rotation: An Example to Teach Rigid Body Motion and Environmental Monitoring

A Fallout of the Exploitation of LARES Satellite Data

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Abstract: The use of satellite laser ranging in combination with other space geodetic techniques allows us to determine Earth's motion with unprecedented accuracy, which is not as simple as usually described in basic textbooks. Besides rotation and revolution there is a wobble of the rotation axis that can be derived by the torque free case in rigid body dynamics. The presence of gravitational perturbations complicates the motion and considering Earth as non-rigid introduces even more variations in the basic Earth motion theory. What is interesting is that also the mass redistribution of air and water on the planet can affect the motion of Earth's rotational axis. Thanks to the millimetre accuracy achievable today, it is possible to correlate very small anomalous rotational axis displacements with global environmental changes such the change in ice melting. The paper will show the experimental motion of the Earth rotation axis and interpret it with the use of the Euler rigid body equations of motion, outlining also the effects of the gravitational perturbations of other bodies in the solar system and of the global climate changes on the Earth rotational axis.

1 INTRODUCTION

The Earth rotation is more complicated than what non-specialists could think, although the main components of the motion are sufficiently intuitive. Before the advent of space age it was not possible to test appropriately freely rotating bodies, so the planetary motion was the best paradigm available.

In the paper it will be first considered the Earth as a rigid body so that part of the wobble of the rotation axis will be explained using rigid body motion equations: the Euler equations. Incidentally we recall that the solution provided by Euler is still valid in the limit case of the rigid Earth. The new techniques available today can position the Earth rotation axis with accuracies at the level of one millimetre. Among those, laser ranging to LARES-type satellites gives a fundamental contribution.

LARES is an Italian Space Agency (ASI) funded program with the aim to improve the measurement of frame dragging or Lense-Thirring effect (Ciufolini et al., 2012a) from about 10% obtained in (Ciufolini and Pavlis, 2004) down at the level of 1% (Ciufolini et al. 2012b). The measurement of this effect is of great interest among scientists. Under development is the GINGER (Gyroscopes IN GEneral Relativity) experiment (Bosi, 2011, di Virgilio, 2014). The apparatus will exploit the Sagnac effect in the ring-lasers to measure frame-dragging.

Contrary to the above missions, LARES is instead a completely passive satellite so it is intrinsically simpler and more reliable. It is covered with Cube Corner Reflectors (CCRs) that have the property of reflecting laser pulses sent from the ground stations of the International Laser Ranging Service (Pearlman et al., 2002). Counting the return time of the pulses one determines the satellite position with few millimetres accuracy.

The accurate reconstruction of the orbit not only is useful for fundamental physics but also for the accurate determination of the position of the centre of mass and rotation axis of the Earth (in one word of the Earth reference frame). The feasibility of

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verifying the General Relativity effect but also the possibility of determining accurately the rotation axis of the Earth rely not only to the high accuracy of the orbit determination reachable with the laser ranging technique. Other factors such as the reduction of the effects on the non-gravitational perturbation achieved with a special design of LARES satellite and its estimation is mandatory, as shown in the Monte Carlo simulations of the experiment reported in (Ciufolini et al., 2013a).

The special design of LARES provides accurate data that integrated with data of other satellites and techniques allow us determine an accurate Earth reference frame (Pavlis et al., 2015a). In particular to reduce the non-gravitational perturbations a very high density material (Paolozzi et al., 2009) have been chosen for the satellite. To further reduce unmodeled effects on the satellite orbit, radiation pressure uncertainties have been limited by avoiding painting the surface of the satellite. Also thermal thrust perturbation have been minimized (Ciufolini et al., 2014) using a different approach with respect to what originally proposed in (Bosco et al., 2007) i.e. reducing the main origin of this perturbation: the CCRs. The surface of the CCRs as compared to the metallic surface of the satellite is the lowest with respect to what obtained in all other laser-ranged satellites (Ciufolini et al., 2013b; Paolozzi et al., 2015).

February 13, 2012 LARES was successfully put in orbit with the VEGA launcher (Paolozzi et al., 2013; Paolozzi et al., 2012b) and data analysis for testing General Relativity is now in progress (Ciufolini et al., 2015).

In this paper we will concentrate on Earth's rotation axis showing that most of its motion can be explained by rigid body dynamics. Relation with climate change is also pointed out and indeed the combination of mechanics and environmental monitoring is believed to be an excellent driver to raise students' interest in both fields.

2 PLANETARY MOTION

In basic textbooks it is reported that planets, and in particular the Earth, have two main motions: revolution, which follows the second Kepler law (equal areas are swept out in equal time) and rotation characterized by a spin axis and angular velocity (considered both constant at first approximation). It is interesting to observe that those two motions are easily predictable under two simplified assumptions:

- Rigid body Earth
- External actions limited to conservative central forces.

If one neglects the non-gravitational perturbations and the gravitational actions of all the other bodies with the exclusion of the Sun, that is the source of the conservative central force, the second hypothesis applies to all planets in the solar system. In this simplified scenario the conservation law of angular momentum provides an endless motion of the planets according to the second Kepler law i.e. the well-known revolution motion.

Concerning the rotation being constant in magnitude and direction, the additional hypothesis of the Earth being infinitely rigid is required. In fact for a non-rigid Earth it could happen that the rotational speed would increase as a result of a reduction of the diameter, exactly in the same way as it would happen to a spinning ice skater when she/he brings the arms closer to the body. Also the direction of the rotational axis could change if the symmetry is broken by mass redistribution on the Earth. In the analogy of the skater also the rotational axis would change if only one arm is retracted. However this hypothesis is not sufficient to guarantee that the spin axis remains constant in inertial space. This aspect will be analyzed in detail later in the paper.

It is customary in rigid body motion analysis to use a reference frame fixed with the body, and this approach is maintained here for the case of the Earth. The Earth reference frame is defined by the International Earth Rotation and Reference Systems Service (IERS) and is called International Terrestrial Reference Frame or ITRF in short. The geographic position of the North and South poles are defined with a complex procedure that uses several techniques including satellite laser ranging on LARES-like satellites. The addition of LARES by the way will improve the accuracy of such a reference frame by about 20% as shown in (Pavlis et al., 2015a; Sindoni et al., 2015). The geographic location of North and South will define a geometric axis that does not coincide with the rotational axis of the Earth. One could think this rotation axis of the Earth is fixed in inertial space and consequently could be considered a realization of an inertial reference frame. But that in general would be a mistake even if the Earth would be an ideal perfect gyroscope as will be shown later in the paper.

In summary there are several reasons why the rotation axis of the Earth cannot be fixed in inertial space, and with respect to the ITRF:

1) Even if the Earth would be an ideal gyroscope, the rotation axis, in general, is not fixed in inertial space. In fact what remains fixed in inertial space is the angular momentum vector (see details later in the paper). In short, Newton's second law, written in an inertial reference frame, will imply the conservation of angular momentum vector $\boldsymbol{L}$ that only under very special situations is parallel to the angular velocity vector $\boldsymbol{\omega}$.
2) The Earth is not a rigid body.
3) There are gravitational actions acting on the Earth other than the solar attraction.
4) There are non-gravitational perturbations acting on the Earth.

In the following section we recall some fundamentals of rigid body dynamics preferring, whenever possible under the limited space available here, a graphical approach and pointing out some pitfalls that usually arise in this area.

3 ANGULAR MOMENTUM AND ANGULAR VELOCITY

First we recall that the angular momentum vector $\boldsymbol{L}$ is a quantity that depends on the rotation speed of the body, and on the mass distribution of the body itself with respect to three arbitrary axes. The angular velocity vector of the body $\boldsymbol{\omega}$ is measured in a chosen reference frame (usually an inertial frame, but not necessarily). The mass distribution is summarized by six independent numbers that are collected in a symmetric matrix called the matrix of inertia I also defined in an arbitrary reference frame (usually the body fixed reference frame). In compact form the rigorous definition of angular momentum in a reference frame where the body is "seen" to rotate with an angular velocity $\boldsymbol{\omega}$ is:

$$\boldsymbol{L}=\boldsymbol{I}\boldsymbol{\omega} \tag{1}$$

If the reference frame, where $\boldsymbol{\omega}$ is measured, is an inertial reference frame and there are no torques applied to the body, the vector $\boldsymbol{L}$ does not change (conservation of angular momentum) so that in general if $\boldsymbol{\omega}$ changes in direction and/or magnitude also $\boldsymbol{I}$ need to change to maintain $\boldsymbol{L}$ constant in time. The change of $\boldsymbol{I}$ is difficult to deal with so one can considerer at each instant an inertial reference frame that is aligned with the body reference frame and centred in the centre of mass of the body. In this series of inertial reference frames the above equation holds, meaning the $\boldsymbol{L}$ is constant although its components change in time because the reference frame is changing. If, in addition, we take the axes of those inertial reference frames parallel to the principal axes of inertia of the body (in general for regular shaped object those coincide with the symmetry axes), then the matrix $\boldsymbol{I}$ becomes diagonal. In Figure 1 the situation is depicted.

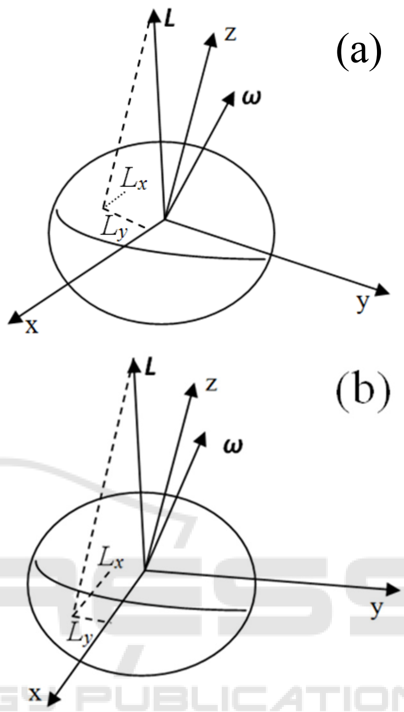

Figure 1: (a) Inertial reference frame at instant $t=t_1$; (b) Inertial reference frame, rotated with respect to (a), at instant $t=t_2$. Vector $\boldsymbol{L}$ is the same as in (a) (conservation of angular momentum) but its components have changed.

For the sake of clarity only x and y components of $\boldsymbol{L}$ have been represented. Note that $\boldsymbol{L}$ has not changed (conservation of angular momentum) from Figures 1a to Figure 1b, while its components on the rotated reference frame have changed. Also $\boldsymbol{\omega}$ from time t_1 (Figure 1a) and time t_2 (Figure 1b) has changed. Also note that in general $\boldsymbol{L}$ and $\boldsymbol{\omega}$ are not parallel in this series of inertial reference frames.
There are however special cases in which the two vectors $\boldsymbol{L}$ and $\boldsymbol{\omega}$ are parallel:

a) All three components of moments of inertia are equal, i.e.

$$I=\begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \tag{2}$$

with $I_{xx}=I_{yy}=I_{zz}=I$.

In this case Eq. 1 reduces to $\boldsymbol{L}=I\boldsymbol{\omega}$, i.e. the two vectors differ in magnitude by the factor given by the moment of inertia I.

b) Only one component of vector $\boldsymbol{\omega}$ is different from zero, with the exclusion of the component corresponding to the direction of the intermediate moment of inertia (because of instability of rotation around the intermediate axis) i.e. suppose $I_{xx}<I_{yy}<I_{zz}$ then it can be $\omega_y=\omega_z=0$ or $\omega_x=\omega_y=0$. In this last case one has: $L_x=L_y=0$ and $L_z=I_{zz}\,\omega_z$.

In both cases $\boldsymbol{\omega}$ is conserved and this fact let erroneously think that this is a general law, which is not, since what is conserved in torque free motion and in an inertial reference frame is the angular momentum vector and not the angular velocity vector.

As an example of the fact that the two vectors are not parallel let us consider the Earth. Using updated data from LARES satellite, in Figure 2 is reported the actual motion of the Earth axis in year 2013 as seen from the ITRF. The motion shown in Figure 2 proves that $\boldsymbol{L}$ and $\boldsymbol{\omega}$ are not parallel. In fact the Earth is not perfectly spherical as shown by the values reported in Table 1 (thus point (a) above does not apply) and $\boldsymbol{\omega}$ has components, though extremely small, also along the two horizontal axes of the ITRF (thus point (b) above does not apply).

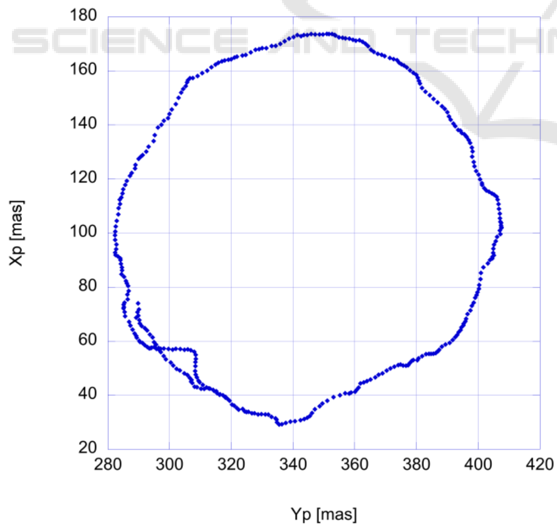

Figure 2: Track of the Earth rotation axis with the Earth surface as seen from an observer over the North pole and fixed in the ITRF. The figure refers to year 2013 and has been obtained including LARES data. 1 milliarcsecond (mas) corresponds approximately to 3 cm on the Earth's surface.

In Figure 3 is reported a sketch where the diameter of the trajectory of Figure 2 has been magnified by a factor of about 10^5.

Table 1: Earth's moments of inertia. Uncertainty on last digit in parenthesis. (from http://hpiers.obspm.fr/eop-pc/models/constants.html#chenshen).

Constant	Symbol	Value	Unit
First equatorial moment of inertia	I_{xx}	8.0101 (2)	10^{37} kg m^2
Second equatorial moment of inertia	I_{yy}	8.0103 (2)	10^{37} kg m^2
Mean equatorial moment of inertia	$I_{mean}=(I_{xx}+I_{yy})/2$	8.010171 (84)	10^{37} kg m^2
Axial moment of inertia	I_{zz}	8.0365 (2)	10^{37} kg m^2
Mean angular velocity of the Earth	Ω	7.2921150 (1)	10^{-5} rad/s
First equatorial moment of inertia	I_{xx}	8.0101 (2)	10^{37} kg m^2

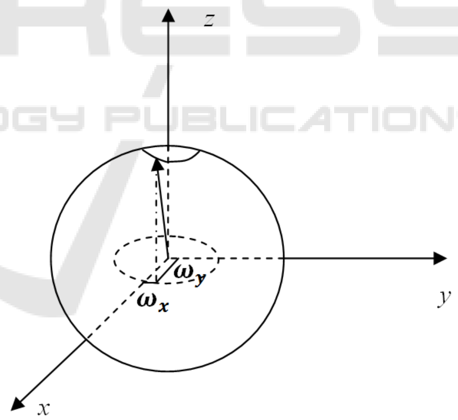

Figure 3: Trajectory (out of scale) of $\boldsymbol{\omega}$ vector in the body fixed reference frame.

4 EQUATIONS OF RIGID BODY MOTION

What is causing the wobble reported in Figure 2 and Figure 3? At the beginning of the paper we listed four possible causes of the movement of the rotation axis of the Earth. It is reasonable to expect all of them contributing to this unexpected movement. Referring to point 2 for instance one can refer to (Creveling J.R.

et al., 2012). But what is the major contributor for the Earth axis rotation? We will see, in the relatively straightforward derivations below, that this wobble is mainly due to the most basic law of mechanics: $\boldsymbol{F}=m\boldsymbol{a}$ (with $\boldsymbol{F}$=0) applied as a total moment to a rigid body. Euler calculated this effect, with a period of about 305 days, back in 1765. The actual experimental value was observed by Chandler as being of about 439 days. The discrepancy was later explained by Newcomb by the fact that the Earth is not rigid (point 2 at beginning of paper). But there are also other effects contributing to this motion as will be mentioned at the end of the paper.

Euler second law in an inertial reference frame states that the rate of change of angular momentum equates the applied torque $\boldsymbol{T}$:

$$\left(\frac{d\boldsymbol{L}}{dt}\right)_{inertial} = \boldsymbol{T} \tag{3}$$

But as mentioned earlier it is convenient to use a body fixed reference frame. So how the previous law will change? To make the graphical representation more understandable let us consider $t=0$ so that angular momentum conservation applies and $\boldsymbol{L}$ is therefore constant. in the body fixed reference frame an observer would see the vector $\boldsymbol{L}$ changing direction (Figure 4). It is easy to see that the rate of change of $\boldsymbol{L}$ is given by $\Omega\ L\ \sin\theta$, or using the vector product notation, $-\boldsymbol{\Omega}\times\boldsymbol{L}$ where $\boldsymbol{\Omega}$ is the angular velocity vector of the body fixed reference frame with respect to an inertial reference frame and ω is the angular velocity magnitude. What described is a transformation valid for calculating the rate of change of any vector in two rotating reference frames. So the rate of change of a vector, and in particular of $\boldsymbol{L}$, as seen from a body fixed reference frame is:

$$\left(\frac{d\boldsymbol{L}}{dt}\right)_{body\ fixed} = \left(\frac{d\boldsymbol{L}}{dt}\right)_{inertial} - \boldsymbol{\Omega}\times\boldsymbol{L} \tag{4}$$

Note: it is useful to observe something that is obvious but that sometimes is confusing. Vector $\boldsymbol{\Omega}$ is measured in the inertial reference frame. Instead the angular velocity as measured from the body fixed reference frame, i.e. the one of the inertial frame with respect to the body frame, is $\boldsymbol{\Omega}_{\mathbf{from\_body}}= -\boldsymbol{\Omega}$. It is also useful to note that those two angular velocities do exist in their respective reference frames. Although the angular velocity $\boldsymbol{\Omega}$ of the relative rotation of the two reference frames equals the angular velocity of the body $\boldsymbol{\omega}$, the two quantities are conceptually different. In fact $\boldsymbol{\omega}$ would be zero as measured from the body fixed reference frame while in this frame the angular velocity of the inertial frame with respect to the body frame $\boldsymbol{\Omega}_{\mathbf{from\_body}}$ is equal to $-\boldsymbol{\Omega}$.

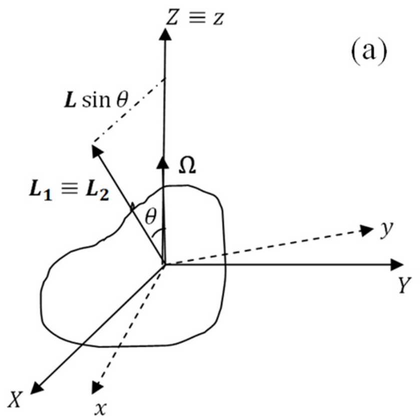

Figure 4a: Vector $\boldsymbol{L}$ as seen from the inertial reference frame (solid line). The body fixed reference frame rotates with angular velocity $\boldsymbol{\Omega}$. At time t_2 it is in the position shown by the dashed lines. Subscripts correspond to the time t_1 and t_2.

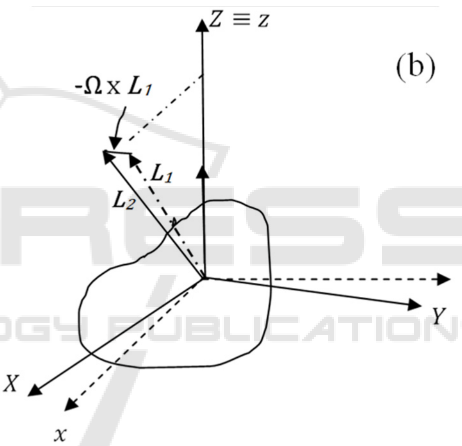

Figure 4b: Vector $\boldsymbol{L}$ as seen from the body fixed reference frame (dashed line). An observer on the moving frame would see the inertial frame rotate in opposite direction so that at t_2 it will be as shown by the solid lines. The rate of change of $\boldsymbol{L}$ is due to the rotation of the axes and is $-\boldsymbol{\Omega}\times\boldsymbol{L}_1$ or to $\boldsymbol{\Omega}_{from\_body}\times\boldsymbol{L}_1$. Subscripts correspond to the time t_1 and t_2.

Rewriting Equation 4 in the general case of torque $\boldsymbol{T}$ different from zero we have:

$$\left(\frac{d\boldsymbol{L}}{dt}\right)_{inertial} = \left(\frac{d\boldsymbol{L}}{dt}\right)_{body\ fixed} + \boldsymbol{\Omega}\times\boldsymbol{L} = \boldsymbol{T} \tag{5}$$

Recalling that the inertia matrix $\boldsymbol{I}$, in the body fixed reference frame, does not change, the definition of $\boldsymbol{L}$ from Equation 1 and that $\boldsymbol{\Omega}=\boldsymbol{\omega}$ (remembering the note reported above) we obtain the Euler equation for rigid body motion in vector form:

$$\boldsymbol{I}\cdot\dot{\boldsymbol{\omega}} + \boldsymbol{\omega}\times(\boldsymbol{I}\cdot\boldsymbol{\omega}) = 0 \tag{6}$$

Having chosen the body axis aligned with the inertia principal axis, the above equation becomes:

$$\begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \begin{Bmatrix} \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \end{Bmatrix} + \begin{Bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{Bmatrix} \times \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \begin{Bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{Bmatrix} = \begin{Bmatrix} T_x \\ T_y \\ T_z \end{Bmatrix} \quad (7)$$

By performing the simple calculations one obtains the Euler equations:

$$\begin{aligned} I_{xx}\dot{\omega}_x - \omega_y\omega_z(I_{yy} - I_{zz}) &= T_x \\ I_{yy}\dot{\omega}_y - \omega_z\omega_x(I_{zz} - I_{xx}) &= T_y \\ I_{zz}\dot{\omega}_z - \omega_x\omega_y(I_{xx} - I_{yy}) &= T_z \end{aligned} \quad (8)$$

Now in the case of the Earth, if we neglect the external actions and the very small differences between I_{xx} and I_{yy}, the above equations simplifies to:

$$\begin{aligned} I_{xx}\dot{\omega}_x &= \omega_y\omega_z(I_{yy} - I_{zz}) \\ I_{yy}\dot{\omega}_y &= -\omega_z\omega_x(I_{xx} - I_{zz}) \\ I_{zz}\dot{\omega}_z &= 0 \end{aligned} \quad (9)$$

with the third equation providing $\omega_z = constant$.

Furthermore by posing:

$$\Omega_C = \frac{I_{zz} - I_{xx}}{I_{xx}}\omega_z \quad (10)$$

and substituting the values reported in Table 1 one obtains:

$$\begin{aligned} \Omega_C &= \frac{8.0365 - 8.0102}{8.0102} 7.2921 \cdot 10^{-5} \frac{rad}{s} \\ &= 2.3942 \cdot 10^{-7} \frac{rad}{s} \\ &= 3.294 \cdot 10^{-3} \frac{rev}{day} \end{aligned} \quad (11)$$

This angular frequency corresponds to a period of 303.6 days and the above equations reduce to:

$$\begin{aligned} \dot{\omega}_x &= -\Omega_C\omega_y \\ \dot{\omega}_y &= \Omega_C\omega_x \end{aligned} \quad (12)$$

The solution to this system of linear differential equations can be easily verified to be:

$$\begin{aligned} \omega_x &= A\cos\Omega_C t \\ \omega_y &= A\sin\Omega_C t \end{aligned} \quad (13)$$

In the *x-y* plane of the body reference frame the projection of the vector $\boldsymbol{\omega}$ will trace in a period of 303.6 days a circle similar to that depicted with dashed line in Figure 3.

So we have shown that a rotating body with no external torques does not have, in general, a rotation axis fixed in inertial space and with respect to the body fixed reference frames. This last motion is referred to as polar motion. Incidentally we observe that it is still not clear what maintains the wobble against the viscoelastic damping of the interior of the Earth (Jenkins, 2015).

5 OTHER EFFECTS AND ENVIRONMENTAL MONITORING.

In this section we will briefly assess the effects of points 2 and 3 mentioned at the beginning of the paper.

We have mentioned that the polar motion is a combination of several effects. The main one has been described in the previous section and is due to the law of mechanics applied to a torque free rigid body. If we add the non-rigid Earth component, the resulting wobble period would change from 303.6 days to about 439 days which is approximately the value measured by Chandler. A more accurate inspection of the motion over a longer period of time will reveal that the radii of the circles (one is shown in Figure 2), varies from about 3 meters to about 15 meters over a 6.5 year period. This variation cannot be simply explained with the torque free motion and the non-rigidity of the Earth because it is due to some external seasonal forcing action that has a period of one year.

But besides the wobbles just described there are other components on the motion of the rotation axis of the Earth. The effects of the gravitational torques mainly of the Moon and the Sun, on the equatorial bulge of the Earth causes the rotational axis of the Earth to precess with a period of 25700 years. This phenomenon is analogous to the precession of a spinning top. Furthermore since the positions of the Moon and the Sun change with time this effect produces a nutation i.e. an oscillation of the Earth spin axis with main period of 18.6 years (lunisolar precession). Also the gravitational perturbations of the planets induce a change of inclination of the Earth axis in the range 22.2 – 24.3 degrees with a mean period of 41.000 years (Berger A.L., 1976).

Possible indications on climate change can be inferred by additional Earth rotation axis shift (Roy and Peltier, 2011; Pavlis et al., 2015b). In fact mass

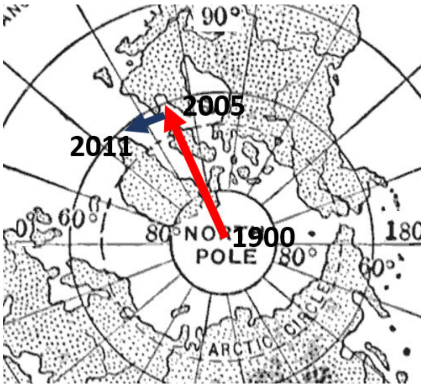

Figure 5: The long arrow (from year 1900 to 2005) corresponds to 12.6m, the short arrow (from 2005 to 2011) is of 1.6 m.

redistribution inside or on the surface of the planet will affect the Length of the Day (LOD) and the rotation axis direction (in one sentence, the angular velocity vector) similarly to what would happen to the skater mentioned above when he/she moves the arms. The mass redistribution on the surface of the planet concerns the atmosphere, the glaciers (Cazenave, A., and Chen J.L. 2010), the oceans, etc… We just would like to mention another small motion of the rotation axis of the Earth that is a secular drift towards East which is partly due to post-glacial rebound i.e. the slow ground rise due to the recovery of the original position of the ground after the melting, and consequently the release of weight, of the enormous quantity of glaciers produced during the ice age. In 2005 it was observed a sudden change of the direction of this secular drift that is attributed to a rapid melting of Greenland glaciers and polar caps (Figure 5) (Chen et al., 2013). Also the El Niño, due to thermal expansion of the Pacific ocean (about 20 cm sea level rise for an extension of thousands of kilometers) produces variation on the Earth Orientation Parameters (EOP) i.e. LOD and rotation axis direction. The variations are very small but the accuracy reached with the laser ranging technique on LARES and other geodetic satellites and other methods such as GNSS and Very Long Baseline Interferometry (VLBI) allow to monitor the axis position with an error of the order of 0.03 milliarcsecond i.e. about 1 mm on the Earth surface.

6 CONCLUSIONS

Taking the planet Earth as an example for a rotating body, the paper describes first the case in which the Earth is considered infinitely rigid. In this limit case the rigid body Euler equations of motion predict a counterintuitive oscillation of the Earth rotation axis, that is the main contributor to the so called Chandler wobble. This oscillation is a remarkable case because it is not due to external torques but simply to the laws of mechanics for a freely rotating body. The discrepancy of the period of the wobble obtained experimentally by Chandler with what obtained by the Euler equations is explained with the non-rigid Earth. Other motions of the axis are due to external gravitational actions of the planets and particularly of the Moon and the Sun. Finally the correlation between the variation on the angular velocity vector or if you like on the Earth Orientation Parameters (EOP) and global climate changes is outlined. In particular it has been observed that rapid Greenland and polar ice melting may be the cause of the sudden change in the polar motion secular drift. Also the Pacific Ocean thermal expansion of El Niño frequently leaves a signature on the EOP. Besides being important for research in the field of global climate change monitoring, Earth rotation studies have recently gained considerable interest with the public, mainly thanks to the proliferation of the web and the increased outreach efforts of all scientists. The combination of two fields, climatology and mechanics, that appear to be so far apart seems to attract very much the interest probably due to the increasing concern on environmental issues. We plan to apply this approach of combining mechanics, Earth rotation and global environmental monitoring in an educational and public outreach context to verify its validity.

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