Single and Multiple Objective Optimization Models for Opportunistic Preventive Maintenance

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Abstract: This paper presents single and multiple objective deterministic optimization models for opportunistic preventive maintenance of multi-component systems. The single objective model develops an aggregate cost objective function encompassing costs of replacement, fixed costs associated with maintenance interventions and costs of component dismounting whenever the replacement of a given component implies disassembling others. Two multiple objective models are proposed, which enable to explore the trade-offs between minimizing costs vs. the number of maintenance interventions and minimizing costs vs. maximizing the remaining lifetime of components at the end of the planning period. Constraints refer to the requirement of replacing each component before the end of its lifetime and consistency restrictions to allow opportunistic maintenance and dismounting requirements.

1 INTRODUCTION

Maintenance generally encompasses the care and servicing by specialized personnel for the purpose of maintaining equipment and facilities in the required operating conditions (Ben-Daya et al., 2009). The primary goal of maintenance is to avoid or mitigate the consequences of failure of equipment. The optimization of maintenance operations and schedule is of utmost importance in industry and services, in particular in equipment-intensive industries and utilities (e.g., aviation, energy, telecommunications, water). Maintenance has a direct impact on equipment reliability and availability, and therefore on operational costs. Also, adequate maintenance and facility management policies and practices are central in sustaining safety and eco-efficiency.

Corrective maintenance actions, i.e. those performed after failure has occurred (run-to-failure), may result in unwanted system disturbances such as too frequent shutdowns with the consequent impacts on costs and quality of service, and even on the environment.

Preventive maintenance is aimed at preserving the equipment operating conditions and preventing their (otherwise costly) failure, involving partial or complete overhauls to preserve and restore equipment reliability. It provides for systematic inspection, detection, and correction of emerging failures before they happen or develop into major faults. It may include tests, measurements, adjustments and component replacement to prevent faults from occurring. In general, preventive maintenance is regularly or condition-based performed on an equipment, or worn components, often still working, to lessen the likelihood of failing. Therefore, production loss, downtime, and safety and environmental hazards are minimized.

Preventive maintenance is generally scheduled based on a time or usage activation signal. An air-conditioner is a typical example of an asset for which a time-based preventive maintenance schedule is performed: e.g., it is checked every year, before the hot season. An example of an asset with a usage-based preventive maintenance schedule is a motor vehicle that should be scheduled for service every 20,000 km. Applications that are generally mentioned as suitable for preventive maintenance

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include those that have a critical operational function, failure modes that can be prevented with regular maintenance, or a likelihood of failure that increases with time or usage.

Since the maintenance schedule should be planned, preventive maintenance is more complex to coordinate than corrective maintenance. In this scope, maintenance should be mainly seen as investment in reliability and availability rather than a cost-inducing activity. Therefore, optimization approaches are required, aimed at encompassing the essential features of real-world maintenance problems in different settings, i.e. taking into account the specificities of each industry and their equipment usage, in order to generate optimal recommendations to planners and decision makers.

In this setting, opportunistic maintenance models are well suited to several real-world problems, thus accommodating flexible strategies for planning maintenance activities (Dekker et al., 1997). Opportunistic models, i.e. preventive maintenance activities at an opportunity, entail deciding whether additional maintenance activities beyond the ones that are strictly required should be performed at a (possibly already planned) maintenance occasion. I.e., if the system is already under maintenance (either working or in a shutdown mode), components may be replaced or maintained at no additional fixed cost for intervention. Opportunistic maintenance optimization using deterministic models has been considered by Epstein and Wilamowsky (1985), Dickman et al. (1990), Nilsson et al. (2009) and Almgren et al. (2012), among others. Stochastic opportunistic replacement models have also been studied by several authors including the recent work by Patriksson et al. (2015).

In this paper single and multiple objective deterministic mathematical models are developed to provide decision support in the scheduling of opportunistic maintenance activities. Decisions to be made involve component replacement and component dismounting whenever the replacement of a given component implies disassembling others. The single objective function aggregates these different costs to determine the optimal solution. The single objective model has been developed from the basic opportunistic replacement model by Almgren et al. (2012) by including component dismounting actions.

Two models with multiple objective functions are then proposed, which enable to explore the trade-offs between minimizing replacement and dismounting costs vs. minimizing the number of maintenance interventions and minimizing total costs vs. maximizing the remaining lifetime of components at the end of the planning period. For multi-objective models the nondominated (Pareto optimal) set is computed. A feasible solution is nondominated if no other feasible solution exists that simultaneously improves all objective function values, i.e. improving an objective function implies worsening the value of at least another objective function value. In the present work the whole nondominated set has been obtained using a procedure based on reference points co-developed by one of the authors (Alves and Clímaco, 2000).

In section 2 a single objective model for optimizing opportunistic maintenance is presented, in which an overall cost objective function is considered. Multi-objective models for decision support in opportunistic maintenance are presented in section 3. Conclusions are drawn and further research is outlined in section 4.

2 A SINGLE OBJECTIVE MODEL FOR OPTIMIZING OPPORTUNISTIC MAINTENANCE

In this section a single objective mathematical model devoted to optimize the maintenance scheduling of a multi-component system is presented. The components must be replaced before they reach the end of their lifetime; this is estimated so that the probability of a component failure within its lifetime is low enough for its intended use and is usually provided by the equipment manufacturer. Hence, a deterministic model is considered under this assumption.

Considering a set of components and a finite planning horizon discretized in time intervals, the model aims at determining the dismounting and replacement schedule of the components during the planning horizon in order to minimize the total cost. A component that is replaced must be firstly dismounted. In addition, the replacement of a given component may imply dismounting others in which the component is embedded, regardless of whether those components require or not maintenance at that time interval.

Opportunistic maintenance is mainly justified when there is a significant fixed cost associated with a maintenance intervention, which is independent of the components that are replaced. The proposed model considers an overall cost objective function including terms related to fixed (opportunity) costs for interventions and costs for component replacement and dismounting.

Constraints refer to:

- the requirement of replacing each component before the end of its lifetime, including considering that at the beginning of the planning period some components may be already worn out (i.e. having some time of use);

- enforcing the consideration of the fixed cost for intervention if at least one component is replaced at a given time interval (to induce maintenance opportunities at no additional fixed cost);

- requirement of dismounting a component if it contains another component that is replaced.

2.1 Single Objective Model

The model inputs are:

- A set of N components, which are the target of the replacement/maintenance actions.

- A set of T time intervals, which result from the discretization of the finite planning horizon.

- A maximum replacement interval L_i for each component $i \in \{1,\ldots,N\}$ corresponding to its estimated lifetime (this maximum replacement interval can also derive from a policy decision, a safety regulation associated with the component's technical life, or a contractual requirement).

L_{0i} is the maximum replacement interval for each component i for the first time in the planning period T, thus taking into account its time of use before $t=0$.

- The replacement cost c_{it} of component $i \in \{1,\ldots,N\}$ at time $t \in \{1,\ldots,T\}$.

- The fixed cost associated with a maintenance intervention (opportunity cost) $d_t \geq 0$ at time $t=1,\ldots,T$, which is independent of the number of components replaced.

- The replacement of a component implies that it should be firstly dismounted. The dismounting cost of component i at time t is a_{it}.

- If component i is embedded into other component(s) then it may happen that in order to dismount component i it is necessary to dismount other component(s) as well. Let $M(i)$ be the set of components j that should be dismounted when component i is dismounted.

Decision variables:

x_{it} = 1, if component $i \in \{1,\ldots,N\}$ is replaced at time $t \in \{1,\ldots,T\}$; 0, otherwise.

y_{it} = 1, if component $i \in \{1,\ldots,N\}$ is dismounted at time $t \in \{1,\ldots,T\}$; 0, otherwise.

w_t = 1, if at least one replacement operation occurs at time $t \in \{1,\ldots,T\}$; 0, otherwise.

The single objective model is:

Model S1

$$\min \sum_{t=1}^{T} \left(\sum_{i=1}^{N} (c_{it} x_{it} + a_{it} y_{it}) + d_t w_t \right) \quad (1)$$

subject to:

$$\sum_{t=1}^{L_{0i}} x_{it} \geq 1, \qquad i = 1,\ldots,N \quad (2)$$

$$\sum_{t=k+1}^{k+L_i} x_{it} \geq 1, \qquad k = 1,\ldots,T-L_i;\; i = 1,\ldots,N \quad (3)$$

$$x_{it} \leq y_{it} \leq w_t, \qquad t = 1,\ldots,T;\; i = 1,\ldots,N \quad (4)$$

$$y_{it} \leq y_{jt}, \qquad t = 1,\ldots,T;\; j \in M(i), \forall i: M(i) \neq \varnothing \quad (5)$$

$$x_{it}, y_{it}, w_t \in \{0,1\} \quad t = 1,\ldots,T; \quad i = 1,\ldots,N \quad (6)$$

The objective function (1) minimizes the total cost considering the component replacement, component dismounting and fixed costs for interventions. Constraints (2) ensure that each component is replaced within its maximum replacement interval for the first time and (3) ensure the replacement of each component before the end of its lifetime for the rest of the planning period. Constraints (4) ensure that a component is dismounted before it is replaced and an intervention operation occurs at that time interval. Constraints (5) impose that each component j in which i is embedded is also dismounted if i is dismounted.

The model (1)-(6) can be simplified to have fewer variables and constraints, and thus minimize the computational effort.

Let $M = \bigcup_{i=1}^{N} M(i)$.

The y_{jt} variables will be defined only for $j \in M$. Therefore, consider the following definitions:

x_{it} = 1, if component $i \in \{1,\ldots,N\}$ is dismounted and replaced at time $t \in \{1,\ldots,T\}$; 0, otherwise.

y_{jt} = 1, if component $j \in M$ is dismounted at time $t \in \{1,\ldots,T\}$; 0, otherwise.

w_t keeps the same definition as above.

The simplified model is:

Model S2

$$\min \sum_{t=1}^{T} \left(\sum_{i=1}^{N} c_{it} x_{it} + \sum_{i \notin M} a_{it} x_{it} + \sum_{j \in M} a_{jt} y_{jt} \right) + \sum_{t=1}^{T} d_t w_t \quad (7)$$

subject to:

$$\sum_{t=1}^{L_{0i}} x_{it} \geq 1, \qquad i = 1,\ldots,N \quad (8)$$

$$\sum_{t=k+1}^{k+L_i} x_{it} \geq 1, \qquad k = 1,...,T - L_i;\ \ i = 1,..., N \qquad (9)$$

$$x_{it} \leq w_t, \qquad t = 1,...,T;\ i = 1,..., N \qquad (10)$$

$$x_{jt} \leq y_{jt} \qquad t = 1,...,T;\ j \in M \qquad (11)$$

$$x_{it} \leq y_{jt}, \qquad t = 1,...,T;\ j \in M(i), \forall i{:}M(i) \neq \varnothing \qquad (12)$$

$$x_{it}, w_t, y_{jt} \in \{0,1\} \ \ t = 1,...,T; \quad i = 1,...,N; j \in M \qquad (13)$$

The model (7)-(13) has the same number of variables and constraints as the model (1)-(6) only if $|M|$=N. Otherwise it has fewer variables and constraints. Constraints (10) and (11) correspond to constraints (4) in *Model S1* and constraints (12) correspond to (5). Remind that x_{it} = 1 means that component i is dismounted and replaced at time t, so variables y_{jt} can be directly related to x_{it} for which y_{it} have not been defined (i.e., for $i \notin M$). The fact that y variables are not defined for all components leads to a different formulation of the cost function (7), associating the dismounting cost with y_{jt} for components $j \in M$ and with x_{it} for $i \notin M$.

As only superfluous variables (and related constraints) are eliminated from *Model S1* to *Model S2,* the two models are equivalent.

2.2 Illustrative Example

Model S2, (7)-(13), has been instantiated with the following data for illustrative purposes: N=5 components and T=50 time intervals.

Table 1 displays the lifetime (L_{0i} and L_i), costs for each component (c_{it} and a_{it}) and dismounting requirements ($M(i)$).

Experiments have been carried out with fixed costs for interventions d_t = 10, 100, 1000, for all t.

Table 1: Lifetime, costs for each component (costs are the same for all time intervals) and dismounting requirements.

Component	1	2	3	4	5
L_{0i}	2	5	11	4	15
L_i	7	10	16	9	20
c_{it}	80	185	160	125	150
a_{it}	20	45	40	30	35
$M(i)$	∅	∅	1	2, 5	∅

These instances have 450 binary variables and 743 constraints. The equivalent *Model S1* would lead to instances with 550 binary variables and 843 constraints.

Tables 2-4 present optimal solutions for d_t = 10, 100, 1000, respectively. “x” denotes component replacement and “o” denotes dismounting without replacement. These solutions were obtained using our software for MultiObjective Mixed-Integer Linear Programming (MOMILP) problems (see section 3), which uses the non-commercial *lpsolve55* software (http://lpsolve.sourceforge.net/5.5/) to solve the integer single-objective problems. The computation times for obtaining the optimal solutions to the problems with d_t = 10, 100 and 1000, in a computer with Intel Core i7-2600K CPU@3.4GHz and 8 GB RAM, were 0.05, 2.65 and 0.08 seconds, respectively.

Table 2: Optimal solution for d_t = 10.

Component → Interval↓	1	2	3	4	5
2	x				
4		x		x	o
9	x		x		
13		x		x	x
16	x	o		x	o
23	x	x	x		
24		o		x	o
30	x				
33		x		x	x
37	x		x		
42		x		x	o
44	x				

The total cost of the optimal solution for d_t = 10 (Table 2) is 4100, 3980 for dismounting and replacing plus 120 for intervention fixed cost, with 12 maintenance interventions being made.

Table 3: Optimal solution for d_t = 100.

Component → Interval↓	1	2	3	4	5
2	x				
4	x	x		x	o
10	x		x		
13		x		x	x
17	x				
22		x		x	o
24	x		x		
31	x	x		x	x
38	x	o	x	x	o
41		x			
44	x	o		x	o

The total cost of the optimal solution for d_t = 100 (Table 3) is 5180, 4080 for dismounting and replacing plus 1100 for intervention fixed cost, with 11 maintenance interventions.

The total cost of the optimal solution for d_t = 1000 (Table 4) is 11690, 4690 for dismounting and replacing plus 7000 for intervention fixed cost, with 7 maintenance interventions.

Table 4: Optimal solution for $d_t = 1000$.

Component → Interval↓	1	2	3	4	5
2	x	x		x	o
9	x	x	x	x	x
16	x	x		x	o
23	x	x	x	x	x
30	x	x		x	o
37	x	x	x	x	x
44	x	x		x	o

The total cost of the optimal solution for d_t = 1000 (Table 4) is 11690, 4690 for dismounting and replacing plus 7000 for intervention fixed cost, with 7 maintenance interventions.

We can observe from these solutions that there is frequent disassembling of component 5 due to the frequent replacement of component 4, which has a short lifetime. Although the replacement of component 3 requires disassembling the component 1, it is not necessary to disassemble component 1 without its replacement in any solution because this component has a short lifetime and low replacement cost. Finally, it should be mentioned that solutions for d_t=10 and d_t=100 (Tables 2-3) have several optimal alternative solutions, i.e. different combinations of component replacement and disassembling in the planning period leading to the same optimal objective function value.

This model requires the specification of fixed costs for interventions d_t, which in some cases may be difficult to estimate. In order to better explore compromises between cost (of replacement and dismounting) and number of maintenance interventions, a bi-objective model is proposed below.

3 MULTIPLE OBJECTIVE MODELS FOR DECISION AID IN OPPORTUNISTIC MAINTENANCE

In this section bi-objective models are proposed, which enable to explore the trade-offs between minimizing replacement and dismounting costs vs. the number of maintenance interventions (*Model M1*) and minimizing total costs vs. maximizing the remaining lifetime of components at the end of the planning period (*Model M2*). The results of the illustrative examples were obtained using the interactive MOMILP software co-developed by one of the authors in Delphi for Windows (Alves and Clímaco, 2004), which includes a reference point-based procedure that is able to compute the whole nondominated front for bi-objective problems.

3.1 Minimizing Costs Vs. Number of Maintenance Interventions

The first objective function (14) of *Model M1* minimizes the replacement and dismounting cost while the second objective function (15) aims at minimizing the number of maintenance interventions.

Model M1

$$\min F_1 = \sum_{t=1}^{T}\left(\sum_{i=1}^{N} c_{it}x_{it} + \sum_{i\notin M} a_{it}x_{it} + \sum_{j\in M} a_{jt}y_{jt}\right) \quad (14)$$

$$\min F_2 = \sum_{t=1}^{T} w_t \quad (15)$$

subject to:

(8) – (13)

Example:

Model M1 has been instantiated with the same data as the single-objective model. The resulting problem has 7 nondominated solutions, which are depicted in Figure 1.

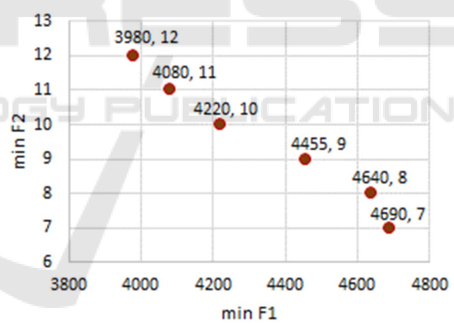

Figure 1: Nondominated front: replacement and dismounting cost vs. number of interventions (*Model M1*).

The nondominated solution that minimizes cost, $(F_1, F_2) = (3980, 12)$, is an alternative optimal solution to the single objective model with the smaller value of d_t, i.e. d_t=10. The nondominated solution that minimizes the number of maintenance interventions, $(F_1, F_2) = (4690, 7)$, is the optimal solution to the single objective model with d_t=1000. The solution where $(F_1, F_2) = (4080, 11)$ is an alternative optimal solution for the single objective model with d_t=100. In addition to these solutions, there are other intermediate nondominated solutions with 8, 9 and 10 interventions. Note that several alternative configurations (decision variable values) can be found for the same nondominated objective point. This happens, in particular, for the solutions that consider a large number of maintenance interventions. However, the nondominated points (objective function values) are only those shown in Figure 1.

3.2 Minimizing Total Cost Vs. Maximizing the Remaining Lifetime of Components at the End of the Planning Period

In addition to the minimization of maintenance costs, the maximization of the value of the assets at the end of the planning period may also be an objective the decision maker wants to accomplish, namely if the planning period should be extended due to any circumstance. The real value of a component depends on its remaining lifetime. The next model (*Model M2*) considers again the overall cost (replacement and dismounting plus the intervention fixed cost) for the first objective function. Thus, its formalization (16) is the same as the objective function (7) of the single objective model. The second objective function aims at maximizing the remaining lifetime of the components, as a proxy for maximizing the value of the assets at the end of the planning period. As the decision maker may want to assign different levels of importance to each component, a weighted sum of the remaining lifetime of the components is considered. This objective function is formalized in (17), where α_i denotes the weight assigned to each component i.

If the component i is replaced at the last time interval of the planning period, t=T, then its remaining lifetime is L_i; if the replacement is at t=T-1, then its remaining lifetime is $L_i - 1$, and so on; thus the remaining lifetime of the component at the end of the planning period can be given by $\sum_{t=T-L_i+1}^{T} v_{it} x_{it}$ with $v_{it} = L_i - k$ for $t = T - k$, provided the model ensures that the component is replaced only once from the time $T - L_i$ to T. Accordingly, the replacement constraints of each component for the last period, i.e. constraints (9) for $k = T - L_i$, are changed to be of type "=" instead of "≥". These are constraints (19) in *Model M2*; the other constraints of (9) are replaced by (18) in this model.

Model M2

$$\min F_1 = \sum_{t=1}^{T}\left(\sum_{i=1}^{N} c_{it}x_{it} + \sum_{i\notin M} a_{it}x_{it} + \sum_{j\in M} a_{jt}y_{jt}\right) + \sum_{t=1}^{T} d_t w_t \quad (16)$$

$$\max F_2 = \sum_{i=1}^{N}\left(\alpha_i \sum_{t=T-L_i+1}^{T} v_{it}x_{it}\right) \quad (17)$$

subject to:

(8), (10) – (13)

$$\sum_{t=k+1}^{k+L_i} x_{it} \geq 1, \qquad k = 1,\ldots,T-L_i-1;\; i = 1,\ldots,N \quad (18)$$

$$\sum_{t=T-L_i+1}^{T} x_{it} = 1, \qquad i = 1,\ldots,N \quad (19)$$

Examples:

Model M2 has been instantiated with the same data as the previous models using d_t =100.

Two experiments were performed. The first one considered α_i =1 for all i. In the second experiment higher weight was given to components with smaller lifetime: α_i =1/L_i. The weights were then normalized so that $\sum_{i=1}^{N} \alpha_i = N$. The normalization enables a better comparison between the two experiments as the weights have equal sum.

In the first experiment 23 nondominated solutions were obtained, which are depicted in figure 2. The overall cost ranges from 5180 to 6230. The solution with minimum cost (solution A in Figure 2, shown in Table 5) presents a sum of remaining lifetime at the end of the planning period of 14. Table 5 also shows the remaining lifetime of each component. This solution is an alternative optimal solution to the single objective *Model S2* with d_t=100. However, the optimal solution of *Model S2* presented in Table 3 has a sum of remaining lifetime of only 9. Solution A is also an alternative to the nondominated solution that minimizes cost in the bi-objective *Model M1* (cost vs. number of maintenance interventions). Likewise, the number of maintenance interventions is equal to 11. However, the sum of remaining lifetime is 12 in the solution obtained for *Model M1*, which is worse than the corresponding value in solution A for *Model M2*. Hence, the nondominated solution that minimizes cost obtained for *Model M2* may be more interesting than the solutions that minimize cost for the single objective *Model S2* and the multiobjective *Model M1*: all these solutions have the same cost and number of maintenance interventions but solution A presents a larger overall remaining lifetime at the end of the planning period.

The nondominated solution that maximizes the sum of remaining lifetime (solution B in Figure 2, shown in Table 6) has a value of 62 for this objective and a total cost of 6230. This solution proposes the replacement of all components at the last time interval of the planning period, i.e. $t = T = 50$, so it ensures the maximum remaining lifetime for all components.

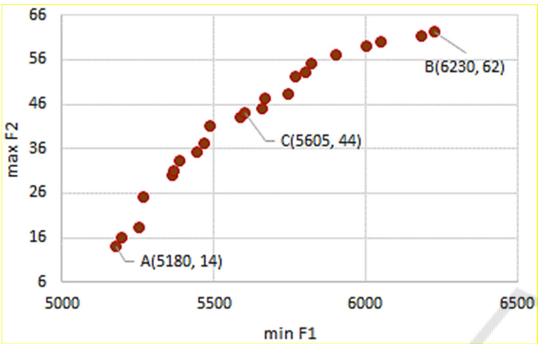

Figure 2: Nondominated front: overall cost vs. sum of remaining lifetime at the end of the planning period (*Model M2* – experiment 1).

Table 5: Solution A that minimizes cost in *Model M2*.

Component → Interval↓	1	2	3	4	5
1	x	o		x	o
5		x			
8	x	o	x	x	o
15	x	x		x	x
22	x		x		
24		x		x	o
29	x				
33		x		x	x
36	x		x		
42	x	x		x	o
49	x				
remaining lifetime	6	2	2	1	3

In the second experiment, considering a weighted-sum for the remaining lifetime objective function, 31 nondominated solutions were obtained, which are depicted in Figure 3. The extreme solutions (A and B), which optimize individually each objective function, are similar in both experiments. The problem solved in the second experiment has more nondominated solutions than the one solved in the first experiment (with equal weights) and some solutions are common, but not all solutions of the first problem belong to the nondominated set of the second problem.

Table 6: Solution B that maximizes the sum of remaining life in *Model M2*.

Component → Interval↓	1	2	3	4	5
2	x	x		x	o
9	x		x		
11		x		x	x
13	x				
20	x	x	x	x	o
27	x	o		x	o
30		x			x
34	x	o	x	x	o
40	x	x			
41		o		x	o
43	x				
50	x	x	x	x	x
remaining lifetime	7	10	16	9	20

An intermediate solution obtained in these experiments with *Model M2* can also be analyzed, e.g. solution C (Figures 2 and 3), common to both experiments. The solution configuration is presented in Table 7. It requires 12 maintenance interventions. The total cost is 5605 and the sum of remaining lifetime is 44 (the weighted sum of remaining lifetime is 39.89 – value of F_2 in the second experiment).

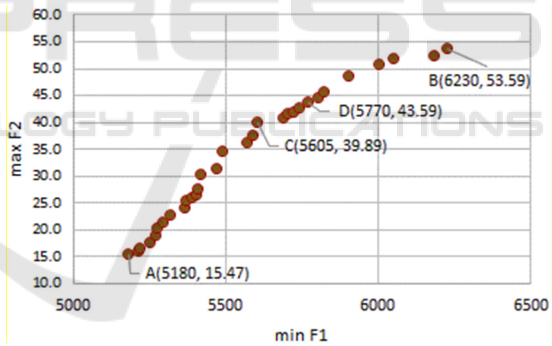

Figure 3: Nondominated front: overall cost vs. weighted sum of remaining life at the end of the planning period (*Model M2* – experiment 2).

We can observe in Table 7 that four of the five components (1, 2, 4 and 5) are replaced near the end of the planning period, i.e., at t=49, so their remaining lifetime is near the respective maximum (L_i–1). However, the last replacement of component 3 is at t=36, so its remaining lifetime at the end of the planning period is only 2. Therefore, although intermediate solutions exist displaying a good trade-off between cost and (weighted) sum of remaining lifetime at the end of the planning period, the individual remaining lifetimes may not be balanced among the various components. Solution D in Figure

3 also needs 12 maintenance interventions. The last replacement of each component occurs at t=48, leading to a remaining lifetime of 5, 8, 14, 7 and 18, respectively for components 1 to 5. The minimum remaining lifetime is 5, thus the remaining lifetimes are more balanced than in solution C. However, solution D presents a higher cost than C (5770 vs. 5605).

Table 7: Solution C to *Model M2*.

Component → Interval↓	1	2	3	4	5
1	x				
4		x		x	o
8	x		x		
13		x		x	x
15	x				
22	x	x	x	x	o
29	x				x
31		x		x	o
36	x		x		
40		x		x	o
43	x				
49	x	x		x	x
remaining lifetime	6	9	2	8	19

4 CONCLUSIONS

Single and multiple objective deterministic optimization models have been presented to support opportunistic preventive maintenance decisions regarding a set of components, which are the target of the replacement/maintenance actions, over a finite planning horizon. The single objective model considers the minimization of an overall cost objective function including fixed costs for interventions and costs for component replacement and dismounting whenever the replacement of a given component implies disassembling others. The multi-objective models enable to explore the trade-offs between minimizing replacement and dismounting costs vs. the number of maintenance interventions and minimizing total costs vs. maximizing the remaining lifetime of components at the end of the planning period. Illustrative examples have been presented using software developed by some of the authors for general multiobjective mixed-integer programming problems to exploit the practical insights offered by the models.

Further research will involve developing adequate sensitivity analysis techniques to take into account the uncertainty associated with the model coefficients to obtain robust solutions, i.e. recommendations that are relatively immune to changes of coefficients within plausible ranges. Moreover, the models will be exploited in real-world settings taking into account the particularities raised by specific situations as well as scalability issues to tackle large-scale problems in systems with hundreds of components.

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