Two Body Dynamic Model for Speed Skating Driven by the Skaters Leg Extension

E. van der Kruk, H. E. J. Veeger, F. C. T. van der Helm and A. L. Schwab
Department of Biomechanical Engineering, Delft University of Technology, Mekelweg 2, Delft, The Netherlands

Keywords: Speed Skating, Multi-Body Dynamics, Mathematical Model, Verification.

1 OBJECTIVES

In speed skating forces are generated by pushing in a sideward direction against an environment, which moves relative to the skater. De Koning et al. (1987) showed that there is a distinct difference in the coordination pattern between (elite) speed skaters. Models can help to give insight in this peculiar technique and ideally find an optimal motion pattern for each individual speed skater. Currently there are three models describing and optimizing the behaviour and performance of skaters, of which only two are relevant in terms of coordination patterns (Allinger and Bogert 1997; Otten 2003). However, none of them have been shown to accurately predict the observed coordination pattern via verification with empirical kinetic and kinematic data. Therefore, the objectives of this study are to present a verified three dimensional inverse skater model with minimal complexity, based on the idea of (Cabrera et al. 2006), modelling the speed skating motion on the straights. The model is driven by the changing distance between the torso and the skate (further referred to as the leg extension), which is also the true input of the skater to generate a global motion. This input, which is indirectly also a measure of the knee extension of the skater, is a variable familiar to the speed skaters and coaches. In this extended abstract we verify this novel model for two strokes (left and right) of one skater through correlation with observed kinematics and forces.

2 METHODS

2.1 Model Description

The model presented in this section simulates the upper body transverse translation of the skater together with the forces exerted by the skates on the ice. The model input is the measured leg extension (coordination pattern). Based on empirical data from previous studies using elite skaters, the double stance phase, the time in which both skates are in contact with the ice, is rather short. For the sake of simplicity, we assume that there is only one skate at a time in contact with the ice, alternating left and right. The point of alternation is defined as the moment in time where the forces exerted on both skates are equal. Furthermore the arm movements and the rotations of the upper body are assumed to be of marginal effect on the overall power and are therefore neglected. Based on these assumptions, the skater can be considered as a combination of two point masses, which are situated at the upper body (mass B) and at each (active) skate (mass S). The body mass of the skater is distributed over the two active masses by a constant mass distribution coefficient (η) to compensate for the shift in the center of mass position during the speed skating movement. Each mass has three degrees of freedom. The set of parameters is restricted to the position coordinates of mass B (x_b, y_b, z_b), two translations in the transverse plane of mass S with the position coordinates (x_s, y_s) (because the skate is assumed to be on the ice, making z_s=0 at all times) and one rotation in the same plane, the steer angle (φ_S). The orientation of the skate is of importance for the constraint forces acting on the skate. All other rotations of the skates are ignored.

Since we want to obtain a model which is driven by generalized (local) coordinates, we introduce a set of generalized coordinates q_i (Figure 2), so the global coordinates can be expressed in terms of leg

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extension via the kinematic relation $x_i = f(q_i)$. These generalized coordinates consist of the leg extension (w_s, u_s, v_s, θ_s)(Figure 2), that is actively controlled by the skater and therefore serves as the input coordinates to the model and the generalized coordinates of the upper body (u_b, v_b), which will be a result of the system dynamics (equal to x_b, y_b)

The equations of motion are expressed in generalized coordinates, so that the constraints are inherently fulfilled. Since we assume no lateral slip, a non-holonomic constraint acting in the lateral direction of the skate was added, causing the undetermined external force λ perpendicular to the skate blade in the transverse plane. This leaves a model with two degrees of freedom in position and only one in velocity. The known external forces acting on the model are the air frictional forces and the ice frictional forces.

2.2 Solving the Model

The model is solved in two steps. First, since the parameters (w_s, u_s, v_s, θ_s) are considered inputs and the air frictional forces acting on the upper body are assumed to be known, the constraint force λ and the transverse position of the upper body (u_b, v_b) can be determined by means of integration (Runge Kutta method), starting from the initial condition $\left(x_{b,0}, y_{b,0}, \dot{x}_{b,0}, \dot{y}_{b,0}\right)$. The constraint is fulfilled for each integration step by a projection method. Hereby a minimization problem was formulated, concerning the distance from the predicted solution to the solution which is on the constraint surface. The global coordinates x_i, which are the global positions of the upper body and the skate, can then be found analytically via the kinematic relation. Finally, with the found upper body position and λ, the local forces acting on the skate can be solved analytically such that a complete two-body dynamic model of the skater has been established.

2.3 Model Verification

The purpose of the model verification is to quantify the error between the simulated data and the measured forces and positions. The forces were measured by a set of instrumented klapskates (van der Kruk et al. 2015). The position of the masses

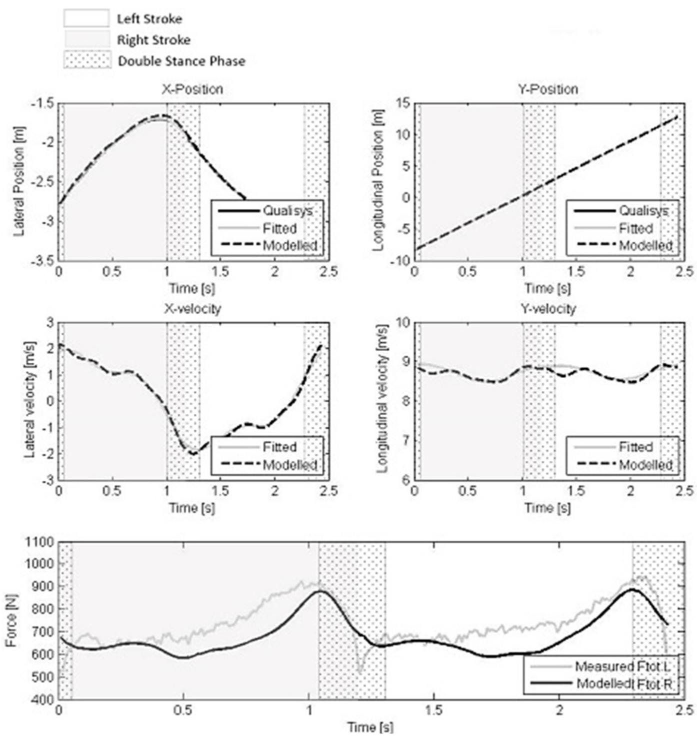

Figure 1: The fitted data of two consecutive strokes for position and velocity of mass B and the total force. The grey area indicates a left stroke, the white area a right stroke, the pattern indicates the double stance phase as measured.. Y is in line with the skate lane, X is perpendicular to the skate lane.

Table 1: Error between the simulated data and the measured data.

	RMSE	Mean Error	SD Error	Jmin
x-position	0.025 [m]	0.019 [m]	0.016 [m]	0.000310
y-position	0.045 [m]	0.033 [m]	0.031 [m]	0.000004
x-velocity	0.116 [m/s]	0.002 [m/s]	0.117 [m/s]	0.0028
y-velocity	0.096 [m/s]	-0.048 [m/s]	0.084 [m/s]	0.000123
F	82 [N]	-54.0 [N]	61.8 [N]	0.0305

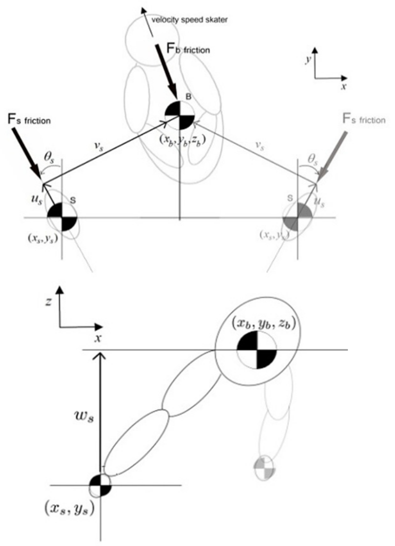

Figure 2: The global and generalized coordinates of the two-mass skater model. Leg extension consists of vertical distance (w_s) and horizontal distance between the mass S and mass B in heading direction (u_s) and perpendicular to heading direction (v_s) and the heading of the skate (θ_s) (orientation).

was measured by a motion capture system on 50 meter of the straight part of the rink, with a passive marker on the Lateral Malleolus (representing mass S) and on the back near the Sacrum (representing mass B). A parametric function was fitted to the recorded data, consisting of a linear and a geometric function, which could be differentiated twice in order to obtain velocity and acceleration data. The air and ice friction were estimated based on previous papers (van Ingen Schenau 1982; De Koning et al. 1992). The body mass was assumed to be distributed equally over mass S and mass B. In this abstract the data of one Dutch elite female speed skater are presented (65kg, 1.75m).

3 RESULTS

The results show that the model estimated the forward position and velocity of mass B the best (Jmin (based on (Cabrera et al. 2006))), followed by the lateral position and velocity, which were all within 1% accuracy. The model was least accurate for the force determination (Table 1). The forces were consistently estimated too low (Figure 2, bottom graph).

4 DISCUSSION

4.1 Kinematic Complexity

Preliminary results presented in this abstract showed that the model, despite the simplicity, was able to simulate the upper body movement accurately. The forces on the skate were underestimated, which can be explained by the simplicity of the model. The skater was considered as a combination of two point masses, which moreover were situated at fixed positions on the body parts, with each a mass half of the total body weight. In reality there might however be a different mass distribution and the CoM of these bodies move throughout the movement. Additionally, the changing distance between the two masses (leg extension), was modelled piston-like without any damping. Optimization of the mass distribution and determination of the true CoM (with a full body marker set) will improve the model estimation. The model would also benefit from improved acceleration measurements by adding IMU's.

Although the double stance phase was neglected based on previous papers, the collected data showed that the double stance phase is apparent in about 13% of the stroke. However the force on the inactive skate is low during this phase and the results do not seem influenced by this assumption.

4.2 Frictional Forces

The estimation of air and ice friction based on previous papers, probably caused an inaccuracy in the model outcome. Moreover, the air friction was assumed to be only dependent on velocity, while the friction coefficient might differ within a stroke, due to change of frontal area and drag. It would be interesting to determine the magnitude and repeatability of this change, in order to relate this to the model error, and to perhaps improve the estimation of the fluctuating character in the forward velocity within one stroke.

4.3 Application

When the model is verified with more data, it will be possible to determine the sensitivity of the model for each parameter, and with that determine the performance-dependent variables in speed skating. This insight will help to provide more valuable feedback on technique to skaters and coaches and via optimization propose individual optimal coordination patterns.

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