

# Adaptive Two-stage Learning Algorithm for Repeated Games

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**Abstract:** In our society, people engage in a variety of interactions. To analyze such interactions, we consider these interactions as a game and people as agents equipped with reinforcement learning algorithms. Reinforcement learning algorithms are widely studied with a goal of identifying strategies of gaining large payoffs in games; however, existing algorithms learn slowly because they require a large number of interactions. In this work, we constructed an algorithm that both learns quickly and maximizes payoffs in various repeated games. Our proposed algorithm combines two different algorithms that are used in the early and later stages of our algorithm. We conducted experiments in which our proposed agents played ten kinds of games in self-play and with other agents. Results showed that our proposed algorithm learned more quickly than existing algorithms and gained sufficiently large payoffs in nine games.

## 1 INTRODUCTION

Humans make a variety of decisions in their daily lives. In social situations in which a person's decision depends on other people, there are complicated mutual relations such as competition and cooperation among people. Many researchers have widely studied game theory that models relations among people as "games" and analyzed rational decision-making in games.

Furthermore, humans have an instinctive desire for survival; therefore, learning through trial-and-error to avoid harmful and unpleasant states and approach beneficial and pleasant states. Such learning occurs given a learning mechanism in the human brain. Many researchers study reinforcement learning algorithms that model this learning mechanism.

Let us consider people who learn their behavior from the result of interacting with others. As a model of this, we discuss reinforcement learning agents that play games. Many (multi-agent) reinforcement learning algorithms that perform well in various games have already been proposed; however, such algorithms typically require a large number of interactions to learn appropriate behavior in the games. People in the real world must quickly learn to make reasonable decisions, because the world changes rapidly. Exist-

ing algorithms have various features, each with its advantages and disadvantages. These algorithms can be complementary. Therefore, in this study, we construct an algorithm that learns quickly and performs well in any type of game by combining features from multiple algorithms.

Aside from this introductory section, the structure of this paper is as follows. In Section 2, we introduce games and learning algorithms used in later sections. In Section 3, we construct a new learning algorithm that learns quickly and performs well in various games by combining two learning algorithms. Next, we evaluate our proposed algorithm by conducting experiments, as described in Section 4. In Section 5, we introduce related works to show the relative position of our algorithm. Finally, in Section 6, we conclude our paper and provide avenues for future work.

## 2 BACKGROUND

In this section, we introduce game theory that models interactions among people and reinforcement learning algorithms that model trial-and-error learning for adaptation to a given environment.

## 2.1 Game Theory

We, as humans, are always making decisions as to what to do next to achieve our desired purpose or goals. In social environments, every decision is affected by the decisions of other people. Game theory (Okada, 2011) mathematically analyzes the relationship among such decisions.

A game in game theory consists of the following four elements (Okada, 2011):

1. Rules that govern the game;
2. Players who decide what to do;
3. Action strategies of the players; and
4. Payoffs given to the players as a result of their decisions.

Game theory analyzes how players behave in an environment in which their actions mutually influence one another. We focus on two-person simultaneous games in this study.

In a two-person simultaneous game, two players simultaneously choose actions based on their given strategies. After both players choose their respective actions, each player is given a payoff determined by the “joint actions” of both players. Since the payoff of each player is determined by not only his or her action but also the other player’s action, it is necessary to deliberate the other player’s action to maximize payoffs.

Note that all games used in this research are non-cooperative game, and players choose their individual strategies based on its own payoffs and all players’ actions. More specifically, after a player observes its payoffs and the action of the other players, he or she can then choose his or her own self-strategy.

A Nash equilibrium is defined as the combination of actions in which no player is motivated to change his or her strategy. Let us consider the “prisoner’s dilemma” game summarized in Table 1. Here, the row and column correspond to the actions of Players 1 and 2, respectively; they gain the left and right payoffs, respectively, corresponding to the joint action in the matrix.

According to the payoff matrix, the player should choose the Defection action regardless of the other player’s action, because it always yields higher payoffs than the Cooperation action. Since the other player considers the same, both players choose Defection, and finally, the combination of actions (i.e., Defection, Defection) becomes a Nash equilibrium.

Conversely, if both players select Cooperation, both payoffs can be raised to 0.6 from 0.2; however, it is very difficult for both players to choose Cooperation, because the combination (i.e., Cooperation, Cooperation) does not yield equilibrium and each player

is motivated to choose Defection. Moreover, even if a player overcomes this motive for a certain reason, he or she will yield a payoff of zero if the partner chooses Defection. The prisoner’s dilemma game shows that the individual’s rationality differs from that of social rationality in a social situation.

Table 1: An example of payoffs in the prisoner’s dilemma game.

	Cooperation	Defection
Cooperation	0.6,0.6	0.0,1.0
Defection	1.0,0.0	0.2,0.2

## 2.2 Reinforcement Learning

Reinforcement learning (Sutton and Barto, 1998) is a learning method that learns strategies by interacting with the given environment. An agent is defined as a decision-making entity, while the environment is everything external to the agent that interacts with the agent. Furthermore, the agent interacts with the environment at discrete time steps, i.e.,  $t = 0, 1, 2, 3, \dots$ . At each time step  $t$ , the agent recognizes current state  $s^t \in S$  of the environment, where  $S$  is a set of possible states, and decides action  $a^t \in A(s^t)$  based on the current state, where  $A(s^t)$  is a set of actions selectable in state  $s^t$ .

At the next step, the agent receives reward  $r^{t+1} \in \mathcal{R}$  as a result of the action and transitions to new state  $s^{t+1}$ . The probability that the agent chooses possible action  $a$  in state  $s$  is shown as strategy  $\pi^t(s, a)$ . Reinforcement learning algorithms update strategy  $\pi^t$  or the action values (not the strategy) at each time step, choosing an action based on the strategy.

## 2.3 Three Foundational Learning Algorithms

Here we introduce three learning algorithms that form the basis for our proposal.

### 2.3.1 M-Qubed

M-Qubed (Crandall and Goodrich, 2011) is an excellent state-of-the-art reinforcement learning algorithm that consists of three strategies that can learn to cooperate with associates (i.e., other players) and avoid being exploited unilaterally in various games. M-Qubed uses Sarsa (Rummery and Niranjan, 1994) to learn action value function  $Q(s, a)$  (called the Q-value), which means the value of action  $a$  in state  $s$ . Here, state is defined as the latest joint action of the agent and its associates. Further,  $Q(s, a)$  is updated by the following

rule:

$$Q^{t+1}(s^t, a^t) = Q^t(s^t, a^t) + \alpha[r^t + \gamma W^t(s^{t+1}) - Q^t(s^t, a^t)], \quad (1)$$

$$V^t(s) = \sum_{a \in A(s)} \pi^t(s, a) Q^t(s, a), \quad (2)$$

where  $r^{t+1} \in [0, 1]$ ,  $\alpha$  is the learning rate, and  $\gamma$  is the discount rate.

After M-Qubed updates the Q-value, it calculates the strategy of the agent from three sub-strategies, i.e., “Profit pursuit”, “Loss aversion”, or “Optimistic search”. Each of these is described below. Further, the *maximin value* is the secured payoff regardless of the associates given by the maximin strategy that maximizes the minimum payoff based on the payoff definition of the game.

#### Profit Pursuit

This sub-strategy takes the action with the maximum Q-value if it is larger than the discounted sum of the maximin value; otherwise, the maximin strategy is used.

#### Loss Aversion

This sub-strategy takes the action with the maximum Q-value if the accumulated loss is less than a given threshold; otherwise, the maximin strategy is used. Here, the accumulated loss is the difference between the accumulated payoffs and the accumulated maximin value. The threshold is set proportional to the number of possible states and joint actions.

#### Optimistic Search

The “Profit pursuit” strategy can acquire high payoffs for the moment; however, it tends to produce myopic actions as a result. The “Loss aversion” strategy cannot lead cooperative strategies with associates, which may yield higher payoffs. To solve these problems, M-Qubed sets the initial Q-values to their highest possible discounted reward  $1/(1 - \gamma)$ , thereby learning wider strategies.

Next, the strategy is a weighted average of “Profit pursuit” and “Loss aversion”, and Q-values are initialized by “Optimistic search”.

However, if all recently visited states have low Q-values, the strategies of the agent and its associates may remain at a local optimum. The agent must explore further to find a solution that may give a higher payoff. Hence, in this case, the strategy is changed to a weighted average of the above strategy and a completely random strategy.

### 2.3.2 Satisficing Algorithm

Satisficing algorithm (S-*alg*) (Stimpson and Goodrich, 2003) is an algorithm that calculates a value called the *aspiration level* of the agent. The agent continues to take an action that gives payoffs more than its aspiration level. The algorithm is shown in Algorithm 1.

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#### Algorithm 1: Satisficing algorithm.

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Parameters:

$t$ : time

$\alpha^t$ : aspiration level at time  $t$

$a^t$ : my action at time  $t$

$r^t$ : payoff at time  $t$

$\lambda \in (0, 1)$ : learning rate

Initialize:

$t = 0$

Set  $\alpha^0$  randomly between the maximum payoff  $R_{max}$  and  $2R_{max}$

**repeat**

  Select  $a^t$

**if**  $r^{t-1} \geq \alpha^{t-1}$  **then**

$a^t \leftarrow a^{t-1}$

**else if** otherwise **then**

$a^t \leftarrow \text{Random}$

**end if**

  Receive  $r^t$  and update  $\alpha^t$

$\alpha^{t+1} \leftarrow \lambda \alpha^t + (1 - \lambda) r^t$

$t \leftarrow t + 1$

**until** Game Over

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S-*alg* is an algorithm that enables an agent to learn to take cooperative actions with its associates.

### 2.3.3 BM Algorithm

Fujita et al. (2016) proposed BM algorithm to maximize payoffs by combining M-Qubed and S-*alg*. BM algorithm comprises the above two learning algorithms and creates a new strategy that combines the strategies derived from the two internal algorithms. Both algorithms simultaneously update their functions, i.e., the Q-value and aspiration level.

BM algorithm uses a Boltzmann multiplication (Wiering and van Hasselt, 2008) as the method of combination. Based on strategy  $\pi_j^t$  of internal algorithm  $j$ , Boltzmann multiplication multiplies strategies of all internal algorithms for each available action and determines the ensemble strategy by the Boltzmann distribution. The preference values of actions are defined as

$$p^t(s^t, a[i]) = \prod_j \pi_j^t(s^t, a[i]) \quad (3)$$

and the resulting ensemble strategy is defined as

$$\pi^t(s^t, a[i]) = \frac{p^t(s^t, a[i])^{\frac{1}{\tau}}}{\sum_k p^t(s^t, a[k])^{\frac{1}{\tau}}}, \quad (4)$$

where  $a[i]$  is a possible action and  $\tau$  is a temperature parameter. After calculating the ensemble strategy, the agent selects an action, and then all internal algorithms learn from the result of this selected action.

Since S-alg yields only pure strategies, all actions except for the chosen one have zero probability. The M-Qubed becomes meaningless when the Boltzmann multiplication is used to combine M-Qubed and S-alg without consideration. Therefore, the S-alg in BM algorithm yields a mixed strategy in which the chosen action is played with probability 0.99.

### 3 PROPOSED ALGORITHM

In this study, we consider reinforcement learning agents that play games. M-Qubed, BM, and S-alg agents perform well in some games, but have problems in other games, as summarized below:

- M-Qubed requires a long time to learn, because it has multiple strategies and needs to decide which one is used. Therefore, the average payoff becomes less than S-alg in the game having only one suitable solution that is in cooperation with the associates.
- BM algorithm performs better than M-Qubed, but it is still slow in search because the internal S-alg cannot completely compensate for the slowness of M-Qubed.
- If the associates are greedy, S-alg is exploited unilaterally, because it decreases the aspiration level, and then S-alg is satisfied with low payoffs.
- Due to insufficient search, S-alg may be satisfied with the second-best payoff.

These algorithms do not have sufficient performance for a variety of reasons; however, their positive abilities are complementary. BM, which is not exploited due to the “Loss aversion” strategy and explores the given environment thoroughly enough, can compensate for the weakness of S-alg that tends to be exploited. Conversely, S-alg, which quickly learns to cooperate, can compensate for the weakness of BM, i.e., its slowness of search. Search in an early stage of interactions and the secured payoff are important when the agent learns. Therefore, we combine BM and S-alg to construct an algorithm that quickly learns good strategies in various games.

We call our proposed algorithm J-algorithm (J-alg). J-alg has an Exploration stage and a Static stage. The Exploration stage is represented by our S’ algorithm, which is a slightly modified version of S-alg. Similarly, the Static stage is represented by BM algorithm. In the following subsections, we introduce our S’ algorithm (i.e., S’-alg) and our proposed two-stage J-alg.

#### 3.1 S’ Algorithm

We focused on S-alg to play a key role in the Exploration stage, because S-alg can learn to cooperate quickly; however, S-alg tends to cover an insufficient search space and falls prey to a myopic strategy. Suppose that the S-alg agents play the Security game (SG) shown in Table 2. If the aspiration level of the row player is smaller than 0.84, the player loses his or her motivation to change his or her action from  $x$ . Consequently, both players are satisfied with a payoff that is not the largest one. S-alg stops searching for other actions and is therefore prone to a shortened search.

Table 2: Security game (SG).

	$z$	$w$
$x$	0.84, 0.33	0.84, 1.0
$y$	0.0, 1.0	1.0, 0.67

To solve this problem, we slightly modify S-alg to add new action  $b$ ; this new algorithm is called the S’ algorithm (i.e., S’-alg). In short, in S’-alg, if the agent receives the maximin value as the last payoff, it selects action  $b$ . The modified algorithm is shown in Algorithm 2.

This change stochastically forces the agent to take other actions to escape from a local solution and potentially find a better one.

#### 3.2 Integrating the Two Stages

J-alg uses S’-alg for the Exploration stage and BM for the Static stage. The J-alg agent starts in the Exploration stage. After the joint action converges, the algorithm switches to the Static stage and resets the Q-values to choose the converged action more often. If the joint action does not converge in the first  $t_c$  rounds, the algorithm simply changes to the Static stage without changing the value. The algorithm is shown in Algorithm 3.

Even though the J-alg agent is exploited in the Exploration stage, it will recover in the Static stage via BM algorithm that can evade a loss by using “Loss aversion” strategy.

**Algorithm 2:** S' algorithm.

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Parameters:  
 $t$ : time  
 $\alpha^t$ : aspiration level at time  $t$   
 $a^t$ : my action at time  $t$   
 $r^t$ : payoff at time  $t$   
 $\lambda \in (0, 1)$ : learning rate  
 $b$ : a random action other than  $a^{t-1}$  with probability  $p$  and a random action selected from all of the actions with probability  $(1 - p)$   
Initialize:  
 $t = 0$   
Set  $\alpha^0$  randomly between the maximum payoff  $R_{max}$  and  $2R_{max}$   
**repeat**  
  Select  $a^t$   
  **if**  $r^{t-1} \geq \alpha^{t-1}$  **then**  
     $a^t \leftarrow a^{t-1}$   
  **else if**  $r^{t-1} = \text{maximin value}$  **then**  
     $a^t \leftarrow b$   
  **else if** otherwise **then**  
     $a^t \leftarrow \text{Random}$   
  **end if**  
  Receive  $r^t$  and update  $\alpha^t$   
   $\alpha^{t+1} \leftarrow \lambda\alpha^t + (1 - \lambda)r^t$   
   $t \leftarrow t + 1$   
**until** Game Over

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**Algorithm 3:** J-alg.

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Parameters:  
 $s$ : state  
 $a$ : action  
 $a_c$ : my action when joint action converges  
 $r_c$ : payoff when joint action converges  
 $\delta \in [0, 1]$ : reduction rate  
Initialize:  
 $t = 1$   
Start the Exploration stage  
**repeat**  
  **if** in the Exploration stage and  $t < t_c$  **then**  
    **if** joint action has not converged **then**  
      Continue the Exploration stage  
    **else if** otherwise **then**  
      **if**  $a = a_c$  **then**  
         $Q(s, a) \leftarrow r_c / (1 - \gamma)$   
      **else if** otherwise **then**  
         $Q(s, a) \leftarrow \delta \times r_c / (1 - \gamma)$   
        for all  $s$ .  
        Start the Static stage  
      **end if**  
    **end if**  
  **else if** otherwise **then**  
    Start the Static stage  
  **end if**  
   $t \leftarrow t + 1$   
**until** Game Over

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## 4 EXPERIMENTS

To confirm sufficient performance of J-alg, we conducted experiments using 10 two-person two-action matrix games used in the M-Qubed paper (Crandall and Goodrich, 2011). Here, the agent can observe only the previous joint action and its own payoff. We also set the M-Qubed parameters to be identical to the original ones noted in Crandall and Goodrich (2011). We set parameter  $\tau$  of the BM algorithm to 0.2. Further, we set learning rate  $\lambda = 0.99$ , probability  $p = 0.3$  of S'-alg, reduction rate  $\delta = 0.99$  and time  $t_c = 500$  for the threshold at which a shift occurs from the Exploration stage to the Static stage. We considered that a certain joint action converged when it had continued 30 rounds. Table 3 shows the games used in our experiments. Here, we call the maximum joint action a joint action by which the sum of both player's payoffs becomes maximum (which is shown in bold italic typeface in the table).

### 4.1 Experiment 1

Our first experiment was conducted in games where both players are agents with the same algo-

algorithm, i.e., in self-play. Agents played one of the 10 games for 50000 rounds, iterating 50 times for each game. We then compared the normalized average payoffs of M-Qubed, S-alg, BM, and J-alg. Note that the earlier the actions of agents converge to the maximum joint action, the larger the normalized average payoffs become. Table 4 shows the normalized average payoffs in the games, each of which is the average payoff divided by that of the maximum joint action. If the normalized average payoff is close to one, it shows that the agent quickly learned the maximum joint action in the game.

From our results, we observe that J-alg gained high payoffs and quickly learned the maximum joint action of nine games in self-play. It was able to learn optimal strategies more quickly than M-Qubed and BM due to the Exploration stage (i.e., S'-alg). M-Qubed and BM particularly gained low payoffs in the prisoner's dilemma (PD) game, because of the mutual defection (b,d) by performing many explorations to learn the cooperative joint action (a,c). S-alg gained high payoffs in eight games, not including the security game (SG) and the offset game (OG). In these games, the aspiration level of S-alg decreased below the second-best payoff because of insufficient ex-

Table 3: Payoff matrices of the two-person two-action matrix games used in our experiments. The row player has two actions, a and b, and the column player has two actions, c and d. The values in the cells show the payoffs the players are given when the joint action appears (i.e., left for the row player and right for the column player). Payoffs in bold italic typeface are those that maximize the sum of the two payoffs.

(a) Common interest game (CIG)		(b) Coordination game (CG)	
	c	d	
a	<b><i>1.0, 1.0</i></b>	0.0, 0.0	
b	0.0, 0.0	0.5, 0.5	
(c) Stag hunt (SH)		(d) Tricky game (TG)	
	c	d	
a	<b><i>1.0, 1.0</i></b>	0.0, 0.75	
b	0.75, 0.0	0.5, 0.5	
(e) Prisoner's dilemma (PD)		(f) Battle of the sexes (BS)	
	c	d	
a	<b><i>0.6, 0.6</i></b>	0.0, 1.0	
b	1.0, 0.0	0.2, 0.2	
(g) Chicken (Ch)		(h) Security game (SG)	
	c	d	
a	<b><i>0.84, 0.84</i></b>	0.33, 1.0	
b	1.0, 0.33	0.0, 0.0	
(i) Offset game (OG)		(j) Matching pennies (MP)	
	c	d	
a	0.0, 0.0	<b><i>0.0, 1.0</i></b>	
b	<b><i>1.0, 0.0</i></b>	0.0, 0.0	

Table 4: Normalized average payoffs when agents played games in self-play. The bold typeface indicates the best results among the four given methods.

	J-alg	M-Qubed	S-alg	BM
CIG	0.999080	0.993804	0.999039	<b>0.999181</b>
CG	0.951450	0.847301	<b>0.997966</b>	0.867965
SH	<b>0.999182</b>	0.995857	0.999149	0.998505
TG	0.998033	0.814068	<b>0.998251</b>	0.828770
PD	0.998569	0.711794	<b>0.998626</b>	0.723831
BS	0.998686	0.910656	<b>0.998700</b>	0.917655
Ch	<b>0.998909</b>	0.934089	0.998900	0.931541
SG	<b>0.998612</b>	0.902031	0.718236	0.907887
OG	0.466739	0.473539	<b>0.499964</b>	0.478196
MP	1.000000	1.000000	1.000000	1.000000
Avg.	<b>0.940926</b>	0.858314	0.920883	0.865353

ploration. For all algorithms, the offset game (OG) proved difficult to learn the optimal strategy.

## 4.2 Experiment 2

In our second experiment, we compared J-alg with M-Qubed, S-alg, and BM in round-robin tournaments in which all combinations of agents were examined. As with Experiment 1, the agents played one of the 10 games 50000 times, iterating 50 times for each game. In asymmetric games, we replaced the position of players and started our experiment again. The

normalized average payoffs of each game are shown in Table 5. Values greater than one indicate that the agent exploited other agents and gained high payoffs as a result. If the average value of the normalized average payoffs became larger, the agent gained high payoffs across many rounds.

Table 5: Normalized average payoffs when agents play games in round-robin tournaments. The bold typeface indicates the best results from among the four given methods.

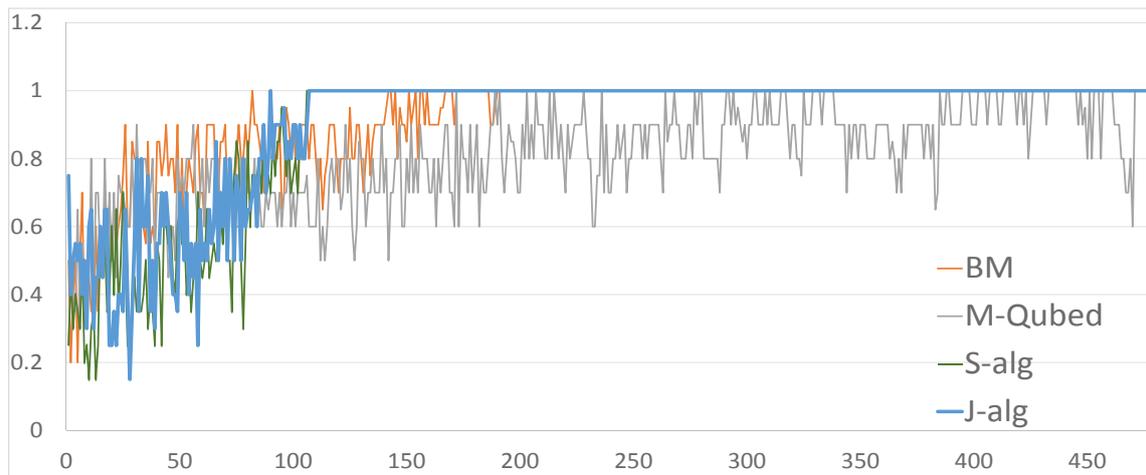
	J-alg	M-Qubed	S-alg	BM
CIG	0.998983	0.995743	<b>0.998991</b>	0.998603
CG	<b>1.048619</b>	0.915732	0.832476	0.905246
SH	0.999098	0.996896	<b>0.999148</b>	0.998226
TG	<b>0.952595</b>	0.863191	0.943683	0.868901
PD	<b>0.900618</b>	0.743598	0.871854	0.754596
BS	0.988460	<b>1.002762</b>	0.921269	0.951607
Ch	0.974938	0.933062	<b>0.975377</b>	0.926621
SG	<b>0.935857</b>	0.881564	0.810752	0.885159
OG	<b>0.555748</b>	0.497452	0.341784	0.538482
MP	1.041080	<b>1.051359</b>	0.858698	1.048864
Avg.	<b>0.939600</b>	0.888136	0.855403	0.887630

J-alg gained high average payoffs in many of the games we used for our experiments. In the coordination game (CG), battle of the sexes (BS), the security game (SG), the offset game (OG), and matching pennies (MP), S-alg was exploited when the associate took a greedy strategy, because its aspiration level decreased too much, and as a result, it was satisfied with small payoffs. M-Qubed and BM were not able to learn optimal strategies quickly and gain high average payoffs because they required a large number of interactions. Note that M-Qubed did indeed gain high average payoffs in battle of the sexes (BS) and matching pennies (MP) by taking a greedy strategy.

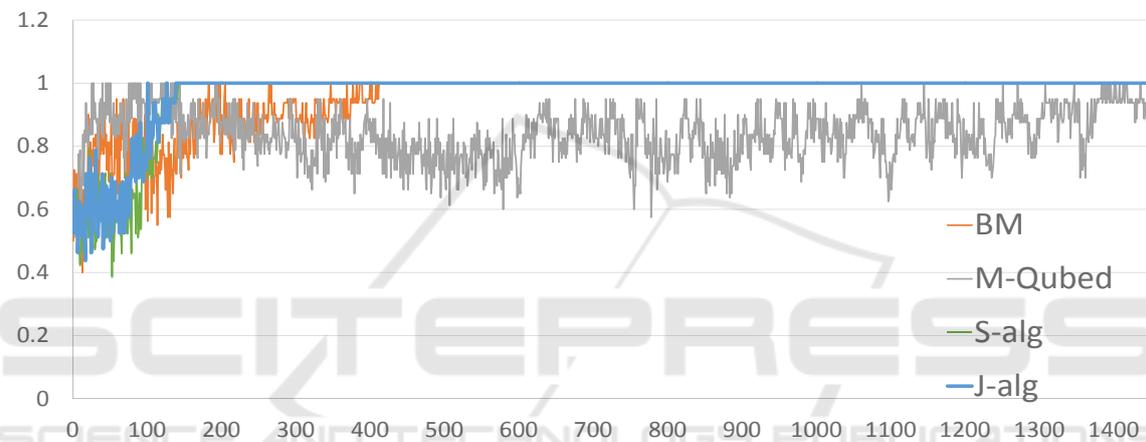
The average payoffs shown in Table 4 and Table 5 show that J-alg gained the highest average payoffs both in self-play and round-robin tournaments. S-alg gained high payoffs in self-play, but it was exploited by greedy players and gained low payoffs in round-robin tournaments. M-Qubed and BM gained better payoffs by exploiting S-alg in round-robin tournaments than in self-play. However, they required a large number of rounds to learn the maximum joint action and therefore gained lower average payoffs than that of J-alg.

## 4.3 Experiment 3

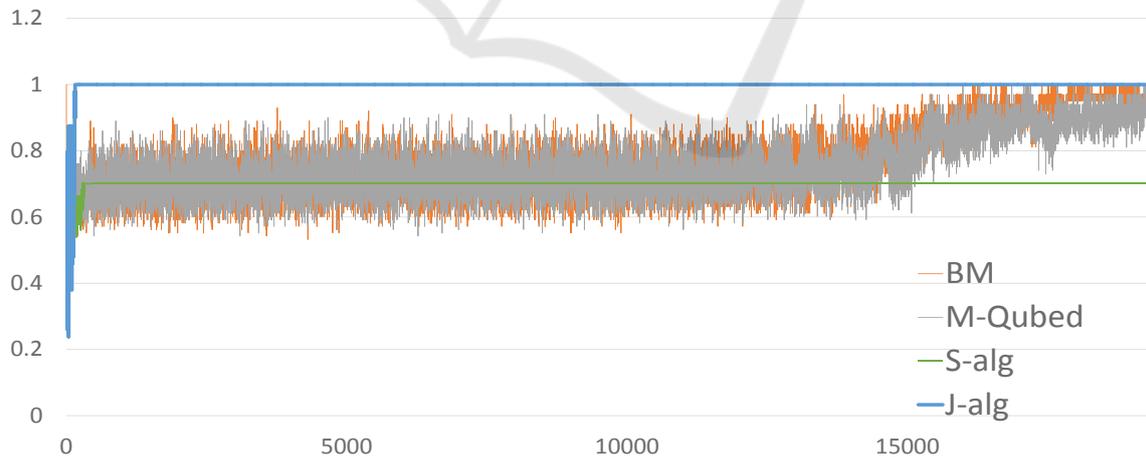
In our third experiment, we investigated whether S'-alg actually contributed to the learning speed of J-alg. Figure 1 shows learning curves of J-alg, M-Qubed, S-alg, and BM in the common interest game (CIG), stag hunt (SH), and the security game (SG). Results of these three games are good examples that show



(a) Common Interest Game (CIG)



(b) Stag Hunt (SH)



(c) Security Game (SG)

Figure 1: Learning curves of the four algorithms in self-play in three games. Blue lines show the curves of J-alg, gray show those of M-Qubed, orange show those of BM, and green show those of S-alg. The  $x$ -axes show the rounds the agents played, while the  $y$ -axes show normalized average payoffs.

how the algorithms comprised in J-alg complement one another.

Figure 1(a), i.e., results of the common interest game (CIG), shows that J-alg and S-alg quickly

learned the maximum joint action almost simultaneously. BM learned the maximum joint action by the 190th round, shortly after J-alg and S-alg. M-Qubed was still learning at the 500th round. Results here show that S'-alg contributed to the learning speed of J-alg.

Results of the stag hunt (SH) game are shown in Fig. 1(b), which shows that J-alg and S-alg finished learning the maximum joint action the quickest. Next, BM finished learning with the average payoff converging to one by the 410th round. Finally, M-Qubed finished learning by the 1500th round. Results here show the same as Fig. 1(a), i.e., J-alg learned the maximum joint action fastest, whereas BM and M-Qubed learned more slowly.

Turning to Fig. 1(c) reveals that BM compensated for a flaw of S-alg by which it failed to learn the maximum joint action. In the figure, we observe that in the security game (SG), S-alg finished learning first, but did not obtain the optimal strategy because S-alg was satisfied with the second-best payoff. As shown in Table 4, S-alg gained smaller payoffs as compared with the other three algorithms in this game. BM and M-Qubed learned the maximum joint action, but they required a large number of interactions and learned slowly. Finally, J-alg successfully learned the quickest.

## 5 RELATED WORKS

Reinforcement learning is widely used as a learning method for agents. What defines reinforcement learning as being different from other learning methods is the combination of trial-and-error searches and delayed rewards. Temporal difference (TD) learning is arguably the most famous reinforcement learning method. TD learning compares the value of the current state with the reward obtained by the current action, updating the given value to decrease deviation. Q-learning (Watkins and Dayan, 1992) and Sarsa (Rummery and Niranjana, 1994) are typical examples of TD learning.

While TD learning was originally proposed as a learning method for a single agent, in a multi-agent environment, Claus and Boutilier (1998) discussed how to incorporate other agents in TD learning. They showed that in payoff matrix games, a "joint action learner" that updates Q-values based on actions of the agent and others is better than an "independent learner" that recognizes others as elements of the environment.

When designing a reinforcement learning agent in a multi-agent environment, we often adopt the knowl-

edge of game theory. Hu and Wellman (2003) proposed Nash Q-learning in which a Nash Q-learning agent explores the game structure from the perspective of the actions and rewards of the agent and others, updating its Q-values under the assumption that every player selects a strategy that leads to Nash equilibrium.

There are also methods that do not update Q-values from immediate joint actions, but instead assign Q-values to a long (fixed-length) interaction history. Burkov and Chaib-draa (2009) proposed a learning approach for an agent when its associates learn its strategies and adapt to its actions. Their agent, called an Adaptive Dynamics Learner, can obtain higher utility than that of an agent at an equilibrium by considering the associates' strategies and assigning Q-values to a long interaction history.

Further, there are reinforcement learning methods that do not use Q-values, instead they use a dynamic threshold called an aspiration level updated by rewards. Masuda and Nakamura (2011) considered a model in which a reinforcement learning agent learns from reinforcement signals calculated from the reward and the aspiration level; they investigated agent behavior when it played the iterated prisoner's dilemma games with other learning agents.

In summary, many researchers have proposed reinforcement learning algorithms for multi-agent environments; however, much of this work has focused only on the convergence of player strategies and thus requires an enormous number of interactions. In this paper, we proposed an algorithm that learns appropriate strategies against the associate's strategy faster than these existing algorithms, because humans decide very quickly in real situations.

## 6 CONCLUSIONS

Many researchers have studied and are studying reinforcement learning algorithms to acquire strategies that maximize payoffs for agents that learn in games. Existing reinforcement algorithms can gain high payoffs in various games, but typically require a large number of interactions to learn an optimal strategy.

To overcome these limitations, we proposed an algorithm called J-alg that learns quickly and gains large payoffs in many games by combining two existing algorithms, namely, BM and S'-alg. S'-alg contributes to learning speed, whereas BM contributes to the prevention of exploitation by greedy opponents. To evaluate our algorithm, we conducted two experiments, i.e., self-play and round-robin, using 10 games. In both sets of experiments, J-alg gained suf-

ficiently large payoffs in nine games and the highest average payoffs among four types of agents.

As future work, we plan to construct an algorithm that can learn the maximum joint action in the offset game (OG). In addition, we plan to extend our work in two-person two-action games and construct an algorithm that can quickly learn optimal strategies in  $n$ -person  $n$ -action games.

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