Preventive Replacement Policies with Aging Failure and Third-part Damage

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Abstract: In general, corrosions, degrading the mechanical strength of pipelines gradually with its age in a stochastic way, is the predominant cause of pipeline leaks. In addition, the third-part damage is the leading cause of pipeline ruptures, which occurs randomly in a statistical sense. Naturally, *corrective replacement* (CR) is done immediately when the pipeline is subjected to a random failure. To reduce the failure probabilities, *preventive replacement* (PR) policies are scheduled to meet the failure due to aging and the third-part damage. We model three PR policies, using the renewal theory in reliability, and obtain their optimal solutions analytically to minimize respective replacement cost rates. Finally, numerical examples are given to compare these policies.

1 INTRODUCTION

The pipeline systems have been increasingly constructed to meet the rapid development of the oil and gas industry. Reliability assessment (Amirat, 2006), failure probability estimation (Xie, et al., 2008), and inspection of damage suffered for shock and corrosion (Sahraoui, et al., 2013) have been applied commonly for preventive replacement actions in pipelines to prevent incidents.

In general, corrosions, degrading the mechanical strength of pipelines gradually with its age in a stochastic way, is the predominant cause of pipeline leaks. In addition, the third-part damage is the leading cause of pipeline ruptures, which occurs randomly in a statistical sense (Pluvinage and Elwany, 2008).

Naturally, *corrective replacement* (CR) (Barlow and Proschan, 1965) should be done immediately when any operating item is subjected to a random failure resulting the loss of productivity. For the pipeline segment, the failure means leaks or ruptures referred above. To reduce the failure probabilities, *preventive replacement* (PR) policies are conducted in a common way such that replacement actions are done preventively at some thresholds or planned measurements such as operating time, usage number, damage level, repair cost, number of faults or repairs, etc. (Osaki, 2002).

When cracks or corrosions are located for the pipeline segment, condition-based maintenances or repairs with minimum reliability requirement are usually conducted to save replacement cost, e.g., structural pipeline repairs using carbon composites (Goertzen and Kessler, 2007), a carbon composite overwrap pipeline repair system (Duell, 2008), and carbon-fiber reinforced composites for pipeline repair (Alexandera and Ochoa, 2010). However, we propose that the pipeline segment would still go into a new degradation level after any repair that is randomly taken place even though it is conducted perfectly. That is, the pipeline segment has aging cycles degrading after repairs, and preventive replacement should be done when it undergoes several cycles of aging for the final renewal.

In this paper, we propose for a pipeline segment that PR are scheduled (i) at a planned time T $(0 < T \leq \infty)$ of operation, which is a classical policy called age replacement, (ii) at the Nth $(N = 1, 2, \cdots)$ cycle of aging, where intervals for aging cycles are random variables Y_n $(n = 1, 2, \cdots)$, (iii) at a number K $(K = 1, 2, \cdots)$ of damages, in order to monitor the fatal third-part damages.

Policies (i) and (ii) are taken to meet the failure due to aging from both determinate and indeterminate viewpoints, meanwhile, policy (iii) is planned to monitor the failure caused by the third-part damage. That is, the pipeline should be replaced preventively before failure at T, N, or K, whichever takes place first. We model the above PR policies, using the renewal theory in reliability (Osaki, 1992), and obtain their optimal solutions analytically to minimize the expected replacement cost rates. Finally, numerical examples

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are given to compare these policies.

In mathematics, we give the following notations to describe the above two failure modes: An item, which is used to denote any product functioning for missions, e.g., pipeline segment mentioned above, fails at a time X due to aging, where X is a random variable and has a cumulative distribution function $F(t) \equiv \Pr\{X \leq t\}$ with density function $f(t)$. Let $r(t) \equiv f(t)/\overline{F}(t)$ denote the instant failure rate of $F(t)$, where $\overline{\Phi}(t) \equiv 1-\Phi(t)$ for any function $\Phi(t)$. The failure rate $r(t)$ increases strictly with t to $r(\infty) \equiv \lim_{t\to\infty} r(t)$.

The third-part damage occurs at a non-homogeneous Poisson process with mean-value function $H(t) \equiv \int_0^t h(u)\mathrm{d}u$, where we denote $p_j(t) \equiv [H(t)^j/j!]\mathrm{e}^{-H(t)}$ $(j = 0,1,2,\cdots)$ be the probability that an exact number j of damages occur in $[0,t]$. The item either fails with probability p or continues to function with probability $q = 1-p$ at each damage. Then, the damage causing the item to be failure occurs in $[0,t]$ with probability

$$F_p(t) = \sum_{j=1}^{\infty} pq^{j-1} \sum_{i=j}^{\infty} \frac{[H(t)]^i}{i!}\mathrm{e}^{-H(t)}$$
$$= 1-\mathrm{e}^{-pH(t)}.$$

In other words, the item fails at age X or at damage p, whichever takes place first. Then, the probability that the unit fails at age X is

$$\int_0^{\infty} \overline{F}_p(t)\mathrm{d}F(t), \tag{1}$$

and the probability that it fails at damage p is

$$\int_0^{\infty} \overline{F}(t)\mathrm{d}F_p(t), \tag{2}$$

where note that (1)+ (2)=1. Thus, the mean time to item failure is

$$\mathrm{MTTF} = \int_0^{\infty} t\overline{F}_p(t)\mathrm{d}F(t) + \int_0^{\infty} t\overline{F}(t)\mathrm{d}F_p(t)$$
$$= \int_0^{\infty} \overline{F}(t)\overline{F}_p(t)\mathrm{d}t. \tag{3}$$

2 REPLACEMENT MODELS

Naturally, *corrective replacement* (CR) is done immediately when the item is subjected to a random failure resulting the loss of productivity. To reduce the failure probabilities for aging X and damage p, the following *preventive replacement* (PR) policies are scheduled:

(a) Replacement done at a planned time T $(0 < T \leq \infty)$ of operation, which is a classical policy called *age replacement* (Barlow and Proschan, 1965) and has been commonly conducted in real applications;

(b) Replacement done at the Nth $(N = 1,2,\cdots)$ cycle of aging, where intervals for aging cycles are random variables Y_n having an identical distribution $G(t) \equiv \Pr\{Y_n \leq t\}$ with density function $g(t)$. Noting that when $N = 1$, the policy degrades to a *random replacement* that was newly proposed for the uncertain need of replacement performance. Denote $G^{(n)}(t)$ $(n = 0,1,2,\cdots)$ be the n-fold Stieltjes convolution of $G(t)$, and $G^{(n)}(t) - G^{(n+1)}(t)$ means the item has entered the $(n+1)$th aging cycle.

(c) Replacement done at a number K $(K = 1,2,\cdots)$ of damages in order to monitor the fatal third-part damages.

Policies (a) and (b) are taken to meet the failure due to aging X from both determinate and indeterminate viewpoints, meanwhile, policy (c) is planned to monitor the failure caused by the third-part damage. That is, the item should be replaced preventively before X or p at T, N, or K, whichever takes place first. In addition, the item is supposed to be replaced at failure p when the Kth damage makes the item fail with probability p.

Let c_p denote the cost for preventive replacement policies at T, N and K, and c_f $(c_f > c_p)$ denote the cost for corrective replacement at failures X and p. Using the theory of renewal process (Osaki, 1992), the expected replacement cost rate can be given by the expected cost per replacement cycle divided by the expected duration of its cycle.

Then, the probability that the item is replaced at time T is

$$\overline{F}(T)[1-G^{(N)}(T)] \sum_{j=0}^{K-1} q^j p_j(T), \tag{4}$$

the probability that it is replaced at aging cycle N is

$$\sum_{j=0}^{K-1} q^j \int_0^T \overline{F}(t)p_j(t)\mathrm{d}G^{(N)}(t), \tag{5}$$

the probability that it is replaced at damage number K is

$$q^K \int_0^T \overline{F}(t)[1-G^{(N)}(t)]p_{K-1}(t)h(t)\mathrm{d}t, \tag{6}$$

the probability that it is replaced at failure for age X is

$$\sum_{j=0}^{K-1} q^j \int_0^T [1-G^{(N)}(t)]p_j(t)\mathrm{d}F(t), \tag{7}$$

and the probability that it is replaced at failure for damage p is

$$\sum_{j=0}^{K-1} pq^j \int_0^T \overline{F}(t)[1-G^{(N)}(t)]p_j(t)h(t)\mathrm{d}t, \tag{8}$$

where note that (4)+(5)+(6)+(7)+(8)=1. Thus, the mean time to replacement is

$$T\overline{F}(T)[1-G^{(N)}(T)]\sum_{j=0}^{K-1}q^j p_j(T)$$
$$+\sum_{j=0}^{K-1}q^j\int_0^T t\overline{F}(t)p_j(t)\mathrm{d}G^{(N)}(t)$$
$$+q^K\int_0^T t\overline{F}(t)[1-G^{(N)}(t)]p_{K-1}(t)h(t)\mathrm{d}t$$
$$+\sum_{j=0}^{K-1}q^j\int_0^T t[1-G^{(N)}(t)]p_j(t)\mathrm{d}F(t)$$
$$+\sum_{j=0}^{K-1}pq^j\int_0^T t\overline{F}(t)[1-G^{(N)}(t)]p_j(t)h(t)\mathrm{d}t$$
$$=\sum_{j=0}^{K-1}q^j\int_0^T \overline{F}(t)[1-G^{(N)}(t)]p_j(t)\mathrm{d}t. \tag{9}$$

Therefore, the expected cost rate is

$$C(T,N,K)=\frac{c_p+(c_f-c_p)\{\sum_{j=0}^{K-1}q^j\int_0^T[1-G^{(N)}(t)]p_j(t)\mathrm{d}F(t)+\sum_{j=0}^{K-1}pq^j\int_0^T\overline{F}(t)[1-G^{(N)}(t)]p_j(t)h(t)\mathrm{d}t\}}{\sum_{j=0}^{K-1}q^j\int_0^T\overline{F}(t)[1-G^{(N)}(t)]p_j(t)\mathrm{d}t}. \tag{10}$$

In particular, when the item is replaced only at failure, i.e., when $T\to\infty$, $N\to\infty$ and $K\to\infty$,

$$C_1\equiv\lim_{T,N,K\to\infty}C(T,N,K)=\frac{c_f}{\int_0^\infty\overline{F}(t)\overline{F}_p(t)\mathrm{d}t}, \tag{11}$$

and when $T\to 0$ for any N and K,

$$C_2\equiv\lim_{T\to 0}C(T,N,K)=\infty. \tag{12}$$

3 OPTIMAL T^*, N^* AND K^*

We have to avoid the high failure cost without any PR in (11). Meanwhile, it is unreasonable to make frequent replacements to waste PR cost, such as an extreme case in (12). In order to minimize the respective cost rates of the above three replacement policies, we next obtain analytically optimum time T^*, aging cycle N^*, and damage number K^*, respectively.

3.1 Optimal T^*

Suppose that the item should be replaced preventively before failure X or p at time T $(0<T\le\infty)$. Then, putting that $N\to\infty$ and $K\to\infty$ in (10), the expected cost rate is

$$C(T)=\frac{c_f-(c_f-c_p)\overline{F}(T)\overline{F}_p(T)}{\int_0^T\overline{F}(t)\overline{F}_p(t)\mathrm{d}t}. \tag{13}$$

Differentiating $C(T)$ with respect to T and setting it equal to zero,

$$[r(T)+ph(T)]\int_0^T\overline{F}(t)\overline{F}_p(t)\mathrm{d}t-[1-\overline{F}(T)\overline{F}_p(T)]=\frac{c_p}{c_f-c_p}. \tag{14}$$

Let $L(T)$ denote the left-hand side of (14),

$$\frac{\mathrm{d}L(T)}{\mathrm{d}T}=\left[\frac{\mathrm{d}r(T)}{\mathrm{d}T}+p\frac{\mathrm{d}h(T)}{\mathrm{d}T}\right]\int_0^T\overline{F}(t)\overline{F}_p(t)\mathrm{d}t.$$

Thus, if $r(T)+ph(T)$ increases strictly with T, then $L(T)$ increases strictly with T from 0 to

$$\lim_{T\to\infty}L(T)=\int_0^\infty\overline{F}(t)\overline{F}_p(t)[r(\infty)+ph(\infty)-r(t)-ph(t)]\mathrm{d}t.$$

Therefore, when $\lim_{T\to\infty}L(T)>c_p/(c_f-c_p)$, there exists a finite and unique T^* $(0<T^*<\infty)$ which satisfies (14), and the resulting cost rate is

$$C(T^*)=(c_f-c_p)[r(T^*)+ph(T^*)], \tag{15}$$

and when $\lim_{T\to\infty}L(T)\le c_p/(c_f-c_p)$, $T^*=\infty$, and the resulting cost rate is given in (11).

When $F(t)=1-\mathrm{e}^{-\alpha t^\beta}$ $(\beta>1)$, $p_j(t)\equiv[(\omega t)^j/j!]\mathrm{e}^{-\omega t}$, and $G^{(N)}(t)=\sum_{j=N}^\infty[(t^\delta)^j/j!]\mathrm{e}^{-t^\delta}$, Table 1 presents optimal T^* and its cost rates $C(T^*)/c_p$ when $\alpha=2.0$, $\beta=1.2$, $\omega=1.0$ and $\delta=0.5$.

Table 1: Optimal T^* and its cost rates for p and $c_p/(c_f-c_p)$.

$\frac{c_p}{c_f-c_p}$	$p=0.01$ T^*	$p=0.01$ $C(T^*)/c_p$	$p=0.1$ T^*	$p=0.1$ $C(T^*)/c_p$
0.01	0.166	37.491	0.167	46.516
0.03	0.424	15.000	0.428	18.025
0.05	0.663	9.821	0.672	11.647
0.07	0.896	7.442	0.913	8.755
0.10	1.245	5.558	1.279	6.487
0.30	4.007	2.332	4.374	2.672
0.50	8.430	1.620	9.910	1.853
0.70	15.883	1.312	19.886	1.500
1.00	36.091	1.080	48.465	1.235

3.2 Optimal N^*

Suppose that the item should be replaced preventively before failure X or p at aging cycle N $(N = 1, 2, \cdots)$. Then, putting that $T \to \infty$ and $K \to \infty$ in (10), the expected cost rate is

$$C(N) = \frac{c_p + (c_f - c_p)\sum_{j=0}^{N-1}\{\int_0^\infty [G^{(j)}(t) - G^{(j+1)}(t)] \times \overline{F}_p(t)\mathrm{d}F(t) + \int_0^\infty [G^{(j)}(t) - G^{(j+1)}(t)]\overline{F}(t)\mathrm{d}F_p(t)\}}{\sum_{j=0}^{N-1}\int_0^\infty [G^{(j)}(t) - G^{(j+1)}(t)]\overline{F}(t)\overline{F}_p(t)\mathrm{d}t}. \quad (16)$$

Forming the inequality $C(N+1) - C(N) \geq 0$,

$$[Q_1(N) + pQ_2(N)]\sum_{j=0}^{N-1}\int_0^\infty [G^{(j)}(t) - G^{(j+1)}(t)]\overline{F}(t)\overline{F}_p(t)\mathrm{d}t - \sum_{j=0}^{N-1}\left\{\int_0^\infty [G^{(j)}(t) - G^{(j+1)}(t)]\overline{F}_p(t)\mathrm{d}F(t) + \int_0^\infty [G^{(j)}(t) - G^{(j+1)}(t)]\overline{F}(t)\mathrm{d}F_p(t)\right\} \geq \frac{c_p}{c_f - c_p}, \quad (17)$$

where

$$Q_1(N) \equiv \frac{\int_0^\infty [G^{(N)}(t) - G^{(N+1)}(t)]\overline{F}_p(t)\mathrm{d}F(t)}{\int_0^\infty [G^{(N)}(t) - G^{(N+1)}(t)]\overline{F}(t)\overline{F}_p(t)\mathrm{d}t},$$

$$Q_2(N) \equiv \frac{\int_0^\infty [G^{(N)}(t) - G^{(N+1)}(t)]\overline{F}(t)\overline{F}_p(t)h(t)\mathrm{d}t}{\int_0^\infty [G^{(N)}(t) - G^{(N+1)}(t)]\overline{F}(t)\overline{F}_p(t)\mathrm{d}t}.$$

Let $L(N)$ denote the left-hand side of (17),

$$L(N+1) - L(N) = [Q_1(N+1) + pQ_2(N+1) - Q_1(N) - pQ_2(N)] \times \sum_{j=0}^{N}\int_0^\infty [G^{(j)}(t) - G^{(j+1)}(t)]\overline{F}(t)\overline{F}_p(t)\mathrm{d}t.$$

Thus, if $Q_1(N) + pQ_2(N)$ increases strictly with N, then $L(N)$ increases strictly with N to

$$\lim_{N\to\infty} L(N) = \int_0^\infty \overline{F}(t)\overline{F}_p(t)[Q_1(\infty) + pQ_2(\infty) - r(t) - ph(t)]\mathrm{d}t.$$

Therefore, when $\lim_{N\to\infty} L(N) > c_p/(c_f - c_p)$, there exists a finite and unique minimum N^* $(1 \leq N^* < \infty)$ which satisfies (17), and the resulting cost rate is

$$(c_f - c_p)[Q_1(N^*-1) + pQ_2(N^*-1)] < C(N^*) \leq (c_f - c_p)[Q_1(N^*) + pQ_2(N^*)]. \quad (18)$$

In particular, when $L(1) \geq c_p/(c_f - c_p)$, $N^* = 1$.

When $F(t) = 1 - \mathrm{e}^{-\alpha t^\beta}$ $(\beta > 1)$, $p_j(t) \equiv [(\omega t)^j/j!]\mathrm{e}^{-\omega t}$, and $G^{(N)}(t) = \sum_{j=N}^\infty [(t^\delta)^j/j!]\mathrm{e}^{-t^\delta}$, Table 2 presents optimal N^* and its cost rates $C(N^*)/c_p$ when $\alpha = 2.0$, $\beta = 1.2$, $\omega = 1.0$ and $\delta = 0.5$.

Table 2: Optimal N^* and its cost rates for p and $c_p/(c_f - c_p)$.

$\frac{c_p}{c_f - c_p}$	$p = 0.01$		$p = 0.1$	
	N^*	$C(N^*)/c_p$	N^*	$C(N^*)/c_p$
0.01	1	47.309	1	55.374
0.03	1	16.725	1	19.495
0.05	1	10.609	1	12.320
0.07	2	7.912	2	9.123
0.10	2	5.772	2	6.646
0.30	7	2.314	8	2.646
0.50	15	1.605	20	1.835
0.70	31	1.300	43	1.487
1.00	80	1.072	120	1.226

3.3 Optimal K^*

Suppose that the item should be replaced preventively before failure X or p at damage number K $(K = 1, 2, \cdots)$. Then, putting that $T \to \infty$ and $N \to \infty$ in (10), the expected cost rate is

$$C(K) = \frac{c_f - (c_f - c_p)q^K\int_0^\infty \overline{F}(t)p_{K-1}(t)h(t)\mathrm{d}t}{\sum_{j=0}^{K-1} q^j \int_0^\infty \overline{F}(t)p_j(t)\mathrm{d}t}. \quad (19)$$

Forming the inequality $C(K+1) - C(K) \geq 0$,

$$[Q_3(K) + pQ_4(K)]\sum_{j=0}^{K-1} q^j \int_0^\infty \overline{F}(t)p_j(t)\mathrm{d}t - \sum_{j=0}^{K-1} q^j\left[\int_0^\infty p_j(t)\mathrm{d}F(t) + p\int_0^\infty \overline{F}(t)p_j(t)h(t)\mathrm{d}t\right] \geq \frac{c_p}{c_f - c_p}, \quad (20)$$

where

$$Q_3(K) \equiv \frac{\int_0^\infty p_K(t)\mathrm{d}F(t)}{\int_0^\infty \overline{F}(t)p_K(t)\mathrm{d}t},$$

$$Q_4(K) \equiv \frac{\int_0^\infty \overline{F}(t)p_K(t)h(t)\mathrm{d}t}{\int_0^\infty \overline{F}(t)p_K(t)\mathrm{d}t}.$$

Let $L(K)$ denote the left-hand side of (20),

$$L(K+1) - L(K) = \left[Q_3(K+1) + pQ_4(K+1) - Q_3(K) - pQ_4(K)\right] \times \sum_{j=0}^{K} q^j \int_0^\infty \overline{F}(t)p_j(t)\mathrm{d}t.$$

Thus, if $Q_3(K) + pQ_4(K)$ increases strictly with K, then $L(K)$ increases strictly with K to

$$\lim_{K\to\infty} L(K) = \int_0^\infty \overline{F}(t)\overline{F}_p(t)[Q_3(\infty) + pQ_4(\infty) - r(t) - ph(t)]\mathrm{d}t.$$

Therefore, when $\lim_{K\to\infty} L(K) > c_p/(c_f - c_p)$, there exists a finite and unique minimum K^* ($1 \le K^* < \infty$) which satisfies (20), and the resulting cost rate is

$$(c_f - c_p)[Q_3(K^*-1) + pQ_4(K^*-1)] < C(K^*) \le (c_f - c_p)[Q_3(K^*) + pQ_4(K^*)]. \quad (21)$$

In particular, when $L(1) \ge c_p/(c_f - c_p)$, $K^* = 1$.

When $F(t) = 1 - \mathrm{e}^{-\alpha t^\beta}$ ($\beta > 1$), $p_j(t) \equiv [(\omega t)^j/j!]\mathrm{e}^{-\omega t}$, and $G^{(N)}(t) = \sum_{j=N}^{\infty}[(t^\delta)^j/j!]\mathrm{e}^{-t^\delta}$, Table 3 presents optimal K^* and its cost rates $C(K^*)/c_p$ when $\alpha = 2.0$, $\beta = 1.2$, $\omega = 1.0$ and $\delta = 0.5$.

Table 3: Optimal K^* and its cost rates for p and $c_p/(c_f - c_p)$.

$\frac{c_p}{c_f - c_p}$	$p = 0.01$		$p = 0.1$	
	K^*	$C(K^*)/c_p$	K^*	$C(K^*)/c_p$
0.01	1	44.545	1	53.545
0.03	1	15.808	1	18.808
0.05	1	10.060	1	11.860
0.07	1	7.597	1	8.883
0.10	2	5.602	2	6.511
0.30	6	2.311	7	2.645
0.50	14	1.605	17	1.835
0.70	29	1.300	37	1.487
1.00	72	1.072	99	1.226

Note that $Q_1(N)$ and $Q_3(K)$ respectively increases with N and K to $r(\infty)$, and $Q_3(N)$ and $Q_4(K)$ respectively increases with N and K to $h(\infty)$. Thus, it is easy to conclude for the above three replacement policies that if

$$\int_0^\infty \overline{F}(t)\overline{F}_p(t)[r(\infty) + ph(\infty) - r(t) - ph(t)]\mathrm{d}t > \frac{c_p}{c_f - c_p},$$

then all of finite and unique T^*, N^* and K^* can be found.

4 OPTIMAL (T^*, N^*), (T^*, K^*) AND (N^*, K^*)

In order to make the functioning item more reliable in real situations, maintenances with double PR scenarios, such as replacement policies scheduled for the parts of an aircraft at a total hours of operation and at a specified number of flights since the last major overhaul, are commonly conducted. In this section, we obtain numerically (T^*, N) for given N, (T, N^*) for given T, (T^*, K) for given K, (T, K^*) for given T, (N^*, K) for given K and (N, K^*) for given N to minimize their respective expected cost rates.

4.1 Optimal (T^*, N^*)

Suppose that the item should be replaced preventively before failure X or p at time T ($0 < T \le \infty$) or at aging cycle N ($N = 1, 2, \cdots$), whichever takes place first. Then, putting that $K \to \infty$ in (10), the expected cost rate is

$$C(T,N) = \frac{c_p + (c_f - c_p)\{\int_0^T [1 - G^{(N)}(t)]\overline{F}_p(t)\mathrm{d}F(t) + \int_0^T [1 - G^{(N)}(t)]\overline{F}(t)\mathrm{d}F_p(t)\}}{\int_0^T [1 - G^{(N)}(t)]\overline{F}(t)\overline{F}_p(t)\mathrm{d}t}. \quad (22)$$

When $F(t) = 1 - \mathrm{e}^{-\alpha t^\beta}$ ($\beta > 1$), $p_j(t) \equiv [(\omega t)^j/j!]\mathrm{e}^{-\omega t}$, and $G^{(N)}(t) = \sum_{j=N}^{\infty}[(t^\delta)^j/j!]\mathrm{e}^{-t^\delta}$, Table 4 and Table 5 present optimal T^* for N, N^* for T, and their cost rates $C(T^*, N)/c_p$ and $C(T, N^*)/c_p$ when $\alpha = 2.0$, $\beta = 1.2$, $\omega = 1.0$, $\delta = 0.5$ and $p = 0.1$.

Table 4: Optimal T^* and its cost rate for N and $c_p/(c_f - c_p)$.

$\frac{c_p}{c_f - c_p}$	$N = 1$		$N = 5$	
	T^*	$C(T^*,N)/c_p$	T^*	$C(T^*,N)/c_p$
0.01	0.198	47.789	0.167	46.511
0.03	0.572	18.906	0.428	18.024
0.05	0.986	12.416	0.672	11.647
0.07	1.457	9.474	0.912	8.755
0.10	2.311	7.176	1.279	6.487
0.30	18.222	3.445	4.391	2.674
0.50	73.222	2.665	10.009	1.856
0.70	191.840	2.278	20.158	1.503
1.00	∞	1.734	49.079	1.238

Table 5: Optimal N^* and its cost rate for T and $c_p/(c_f - c_p)$.

$\frac{c_p}{c_f - c_p}$	$T = 1.0$		$T = 15.0$	
	N^*	$C(T,N^*)/c_p$	N^*	$C(T,N^*)/c_p$
0.01	1	52.846	1	57.711
0.03	3	18.736	1	20.287
0.05	6	11.756	2	12.659
0.07	23	8.759	2	9.290
0.10	∞	6.511	3	6.743
0.30	∞	3.014	9	2.674
0.50	∞	2.315	22	1.851
0.70	∞	2.015	∞	1.499
1.00	∞	1.790	∞	1.234

4.2 Optimal (T^*, K^*)

Suppose that the item should be replaced preventively before failure X or p at time T $(0 < T \le \infty)$ or at damage number K $(K = 1, 2, \cdots)$, whichever takes place first. Then, putting that $N \to \infty$ in (10), the expected cost rate is

$$C(T,K) = \frac{c_p + (c_f - c_p)\sum_{j=0}^{K-1} q^j \left[\int_0^T p_j(t)\mathrm{d}F(t) + p\int_0^T \overline{F}(t)p_j(t)h(t)\mathrm{d}t\right]}{\sum_{j=0}^{K-1} q^j \int_0^T \overline{F}(t)p_j(t)\mathrm{d}t}. \quad (23)$$

When $F(t) = 1 - \mathrm{e}^{-\alpha t^\beta}$ $(\beta > 1)$, $p_j(t) \equiv [(\omega t)^j/j!]\mathrm{e}^{-\omega t}$, and $G^{(N)}(t) = \sum_{j=N}^{\infty}[(t^\delta)^j/j!]\mathrm{e}^{-t^\delta}$, Table 6 and Table 7 present optimal T^* for K, K^* for T, and their cost rates $C(T^*, K)/c_p$ and $C(T, K^*)/c_p$ when $\alpha = 2.0$, $\beta = 1.2$, $\omega = 1.0$, $\delta = 0.5$ and $p = 0.1$.

Table 6: Optimal T^* and its cost rate for K and $c_p/(c_f - c_p)$.

$\frac{c_p}{c_f - c_p}$	$K=1$		$K=5$	
	T^*	$C(T^*,K)/c_p$	T^*	$C(T^*,K)/c_p$
0.01	0.173	46.768	0.167	46.512
0.03	0.469	18.299	0.428	18.024
0.05	0.781	11.943	0.671	11.647
0.07	1.128	9.073	0.912	8.754
0.10	1.748	6.840	1.278	6.486
0.30	13.067	3.244	4.422	2.677
0.50	52.892	2.510	10.339	1.867
0.70	149.037	2.173	21.277	1.518
1.00	∞	1.902	52.995	1.256

Table 7: Optimal K^* and its cost rate for T and $c_p/(c_f - c_p)$.

$\frac{c_p}{c_f - c_p}$	$T=1.0$		$T=15.0$	
	K^*	$C(T,K^*)/c_p$	K^*	$C(T,K^*)/c_p$
0.01	1	52.470	1	55.486
0.03	2	18.695	1	19.457
0.05	3	11.755	1	12.251
0.07	12	8.511	2	9.084
0.10	∞	6.511	2	6.631
0.30	∞	3.014	7	2.673
0.50	∞	2.315	18	1.851
0.70	∞	2.015	∞	1.499
1.00	∞	1.790	∞	1.234

4.3 Optimal (N^*, K^*)

Suppose that the item should be replaced preventively before failure X or p at aging cycle N $(N = 1, 2, \cdots)$ and at damage number K $(K = 1, 2, \cdots)$, whichever takes place first. Then, putting that $T \to \infty$ in (10), the expected cost rate is

$$C(N,K) = \frac{c_p + (c_f - c_p)\sum_{j=0}^{K-1} q^j \left\{\int_0^\infty [1 - G^{(N)}(t)]p_j(t)\mathrm{d}F(t) + p\int_0^\infty \overline{F}(t)[1 - G^{(N)}(t)]p_j(t)h(t)\mathrm{d}t\right\}}{\sum_{j=0}^{K-1} q^j \int_0^\infty \overline{F}(t)[1 - G^{(N)}(t)]p_j(t)\mathrm{d}t}. \quad (24)$$

When $F(t) = 1 - \mathrm{e}^{-\alpha t^\beta}$ $(\beta > 1)$, $p_j(t) \equiv [(\omega t)^j/j!]\mathrm{e}^{-\omega t}$, and $G^{(N)}(t) = \sum_{j=N}^{\infty}[(t^\delta)^j/j!]\mathrm{e}^{-t^\delta}$, Table 8 and Table 9 present optimal K^* for N, N^* for K, and their cost rates $C(N, K^*)/c_p$ and $C(N^*, K)/c_p$ when $\alpha = 2.0$, $\beta = 1.2$, $\omega = 1.0$, $\delta = 0.5$ and $p = 0.1$.

Table 8: Optimal K^* and its cost rate for N and $c_p/(c_f - c_p)$.

$\frac{c_p}{c_f - c_p}$	$N=1$		$N=5$	
	K^*	$C(N,K^*)/c_p$	K^*	$C(N,K^*)/c_p$
0.01	1	49.245	1	53.503
0.03	1	18.224	1	18.797
0.05	2	12.015	1	11.855
0.07	2	9.133	1	8.881
0.10	4	6.918	2	6.508
0.30	32	3.350	7	2.647
0.50	∞	2.632	18	1.839
0.70	∞	2.325	38	1.492
1.00	∞	2.094	104	1.232

Table 9: Optimal N^* and its cost rate for K and $c_p/(c_f - c_p)$.

$\frac{c_p}{c_f - c_p}$	$K=1$		$K=5$	
	K^*	$C(N^*,K)/c_p$	K^*	$C(N^*,K)/c_p$
0.01	1	49.245	1	54.974
0.03	1	18.224	1	19.382
0.05	2	11.788	1	12.263
0.07	3	8.873	1	9.213
0.10	∞	6.650	1	6.925
0.30	∞	3.176	∞	2.648
0.50	∞	2.481	∞	1.849
0.70	∞	2.184	∞	1.506
1.00	∞	1.960	∞	1.250

5 CONCLUSIONS

We have considered two failures of aging and the third-part damage in pipeline segments, where aging failure time is a random variable with cumulative distribution $F(t)$, and the third-part damage occurs at a non-homogeneous Poisson process and cause pipeline failure with probability p. The proposed three preventive replacement models, i.e., policies done at time T

and at the Nth aging cycle are planned to meet failures due to aging, and the policy done at number K of third-part damage is planned to monitor the fatal damage with p, have been formulated using the renewal theory in reliability. The three PR policies have been optimized to minimize their respective replacement cost rates, and their optimal joint PR polices with two variables have also been obtained. Numerical examples have been given to shown the analytical optimum solutions.

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