A Practical-time Attack on Reduced-round MISTY1

Nobuyuki Sugio[1], Yasutaka Igarashi[2], Toshinobu Kaneko[2] and Kenichi Higuchi[2]
[1]*NTT DOCOMO, INC., 3-6 Hikarinooka, Yokosuka, Kanagawa, 239-8536, Japan*
[2]*Tokyo University of Science,2641 Yamazaki, Noda, Chiba, 278-8510, Japan*

Keywords: MISTY1, Symmetric Key Algorithm, Block Cipher, Higher Order Differential Attack.

Abstract: MISTY1 is a symmetric key algorithm which has been standardized by ISO and that its modified version is used in GSM and 3G mobile networks. MISTY1 is a 64-bit block cipher supporting key length of 128 bits. In this paper, we focused on evaluating the security of MISTY1 against higher order differential attack. We show 6-round MISTY1 with 4 FL layers is attackable with 2^{43} blocks of chosen plaintexts and $2^{43.31}$ times of data encryption. This is the best practical-time attack on reduced-round MISTY1.

1 INTRODUCTION

MISTY1 is one of the symmetric key algorithms. MISTY1 is a 64-bit block cipher supporting key length of 128 bits. MISTY1 was proposed by Matsui in 1997 (Matsui, 1997). The number of rounds is 8. MISTY1 achieves a provable security against differential cryptanalysis and linear cryptanalysis with round function FO. Designer adds on an auxiliary function FL in order to become secure against other attacks. MISTY1 was selected as one of the NESSIE-recommended ciphers portfolio and was adopted as the international standard by ISO/IEC 18033-3 (ISO, 2010). CRYPTREC project has chosen MISTY1 as one of the e-Government Recommended candidate ciphers in 2013 (CRYPTREC, 2013). Furthermore, the block cipher KASUMI designed as a slight modification of MISTY1 is used in the GSM/3G mobile networks, which makes it one of the most widely used block ciphers today.

Up to now, many cryptanalytic methods were used to evaluate the security of MISTY1 such as higher order differential attack, impossible differential attack, integral attack, and multi-dimensional zero correlation linear attack. The main previous attacks are as follows. Tsunoo et. al. proposed 46-th order differential and showed 7-round MISTY1 with 4 FL layers was attackable with $2^{54.1}$ chosen plaintexts and $2^{120.7}$ encryptions (Y. Tsunoo and Kawabata, 2008). Jia et. al. constructed a 7-round impossible differential and mounted impossible differential attack on 7-round MISTY1 with 3 FL layers (Jia and Li, 2012). Yi presented zero-correlation linear attack on 7-round MISTY1 with 4 FL layers, that requires $2^{62.9}$ known plaintexts and 2^{118} encryptions (Yi and Chen, 2014). Todo introduced Integral attack by division property, and showed that the secret key of the full MISTY1 can be recovered with $2^{63.58}$ chosen plaintexts and 2^{121} time complexity (Todo, 2015). Bar On improved the attack proposed by Todo, and presented full MISTY1 was attackable with 2^{64} chosen plaintexts and $2^{69.5}$ encryptions (Bar-On, 2015a).

Most of the previous attacks aimed at maximizing the number of attacked rounds, and as a result, their complexities are highly impractical. In this paper, we focused on evaluating the security of MISTY1 in terms of practical-time complexity. The previous practical-time attack was proposed by Hatano et. al. (Y. Hatano and Kaneko, 2004), and Dunkelman et. al. (Dunkelman and Keller, 2013), respectively. The best practical-time attack was higher order differential attack on 5-round MISTY1 with 4 FL layers.The necessary computational complexity by using higher order differential can be estimated as sum of the following 2-steps.

1. Preparation of data
2. Key recovery

The order of differential affects both steps. Therefore, it is very important to discover the lower order differential characteristics to reduce the complexity for an attack. The results we obtain are the following.

1. We implemented the 46th-order differential for 4-round MISTY1 introduced in (Y. Tsunoo and Kawabata, 2008) on a computer which mounted Graphics Processing Unit (GPU) co-processors

Sugio, N., Igarashi, Y., Kaneko, T. and Higuchi, K.
A Practical-time Attack on Reduced-round MISTY1.
DOI: 10.5220/0005652202350242
In *Proceedings of the 2nd International Conference on Information Systems Security and Privacy (ICISSP 2016)*, pages 235-242
ISBN: 978-989-758-167-0

Table 1: Summary of single-key attacks on MISTY1.

Rounds	FL layers	Data	Time	Attack algorithm	Reference
5	4	2^{22} CP	2^{28}	Higher Order Differential	(Y. Hatano and Kaneko, 2004)
6	4	2^{51} CP	$2^{123.4}$	Impossible Differential	(Dunkelman and Keller, 2008)
6	4	$2^{53.7}$ CP	$2^{53.7}$	Higher Order Differential	(Y. Tsunoo and Kawabata, 2008)
6	4	2^{43} CP	$2^{43.31}$	Higher Order Differential	**Section5**
7	0	$2^{50.2}$ KP	$2^{114.1}$	Impossible Differential	(Dunkelman and Keller, 2008)
7	3	2^{58} KP	$2^{124.4}$	Impossible Differential	(Jia and Li, 2012)
7	4	$2^{62.9}$ KP	2^{118}	Multi-Zero Correlation	(Yi and Chen, 2014)
7	4	$2^{54.1}$ CP	$2^{120.7}$	Higher Order Differential	(Y. Tsunoo and Kawabata, 2008)
7	5	$2^{51.45}$ CP	2^{121}	Higher Order Differential	(Bar-On, 2015b)
8	5	$2^{63.58}$ CP	2^{121}	Integral by division property	(Todo, 2015)
8	5	2^{64} CP	$2^{69.5}$	Integral by division property	(Bar-On, 2015a)

CP: Chosen Plaintexts, KP : Known Plaintexts.

Table 2: The Key Scheduling of MISTY1.

KO_{i1}	KO_{i2}	KO_{i3}	KO_{i4}	KI_{i1}	KI_{i2}	KI_{i3}	KL_{i1}	KL_{i2}
K_i	K_{i+2}	K_{i+7}	K_{i+4}	K'_{i+5}	K'_{i+1}	K'_{i+3}	$K_{\frac{i+1}{2}}$ (odd i) $K'_{\frac{i}{2}+2}$ (even i)	$K'_{\frac{i+1}{2}+6}$ (odd i) $K_{\frac{i}{2}+4}$ (even i)

and found 16-bits of the above characteristic was always 0. We gradually reduced the order of differentials for 4-round MISTY1 by computer experiment, and discovered new 38-th order differential characteristics for 4-round MISTY1 which held 7-bits of those differential characteristics 0[1].

2. We can attack 6-round MISTY1 with 4 FL layers by using the 38-th order differential characteristic. The complexity for the attack needs 2^{43} chosen plaintexts and $2^{43.31}$ encryptions. Our method can reduce the necessary number of chosen plaintexts and the computational cost for the attack of 6-round MISTY1 with 4 FL layers illustrated in (Y. Tsunoo and Kawabata, 2008) by a factor of 2^{10}. This is the best practical-time attack on 6-round MISTY1. Summary of main attacks on MISTY1 are shown in Table 1.

The remainder of this paper is organized as follows. Section 2 gives a brief introduction of MISTY1. Section 3 explains higher order differentials and its application for an attack. Section 4 shows previous higher order differentials and presents a new higher order differential for 4-round MISTY1. Section 5 proposes higher order differential attack on 6-round MISTY1 with 4 FL layers. Section 6 summarizes this paper.

2 MISTY1

MISTY1 is a Feistel type 64-bit block cipher supporting secret key length of 128 bits. MISTY1 was proposed by Matsui in 1997 (Matsui, 1997). The number of rounds which designer recommends is 8. MISTY1 achieves a provable security against differential cryptanalysis and linear cryptanalysis with round function FO. Designer adds on an auxiliary function FL in order to become secure against other attacks.

Figure 1 shows the main structure and components of the cipher. The round function FO_i $(1 \leq i \leq 8)$ is a variant of a 3-round Feistel construction which has 16-bit bijective function FI_{ij} $(1 \leq j \leq 3)$ and 16-bit extended key KO_{ij} $(1 \leq j \leq 4)$, KI_{ij} $(1 \leq j \leq 3)$. FI_{ij} is a variant of a 3-round Feistel construction and its input is divided into left 9-bit data and right 7-bit data, which are transformed by bitwise XOR operations denoted by the symbol $\oplus$ and substitution tables S7 and S9. KI_{ij1} and KI_{ij2} are left 7-bit data and right 9-bit data of KI_{ij}, respectively. The key dependent linear function FL_i are composed of bitwise AND operation denoted by the symbol $\cap$, OR operation denoted by the symbol $\cup$, XOR operations and KL_{ij} $(1 \leq j \leq 2)$.

The key schedule of MISTY1 takes the 128-bit secret key K to generate extended keys. Let K_i $(1 \leq i \leq 8)$ be the i-th (from left) 16-bit data of the secret key K, and let KO_i $(1 \leq i \leq 8)$ be the output of FI_{ij} where the input of FI_{ij} is K_i and the key KI_{ij} is K_{i+1}. Also, identify K_9 with K_1. The correspondence between the

[1] The characteristic of 38-th order differential equals to the characteristic of 46-th order differential for 4-round MISTY1 estimated in (Y. Tsunoo and Kawabata, 2008)

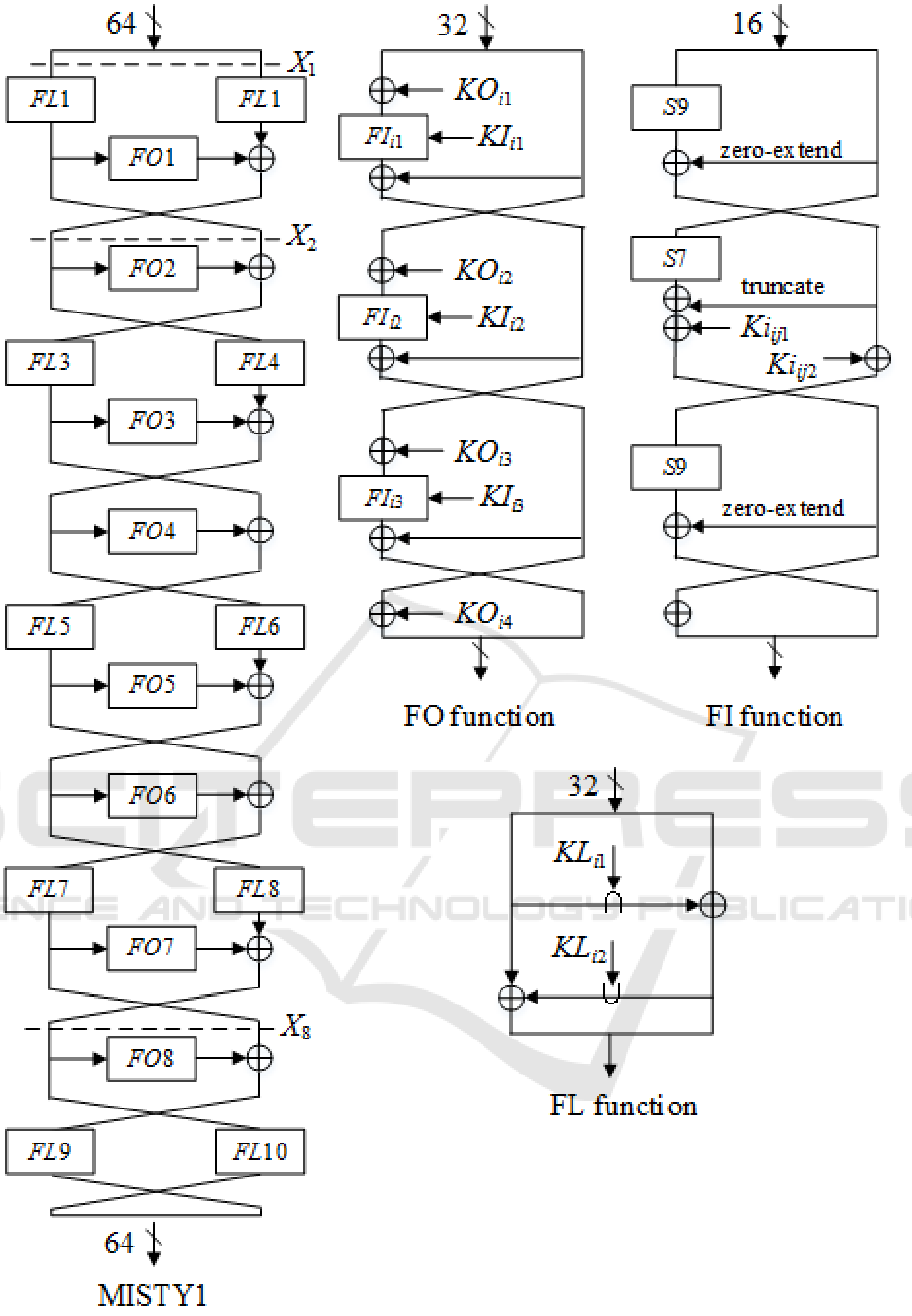


Figure 1: Outline of MISTY1.

symbols KO_{ij}, KI_{ij}, KL_{ij} and the actual key is shown in Table 2. Here, K_i' is the output of $FI_{i,j}$ where the input is K_i and the key is K_{i+1}.

2.1 Notations Used in This Paper

We use the following notations for intermediate values during the MISTY1 encryptions process.

- The plaintext and ciphertext are denoted by P and C. The left 32-bit value of P is denoted by P_L and the right 32-bit value of P is denoted by P_R. The left 32-bit value of C is denoted by C_L and the right 32-bit value of C is denoted by C_R, respectively.
- The input of i-th round $(1 \leq i \leq 8)$ are denoted by X_i. We denote the intermediate value after appli-

cation of FL functions by X_i'.

- Let Z be a intermediate variable. $Z[k]$ denotes k-th bit of Z and $Z[k-l]$ denotes bits from k to l of Z respectively.

3 HIGHER ORDER DIFFERENTIAL ATTACK

This section gives an overview of higher order differential attack.

3.1 Higher Order Differential (Lai, 1994)

Let $E(\cdot)$ be an encryption function as follows.

$$Y = E(X;K) \tag{1}$$

where $X \in \mathrm{GF}(2)^n$, $K \in \mathrm{GF}(2)^s$, $Y \in \mathrm{GF}(2)^m$. For a block cipher, X, K and Y denote plaintext, key and ciphertext respectively. Let $\{a_1, a_2, \cdots, a_i\}$ be a set of linearly independent vectors in $\mathrm{GF}(2)^n$ and $V^{(i)}$ be a sub-space spanned by these vectors. We define $\Delta^{(i)}_{V^{(i)}} E(X;K)$ as an i-th order differential of $E(X;K)$ with respect to X as follows.

$$\Delta^{(i)}_{V^{(i)}} E(X;K) = \bigoplus_{A \in V^{(i)}} E(X \oplus A; K) \tag{2}$$

In the following, we abbreviate $\Delta^{(i)}_{V^{(i)}}$ as $\Delta^{(i)}$, when it is clearly understood. In this paper, we use the following properties of the higher order differential.

Property 1. If the degree of $E(X;K)$ with respect to X equals to d, then

$$deg_X\{E(X;K)\} = d \Leftrightarrow \begin{cases} \Delta^{(d+1)} E(X;K) = 0 \\ \Delta^{(d)} E(X;K) = const \end{cases} \tag{3}$$

Property 2. Higher order differential has a linear property on XOR sum. That means d-th order differential of the sum of each function equals to the sum of d-th order differential of each function.

$$\begin{aligned} &\Delta^{(d)}\{E(X_1;K_1) \oplus E(X_2;K_2)\} = \\ &\Delta^{(d)} E(X_1;K_1) \oplus \Delta^{(d)} E(X_2;K_2) \end{aligned} \tag{4}$$

3.2 Attack Equation

Consider an R-round iterative block cipher. Let $\mathrm{H}_{R-1}(X) \in \mathrm{GF}(2)^m$ be a part of the $(R-1)$-th round output and $C(X) \in \mathrm{GF}(2)^n$ be the ciphertext corresponding to the plaintext $X \in \mathrm{GF}(2)^n$. $\mathrm{H}_{R-1}(X)$ is expressed as follows.

$$\mathrm{H}_{R-1}(X) = F_{R-1}(\cdots F_2((F_1(X;K_1);K_2), \cdots, K_{R-1}) \tag{5}$$

where $K_i \in \mathrm{GF}(2)^s$ be the i-th round key and $F(\cdot)$ be a function of $GF(2)^n \times GF(2)^s \rightarrow GF(2)^m$.

If the degree of $\mathrm{H}_{R-1}(X)$ with respect to X is d, we have the following equation from Property 1.

$$\Delta^{(d+1)} \mathrm{H}_{R-1}(X) = 0 \tag{6}$$

This equation holds with probability 1.

Let $\tilde{F}(\cdot)$ be a decoding function that calculates $\mathrm{H}_{R-1}(X)$ from a ciphertext $C(X) \in \mathrm{GF}(2)^n$.

$$\mathrm{H}_{R-1}(X) = \tilde{F}(C(X);K_R) \tag{7}$$

where $K_R \in \mathrm{GF}(2)^s$ denotes the R-th round key to decode $\mathrm{H}_{R-1}(X)$ from $C(X)$. From equation (6), (7) and (2), we can derive following equation.

$$\bigoplus_{A \in V^{(d+1)}} \tilde{F}(C(X \oplus A);K_R) = 0 \tag{8}$$

We can determine K_R by solving (8). In the following, we refer to equation (8) as an attack equation for key recovery.

3.3 Algebraic Method

Shimoyama et al. proposed an effective method of solving equation (8) (T. Shimoyama and Tsujii, 1999). This method, called algebraic method in this paper, expands equation (8) as boolean polynomials over GF(2), and linearizes by treating every higher order variables like $k_i k_j$ with new independent variables like k_{ij}. In the following, we use the term *linearized attack equation* to refer to an attack equation that is regarded as a linear equation.

Let L be the number of unknowns in the linearized attack equation (8). Since the equation (8) is derived by using an m-bit sub-block, we can rewrite equation (8) as follows.

$$\mathbf{Ak} = \mathbf{b}\ ,\ \mathbf{k} = {}^t(k_1, k_2, \ldots, k_1k_2, \ldots, k_1k_2k_3, \cdots) \tag{9}$$

where $\mathbf{A}$, $\mathbf{b}$, and $\mathbf{k}$ are the $m \times L$ coefficient matrix, the m-dimensional vector, and the L-dimensional vector over GF(2). $\mathbf{k}$ denotes linearized unknowns that are expressed as monomials of the R-th round key K_R.

We can obtain m linearized attack equations from one $(d+1)$-th order differential because equation (8) is an m-bit equation. Therefore we need $\lceil L/m \rceil$ sets of the $(d+1)$-th order differential for the unique solution.

Since one set of $(d+1)$-th order differential requires 2^{d+1} chosen plaintexts, the necessary number of plaintexts D for the determination of a key is estimated as

$$D = 2^{d+1} \times \left\lceil \frac{L}{m} \right\rceil \tag{10}$$

If we use the same technique shown in (T. Shimoyama and Tsujii, 1999), equation (9) requires $2^{d+1} \times (L+1)$ times of $\tilde{F}(\cdot)$ calculations. Since we have to prepare $\lceil L/m \rceil$ sets of $(d+1)$-th order differentials to determine $\mathbf{k}$, the computational cost[2] is estimated as

$$T = 2^{d+1} \times (L+1) \times \left\lceil \frac{L}{m} \right\rceil \tag{11}$$

Hatano et. al. proposed the optimization for algebraic method by analyzing the number of independent unknowns $l(\leq L)$ in equation (8) (Y. Hatano and Kaneko, 2004). If we can analyze the number of independent unknowns $l(\leq L)$ in equation (8), the $\lceil l/m \rceil \times l$ coefficient matrix $\mathbf{A}_{op}$ and the $\lceil l/m \rceil$-dimensional vector $\mathbf{b}_{op}$ is sufficient for solving the linearized attack equation.

4 HIGHER ORDER DIFFERENTIALS FOR REDUCED ROUND MISTY1

This section explains the previous results of higher order differential characteristic of MISTY1 and illustrates new higher order differential characteristics which we discovered. Here, α and β denote fixed and variable sub-block respectively. For example, 64-bit variable Y consisting of a 3-bit variables a-th, b-th, and c-th bit of sub-block β $(0 \leq a,b,c \leq 63,\ a \neq b \neq c)$ can be denoted as $Y = \{Y[i] \in \alpha,\ Y[j] \in \beta \mid i \neq j,\ j = a,b,c,\ a \neq b \neq c\}$. A 3-rd order differential of intermediate variable $Z[k-l]$ by using Y can be denoted as

$$\begin{gathered} Y = \{Y[i] \in \alpha,\ Y[j] \in \beta \,|\, i \neq j, j = a,b,c, a \neq b \neq c\} \\ \Delta^{(3)}_{V^{(3)}} Z[k-l] \end{gathered} \tag{12}$$

where $0 \leq i,j,k,l \leq 63$ and $V^{(3)}$ is a subspace based on variable sub-block $Y[j]$.

4.1 Previous Results

Hatano et. al. proposed 14-th order differential of 3-round MISTY1 with FL functions (Y. Hatano and Kaneko, 2004).

$$\begin{gathered} P = \{P[i] \in \alpha,\ P[j] \in \beta \mid i \neq j,\ 0 \leq j \leq 6, \\ 16 \leq j \leq 22\} \\ \Delta^{(14)}_{V^{(14)}} X_4[63-57] = 0 \end{gathered} \tag{13}$$

where $V^{(14)}$ is the subspace based on variable sub-block $P[22-16, 6-0]$.

Tsunoo et. al. proposed 46-th order differential which was a 1-round extension of the 14-th order differential to the direction to plaintext. Due to the Feistel structure of MISTY1, the 3-round 14-th order differential can be extended to a 4-round 46-th order differential by taking all the 2^{32} possible values in the previous round. Thus, the following theorem could be derived.

Theorem. For four consecutive rounds of MISTY1 with FL functions that starts at 1-st round, the following equation independently holds under any fixed value of the key, constant value of the $P[i]$ $(39 \leq i \leq 47,\ 55 \leq i \leq 63)$ and $V^{(46)}$ is the subspace based on variable sub-blockP_R, $P[j]$ $(\ 32 \leq j \leq 38,\ 48 \leq j \leq 54)$.

$$\begin{gathered} P = \{P[i] \in \alpha,\ P[j], P_R \in \beta \mid 39 \leq i \leq 47, \\ 55 \leq i \leq 63,\ 32 \leq j \leq 38,\ 48 \leq j \leq 54\} \\ \Delta^{(46)}_{V^{(46)}} X_5[63-57] = 0 \end{gathered} \tag{14}$$

4.2 New Higher Order Differentials for MISTY1

In this subsection, we describe the new higher order differential characteristics for MISTY1 which were discovered by computer experiment.

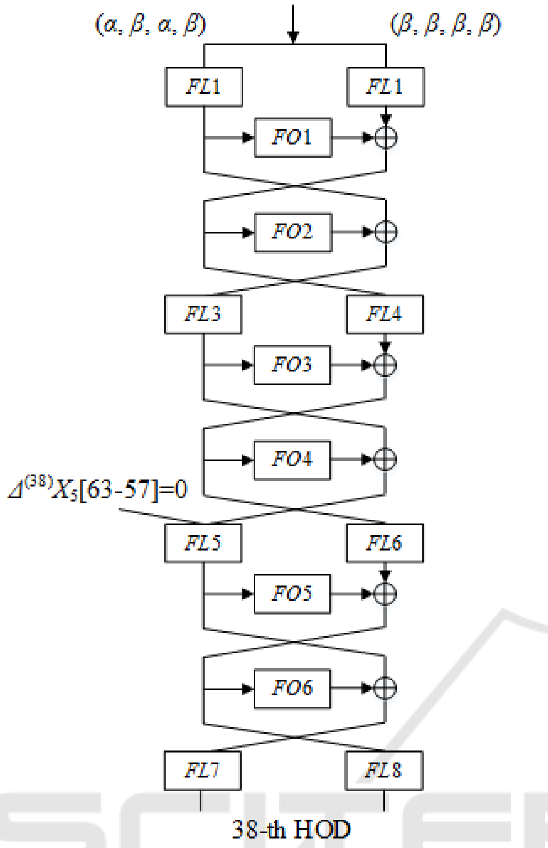
46-th Order Differential Characteristic. We implemented 46-th order differential described in equation (14) on a computer which mounts Graphics Processing Unit (GPU) co-processors and found the following 46-th order differential characteristic.

$$\Delta^{(46)}_{V^{(46)}} X_5[63-48] = 0 \tag{15}$$

Although we verified equation (15) with 10 different keys and fixed sub-blocks, equation (15) always held. Since the provability of 10 sets 16-bit random variables incidentally becomes 0 is 2^{-160}, we don't think equation (15) accidentally holds.

38-th Order Differential Characteristic. We gradually reduced the order of differentials in equation (15) and discovered the new higher order

[2] This computational cost ignores the complexity of solving linearized attack equation, because the computational cost is negligible as long as coefficient matrix size is small.



β shown in 32-bit state (α, β, α, β) are 3-bits variables out of 7-bits sub-blocks, respectively.

Figure 2: A new 38th order differential of MISTY1.

differential characteristics as follows.

$$\begin{gathered} P = \{P[i] \in \alpha, P[j,k], P_R \in \beta \mid i \neq j, i \neq k \\ 32 \leq i \leq 63,\ j = k+16, \\ k = h_1, h_2, h_3\ (32 \leq h_1 < h_2 < h_3 \leq 38)\} \\ \Delta^{(38)}_{V^{(38)}} X_5[63-57] = 0 \end{gathered} \tag{16}$$

where h_1, h_2 and h_3 are bit patterns of variable sub-block. The variations of 38-th order differential illustrated in equation (16) exist $\binom{7}{3}$=35 patterns.

Other Characteristics. We searched for the new higher order differential characteristics on 3-round, 4-round and 5-round MISTY1 respectively. The results are presented in Table 3.

5 38-TH ORDER DIFFERENTIAL ATTACK ON REDUCED ROUND MISTY1

In this section we apply 38-th order differential characteristic to attack 6-round MISTY1 with 4 FL layers.

Table 3: The results of higher order differential characteristics on MISTY1.

*We abbreviate $\Delta^{(i)}_{V^{(i)}} X_4[63-57] = 0$ $(i = 7, 14)$ as $X_4[63-57] = 0$ in the table. This abbreviation is same other characteristics. The symbol 'N/A' means that we couldn't find a higher order differential characteristic which satisfied $\Delta^{(i)}_{V^{(i)}} X_4[63-41] = 0$ $(1 \leq i \leq 31)$. (*1) is discovered by Hatano (Y. Hatano and Kaneko, 2004). (*2)is discovered by Tanaka (H. Tanaka and Kaneko, 1999). (*3)is discovered by Igarashi (Igarashi and Kaneko, 2008).*

Rounds	*i*-th order diff.		Output
	FL	without FL	
3	$14^{(*1)}$	$7^{(*2)}$	$X_4[63-57] = 0$
3	18	10	$X_4[63-48] = 0$
3	N/A	26	$X_4[63-41] = 0$
3	32	31	$X_4[63-32] = 0$
4	38	$32^{(*3)}$	$X_5[63-57] = 0$
4	44	36	$X_5[63-48] = 0$
4	50	47	$X_5[63-41] = 0$
4	-	48	$X_5[63-32] = 0$
5	-	~53	**Unknown**

We estimate the complexity for an attack by means of same procedure described in (Y. Tsunoo and Kawabata, 2008). Using the chosen plaintext denoted in equation (16), we have the following attack equation by assuming $KL_{52}[15-9] = 0x7f$. (See figure 3.)

$$\begin{gathered} \bigoplus_{A \in V^{(38)}} \{FO_6(X_6[63-32]; KO_{61}, KO_{62})[31-25] \oplus \\ FL_7^{-1}(C_L(X \oplus A); KL_{72})[31-25]\} = 0, \\ X_6[63-32] = FL_8^{-1}(C_R(X \oplus A); KL_{81}, KL_{82}) \end{gathered}$$

where FL^{-1} means an inverse function of FL. We divide the attack equation into seven kinds of 1-bit attack equations in order to increase the success probability. Assuming $KL_{52}[15-9] = 0x7f$, the seven kinds of 1-bit attack equations are written as

$$\begin{gathered} \bigoplus_{A \in V^{(38)}} \{FO_6(X_6[63-32]; KO_{61}, KO_{62})[i] \oplus \\ FL_7^{-1}(C_L(X \oplus A); KL_{72})[i]\} = 0, \\ X_6[63-32] = FL_8^{-1}(C_R(X \oplus A); KL_{81}, KL_{82}) \end{gathered} \tag{17}$$

where $25 \leq i \leq 31$. Each 1-bit attack equation holds with probability 2^{-1}. Using those equations, we can determine the key with probability $1 - 2^{-7}$, which means $KL_{52}[15-9] \neq 0x00$. Otherwise we can determine $KL_{52}[15-9] = 0x00$.

After linearization of the attack equation which consists of 7-bits width, we obtain the total number of unknown variables $L = 1665$ in a system of linear equations. If any of the unknown variables have linear sum relations, the complexity for attack can be reduced. In this paper, we chose independent un-

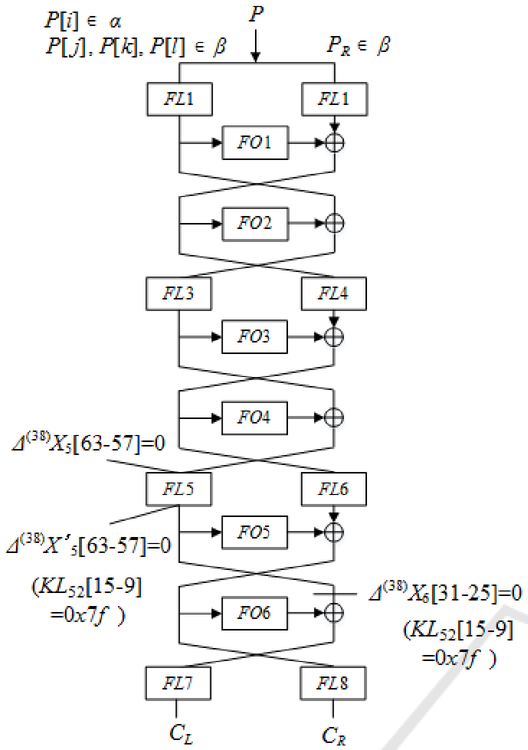


Figure 3: An attack on 6-round MISTY1 by using 38-th order differential.

known variables[3] $l=189$ as worst-case. Because every linearized attack equation was derived by 1-bit key $KL_{52}[i]$ assumption, additional 10 linear equations are needed to remove all false keys. From equation (10), the necessary number of chosen plaintexts D is estimated as

$$D=2^{38}\times\left\lceil\frac{189+10}{m}\right\rceil$$

where $m=1$ since the attack solves equation (17) for each bit. Namely, $D=2^{38}\times 199\approx 2^{45.7}$. Here, we can reduce the number of plaintexts D as follows. Let us consider a 43-rd order differential denoted equation (18).

$$\begin{aligned}&P=\{P[i]\in\alpha,P[j,k,l],P_R\in\beta\,|\,i\neq j,k,l,j=k+16\}\\&k=h_1,\sim,h_5\ (32\leq h_1<h_2<h_3<h_4<h_5\leq 38)\\&l=h_6\ (32\leq h_6\leq 38,\ \text{or}\ 48\leq h_6\leq 54)\end{aligned}\tag{18}$$

A 43-rd order differential described in equation (18) can be used to construct $2^5\times\binom{5}{3}$=320 sets of 38-th order differential. The explanation is as follows. The combination of choosing variable bit sub-blocks of 38-th order differential in equation (18) is $\binom{5}{3}$, because we have to choose 3-bits from $(h_1,\sim,h_5)$ as variable bit sub-blocks. When we regard one set of 43-rd order differential as some sets of 38-th order differential, the number of fixed bit sub-blocks in 43-rd order differential is five. 5-bits pattern includes 2^5 possible values. Thus, a 43-rd order differential illustrated in equation (18) generates $2^5\times\binom{5}{3}$=320 $>$ 199 sets of 38-th order differential. Therefore, the necessary number of chosen plaintexts D for key recovery is estimated as 2^{43}.

Now, we consider the time complexity for this attack. The necessary computational cost for an attack can be estimated as sum of the following 2-steps.

1. Preparation of ciphertexts
2. Key recovery

The time complexity for a preparation of ciphertexts T_C is estimated as 2^{43} 6-round MISTY1 encryptions. From equation (11), the complexity for key recovery is estimated as follows.

$$T_M=2^{38}\times(1665+1)\times\left\lceil\frac{189+10}{1}\right\rceil$$

We can reduce the complexity by using a modulo 2 frequency distribution table. This table counts ciphertext values appearing an odd number of times since performing an XOR operation on the same value an even number of times results in a value of 0. We prepare 2 kinds of tables whose size are 2^{18} with respect to 18-bits $C_L[32-23,15-7]$ for 2 S9-boxes and 2^{14} with respect to 14-bits $C_L[22-16,6-0]$ for 2 S7-boxes respectively. The cost for generating these tables is

$$T_t=2^{38}\times 2\times 199$$

table look-ups. If the computational cost for one table look-up equals to the cost for a S7, S9 look-up, T_t is estimated as

$$T_t=\frac{2^{38}\times 2\times 199}{6\times 9}\approx 2^{40.9}$$

encryptions because 6-round MISTY1 has 6×9 S-boxes. The computational cost from equation (11) by using these tables is estimated as

$$\begin{aligned}T'_M=&(2\times 2^{18}+2\times 2^{14})\times(1665+1)\times\left\lceil\frac{189+10}{1}\right\rceil\\&\approx 2^{37.5}\end{aligned}$$

S-box look-ups. T'_M is estimated as $\frac{2^{37.5}}{6\times 9}\approx 2^{31.8}$ encryptions. The complexity for solving a system of equations resulting from linearization is negligible. The overall time complexity T is estimated as

$$T=T_C+T_t+T'_M=2^{43}+2^{40.9}+2^{31.8}\approx 2^{43.31}$$

times of 6-round MISTY1 encryptions. The time complexity T is dominated by the encryption of plaintexts.

[3] It should be noted that the number of independent unknown variables depends on the exact bit in attack equation. See (Y. Tsunoo and Kawabata, 2008) in detail.

6 CONCLUSIONS

In this paper, we focused on evaluating the security of MISTY1 in terms of practical-time complexity. We implemented the 46th-order differential characteristic for 4-round MISTY1 introduced in (Y. Tsunoo and Kawabata, 2008) on a computer which mounts GPU co-processors. We found 16-bits of 46-th order differential characteristic was 0. We discovered the new 38-th order differential characteristic for 4-round MISTY1 whose characteristic is equal to the characteristic estimated in (Y. Tsunoo and Kawabata, 2008).

We applied the 38-th order differential characteristic to attack 6-round MISTY1 with 4 FL layers. The complexity for attack needs 2^{43} chosen plaintexts and $2^{43.31}$ encryptions. By using 38-th order differential, we can reduce the necessary number of data and time complexity for an attack on 6-round MISTY1 with 4 FL layers by a factor of 2^{10}. This is the best practical-time attack on 6-round MISTY1.

REFERENCES

Bar-On, A. (2015a). A 2^{70} attack on the full misty1. In *IACR ePrint Archive, 2015/746*. IACR.

Bar-On, A. (2015b). Improved higher-order dierential attacks on misty1. In *Fast Software Encryption 22nd International Workshop*. Springer.

CRYPTREC (2013). CRYPTREC ciphers list, http://www.cryptrec.go.jp/english/method.html.

Dunkelman, O. and Keller, N. (2008). An improved impossible differential attack on misty1. In *ASIACRYPT, volume 5350 of LNCS, pages 441-454.* Springer.

Dunkelman, O. and Keller, N. (2013). Practical-time attacks against reduced variants of misty1. In *IACR ePrint Archive, 2013/431*. IACR.

H. Tanaka, K. H. and Kaneko, T. (1999). Strength of misty1 without fl function for higher order differential attack. In *Applied Algebra, Algebraic Algorithms and Error-Correcting Codes, volume 1719 of LNCS, pp 221-230.* Springer.

Igarashi, Y. and Kaneko, T. (2008). The 32nd-order differential attack on misty1 without fl functions. In *2008 International Symposium on Information Theory and its Applications, No.W-TI-4-4, pages 1503-1508.* IEICE.

ISO (2010). ISO/IEC 18033-3 information technology - security techniques - encryption algorithms - part 3: Block ciphers.

Jia, K. and Li, L. (2012). Impossible differential attacks on reduced-round misty1. In *13th International Workshop, WISA, volume 7690 of LNCS pages 1527.* Springer.

Lai, X. (1994). Higher order derivatives and differential cryptanalysis. In *Communications and Cryptography, pages 227-233.*

Matsui, M. (1997). New block encryption algorithm misty. In *Fast Software Encryption 4th International Workshop*. Springer.

T. Shimoyama, Siho Moriai, T. K. and Tsujii, S. (1999). Improving higher order differential attack and its application to nyberg-knudesen's designed block cipher. In *IEIEC Transactions, Fundamentals, Vol.E82-A, No.9, pages 1971-1980.* IEICE.

Todo, Y. (2015). Integral cryptanalysis on full misty1. In *CRYPTO 2015 volume 9215 of LNCS, pages 413-432.* Springer.

Y. Hatano, H. T. and Kaneko, T. (2004). Optimization for the algebraic method and its application to an attack of misty1. In *IEIEC Transactions, Fundamentals, Vol.E87-A, No.1, pages 18-27.* IEICE.

Y. Tsunoo, Teruo Saito, M. S. and Kawabata, T. (2008). Higher order differential attacks on reduced-round misty1. In *ICISC, volume 5461 of LNCS, pages 415-431.* Springer.

Yi, W. and Chen, S. (2014). Multidimensional zero-correlation linear attacks on reduced-round misty1. In *CoRR, abs/1410.4312*.