Bayesian Logistic Regression using Vectorial Centroid for Interval Type-2 Fuzzy Sets

Ku Muhammad Naim Ku Khalif and Alexander Gegov
School of Computing, University of Portsmouth, Portsmouth, PO1 3HE, U.K.

Keywords: Interval Type-2 Fuzzy Sets, Uncertainty, Defuzzification, Vectorial Centroid, Machine Learning, Bayesian Logistic Regression, Human Intuition.

Abstract: It is necessary to represent the probabilities of fuzzy events based on a Bayesian knowledge. Inspired by such real applications, in this research study, the theoretical foundations of Vectorial Centroid of interval type-2 fuzzy sets with Bayesian logistic regression is introduced. This includes official models, elementary operations, basic properties and advanced application. The Vectorial Centroid method for interval type-2 fuzzy set takes a broad view by exampled labelled by a classical Vectorial Centroid defuzzification method for type-1 fuzzy sets. Rather than using type-1 fuzzy sets for implementing fuzzy events, type-2 fuzzy sets are recommended based on the involvement of uncertainty quantity. It also highlights the incorporation of fuzzy sets with Bayesian logistic regression allows the use of fuzzy attributes by considering the need of human intuition in data analysis. It is worth adding here that this proposed methodology then applied for BUPA liver-disorder dataset and validated theoretically and empirically.

1 INTRODUCTION

Uncertainty problems are frequently described in complex systems. In dealing with uncertainty, a lot of techniques have drawn the attentions of researchers and applied scientists over last decade. Decisions are made based on information given which known as data. However, information about decision is always uncertain. In real-world phenomena, the uncertain information may consist of randomness, vagueness and fuzziness. Machine learning has always been considered as an integral part of the field of artificial intelligence. In artificial intelligence research area, the main problems that always arise are: how to represent the uncertain information precisely: and how to reason using uncertain information (Tang et al., 2002). Machine learning is certainly one of the most significant subfields of modern artificial intelligence. In recent years, machine learning systems have been adopted standard framework to deal with imprecision in data analysis.

In describing imprecise, type-1 fuzzy sets are used as a tool to erase these imprecision properly. Uncertainty is closely related with probability, which establishes the formal framework in machine learning systems. Uncertainty and fuzziness are well-known phenomena in many application areas in science and engineering, where are often not crisp but there exist various degree of membership grade that practical automatically occurs in machine learning. Type-2 fuzzy sets are suitable for uncertainty or approximate reasoning, especially for the machine learning systems with a mathematical model that is difficult to derive. Klir and Yuan (1995) claim that type-1 fuzzy sets only describe imprecise not uncertainty. On particular motivation for the further interest in type-2 fuzzy sets that its' provide a better scope for modelling uncertainty than type-1 fuzzy sets (Wagner and Hagras, 2010).

In the literature of fuzzy sets, Zadeh (1965) was introduced fuzzy set theory in representing vagueness or imprecision in a mathematical approach. In order to do so, the main motivation of using fuzzy sets shows its ability in appropriately dealing with imprecise numerical quantities and subjective preferences of decision makers (Deng, 2013). According to Zimmermann (2000), he claims that the fuzzy numbers are represented as possibility distribution where most of the real-world phenomena that exist in nature are fuzzy rather than probabilistic or deterministic. Fuzzy set theory was specifically designed to mathematically represent to uncertainty and vagueness. It also provide formalised tools for dealing with imprecision

Khalif, K. and Gegov, A..
Bayesian Logistic Regression using Vectorial Centroid for Interval Type-2 Fuzzy Sets.
In *Proceedings of the 7th International Joint Conference on Computational Intelligence (IJCCI 2015) - Volume 2: FCTA*, pages 69-79
ISBN: 978-989-758-157-1

essential to many real problems nowadays. Technologies nowadays have been developed in fuzzy sets that have potential to support all of the steps that encompass a process of model orientation and knowledge discovery. In particular, fuzzy sets theory can be used in data analysis to model vague data in terms of fuzzy sets. There are some contributions that fuzzy sets can make to machine learning which are: 1) graduality; 2) granulity; 3) interpretability; 4) robustness; 5) representation of uncertainty; 6) incorporation of background knowledge and; 7) aggregation, combination, and information fusion (Hullermeier, 2011).

The concept of type-2 fuzzy set was introduced by Zadeh (1975) as an extension of the type-1 fuzzy set. According to Karnik and Mendel (2001), they claim that type-2 fuzzy set can be characterised as fuzzy membership function where the membership value for each element in type-2 is a fuzzy set in *[0,1]*, unlike type-1 where the membership value is a crisp value in *[0,1]*. The interval type-2 fuzzy sets are widely used type-2 fuzzy sets in many practical science and engineering areas (Mendel et al., 2006). The involvement of higher level uncertainty of type-2 fuzzy sets compared to type-1, provide additional degrees of freedom to represent the uncertainty and the fuzziness of real world problems. There are two types of uncertainty which are inter and intra personal uncertainties, in improvising the representation of type-1 fuzzy sets in the literature of fuzzy sets. This is also supported by Wallsten and Budescu (1995) where there are supposedly two kinds of uncertainties that are related to linguistic characteristics namely intra-personal uncertainty and inter-personal uncertainty. In particular, a lot of experts have applied interval type-2 fuzzy sets in machine learning systems analysis. Due to implementing interval type-2 fuzzy sets in real problems, the way to handle is different and much more complex compared to type-1 fuzzy sets. The contribution of centroid of type-2 fuzzy sets till now commonly-used uncertainty measure for modelling problems.

The implementation of defuzzification plays an important role in the performance of fuzzy system's modelling techniques (Yager and Filev, 1994). Defuzzification process is guided by the output fuzzy subset that one value would be selected as a single crisp value as the system output. There are variety defuzzification methods have largely developed, however they have difference performances in difference applications and there is a general method can satisfactory performance in all conditions (Mogharreban and Dilalla, 2006). The centroid defuzzification methods of fuzzy numbers have been explored for the last decade that commonly used and have been applied in various discipline areas. The computation complexity of type-2 fuzzy set is very difficult to handle into practical applications because of characterised by their footprint of uncertainty (Mendel, 2001). There are two typical paths in computing type-2 fuzzy sets which are: 1) type-reduction (Karnik and Mendel, 2001), (Mendel, 2001) (Liu, 2008) and; 2) direct defuzzification (Gong et al., 2015). Most experts applied type-reduction methods in handling the complexity of type-2 fuzzy sets by finding equivalent type-1 fuzzy sets. However, direct defuzzification for type-2 fuzzy sets is still under study.

The concept of possibility mean value for interval fuzzy sets was introduced by Carlsson and Fuller (2001) where the notations of lower possibilistic and upper possibilistic mean values is defined the interval-valued possibilistic mean. From probabilistic viewpoint, the possibility mean value of fuzzy sets can be represented as expected values which is same function as direct defuzzification method where it doesn't need type-reduction stage to get the outputs. Gong et al. (2015) extends the concept of Carlsson and Fuller (2001) about possibility mean value of type-1 fuzzy sets which introduce the lower and upper possibility mean value for interval type-2 fuzzy sets. In this paper, the comparative simulation results and between the proposed pf the extension of Vectorial Centroid (Ku Khalif and Gegov, 2015) and possibility mean value that proposed by Gong et al. (2015) for interval type-2 fuzzy sets is discussed. There are some limitations exist in implementing Gong et al. (2015) method for interval type-2 fuzzy sets, where in some cases it will give illogical results that not consistent with human intuition. This method also can cater all possible cases of interval type-2 fuzzy sets properly since some of the results are dispersed far away from the closed interval bounded by the expectations calculated from its upper and lower distribution functions.

Due to growths in computational capability and technology development, data are being generated for understanding details real world problems in health nowadays that associated with clinical tests, diseases, disorder, genetic cases and so forth (Chen et al., 2011). However, with the availability of large datasets become the essential challenges of a new methods of statistical analysis and modelling. Logistic regression model is one of machine learning technique that used in handling these problems with high-dimensional data. The dataset that represents binary dependent attribute where it uses logit transform to predict probabilities directly. Logistic regression is a model-based approach to mapping observers' distribution. When applied within Bayesian setting, logistic regression provides a useful platform for integrating expert knowledge, in the form of a prior, with empirical data (Choy, 2013). Probability is complete with parametric models that let us characterize random uncertainty (Mendel and Wu, 2006).

Issues with respects to representation capability of fuzzy sets in machine learning systems on uncertainty become one of the important problems in decision making environments. The main objective of the present paper is to illustrate the extension of Vectorial Centroid (Ku Khalif and Alex, 2015) method for interval type-2 fuzzy sets that consider the illustration of Bayesian algorithm about the parameters of a logistic regression model. Aiming at the problems pointed out above, new centroid defuzzification for interval type-2 fuzzy sets is proposed that easy to understand, more flexible and more intelligent compared to existing methods. The proposed method also considers the need of human intuition and gives logical results while dealing with machine learning systems. In this research study, classification dataset with binary dependent attribute is used. The observations in this dataset, we will work on "BUPA liver-disorder" that were sampled by BUPA Medical Research Ltd. There are 7 attributes that consist of six independent attributes and one binary dependent attribute. The BUPA liver-disorder dataset represents blood tests indicating a property of liver disorders that may increase from excessive alcohol consumption.

The remainder of this paper is organised as follows: Section II introduces the concepts of type-2 fuzzy set, interval type-2 fuzzy set, centroid method that proposed by Gong et al. (2015) and Bayesian logistic regression. Section III views the proposed new centroid method for interval type-2 fuzzy sets using Vectorial Centroid method. Section IV illustrates the implementation of proposed method with Bayesian logistic regression in BUPA liver-disorder and compares the results with Gong et al. (2015) method. Section V summarises the main results and draws conclusion.

2 PRELIMINARIES

In this section, we briefly review some concepts of interval type-2 fuzzy sets (IT2 FSs), Bayesian logistic regression and interval-value possibility mean

2.1 Interval Type-2 Fuzzy Sets

***Definition 1*:** A type-2 fuzzy set (T2 FS) $\tilde{\tilde{A}}$ in the universe of discourse X represented by the type-2 membership function μ. If all $\mu_{\tilde{\tilde{A}}}(x,u)=1$, then $\tilde{\tilde{A}}$ is called an interval type-2 fuzzy sets (IT2 FSs). An IT2 FS can be considered as a special case T2 FS, denoted as follows (Deng, 2013):

$$\tilde{\tilde{A}} = \int_{x\in X} \int_{u\in J_x} 1/(x,u) \tag{1}$$

where $J_x \subseteq [0,1]$.

***Definition 2*:** The upper and lower membership function of an IT2 FS are type-1 fuzzy sets (T1 FSs) membership functions, respectively. A trapezoidal interval type-2 fuzzy sets can be represented by $\tilde{\tilde{A}}_i = (\tilde{A}_i^U, \tilde{A}_i^L) = ((a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U; H_1(\tilde{A}_i^U), \tilde{H}_2(\tilde{A}_i^L)), (a_{i1}^L, a_{i2}^L, a_{i3}^L, a_{i4}^L; H_1(\tilde{A}_i^L), H_2(\tilde{A}_i^L)))$ where can be shown in Figure 1 (Gong et al., 2015). The $\tilde{A}_i^U$ and $\tilde{A}_i^L$ are T1 FSs, $a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U, a_{i1}^L, a_{i2}^L, a_{i3}^L$ and a_{i4}^L are the reference points of the IT2 FS $\tilde{\tilde{A}}$, $H_j(\tilde{A}_i^U)$ denote the membership value of the element $a_{i(j+1)}^U$ in the upper trapezoidal membership function $\tilde{A}_i^U$, $1 \le j \le 2$, $H_j(\tilde{A}_i^L)$ denotes the membership value of the element $a_{i(j+1)}^L$ in the lower trapezoidal membership function $\tilde{A}_i^L$, $1 \le j \le 2$, and for $H_1(\tilde{A}_i^U) \in [0,1]$, $H_2(\tilde{A}_i^U) \in [0,1]$, $H_1(\tilde{A}_i^L) \in [0,1]$, $H_2(\tilde{A}_i^L) \in [0,1]$ and $1 \le i \le n$, $H_2(\tilde{A}_i^U) \in [0,1]$ (Deng, 2013).

***Definition 3*:** The arithmetic additional operation between the trapezoidal IT2 FSs $\tilde{\tilde{A}}_1 = (\tilde{A}_1^U, \tilde{A}_1^L) = ((a_{11}^U, a_{12}^U, a_{13}^U, a_{14}^U; H_1(\tilde{A}_1^U), H_2(\tilde{A}_1^U)), (a_{11}^L, a_{12}^L, a_{14}^L; H_1(\tilde{A}_1^L), H_2(\tilde{A}_1^L)))$ and $\tilde{\tilde{A}}_2 = (\tilde{A}_2^U, \tilde{A}_2^L) = ((a_{21}^U, a_{22}^U, a_{23}^L, a_{24}^L; H_1(\tilde{A}_2^L), a_{23}^U, a_{24}^U; H_1(\tilde{A}_2^U), H_2(\tilde{A}_2^U)), (a_{21}^L, a_{22}^L$ $H_2(\tilde{A}_2^L)))$ is defined as follows (Lee and Chen, 2008):

$$\begin{aligned}\tilde{\tilde{A}}_1+\tilde{\tilde{A}}_2&=(\tilde{A}_1^U,\tilde{A}_1^L)+(\tilde{A}_2^U,\tilde{A}_2^L)\\&=((a_{11}^U+a_{21}^U,a_{12}^U+a_{22}^U,a_{13}^U+a_{23}^U,a_{14}^U+a_{24}^U;\\&\min(H_1(\tilde{A}_1^U),(H_1(\tilde{A}_2^U)),\min(H_2(\tilde{A}_1^U),(H_2(\tilde{A}_2^U)),\\&=((a_{11}^L+a_{21}^L,a_{12}^L+a_{22}^L,a_{13}^L+a_{23}^L,a_{14}^L+a_{24}^L;\\&\min(H_1(\tilde{A}_1^L),(H_1(\tilde{A}_2^L)),\min(H_2(\tilde{A}_1^L),(H_2(\tilde{A}_2^L)))\end{aligned} \tag{2}$$

Definition 4: The arithmetic substraction operation between the trapezoidal IT2 FSs $\tilde{\tilde{A}}_1=(\tilde{A}_1^U,\tilde{A}_1^L)=((a_{11}^U,a_{12}^U,a_{13}^U,a_{14}^U;H_1(\tilde{A}_1^U),H_2(\tilde{A}_1^U)),(a_{11}^L,$ $a_{12}^L,a_{14}^L;H_1(\tilde{A}_1^L),H_2(\tilde{A}_1^L)))$ and $\tilde{\tilde{A}}_2=(\tilde{A}_2^U,\tilde{A}_2^L)=((a_{21}^U,$ $a_{22}^U,a_{23}^L,a_{24}^L;H_1(\tilde{A}_2^L),a_{23}^U,a_{24}^U;H_1(\tilde{A}_2^U),H_2(\tilde{A}_2^U)),(a_{21}^L,a_{22}^L$ $H_2(\tilde{A}_2^L)))$ is defined as follows (Lee and Chen, 2008):

$$\begin{aligned}\tilde{\tilde{A}}_1-\tilde{\tilde{A}}_2&=(\tilde{A}_1^U,\tilde{A}_1^L)-(\tilde{A}_2^U,\tilde{A}_2^L)\\&=((a_{11}^U-a_{21}^U,a_{12}^U-a_{22}^U,a_{13}^U-a_{23}^U,a_{14}^U-a_{24}^U;\\&\min(H_1(\tilde{A}_1^U),(H_1(\tilde{A}_2^U)),\min(H_2(\tilde{A}_1^U),(H_2(\tilde{A}_2^U)),\\&=((a_{11}^L-a_{21}^L,a_{12}^L-a_{22}^L,a_{13}^L-a_{23}^L,a_{14}^L-a_{24}^L;\\&\min(H_1(\tilde{A}_1^L),(H_1(\tilde{A}_2^L)),\min(H_2(\tilde{A}_1^L),(H_2(\tilde{A}_2^L)))\end{aligned} \tag{3}$$

Definition 5: The arithmetic multiplication operation between the trapezoidal IT2 FSs $\tilde{\tilde{A}}_1=(\tilde{A}_1^U,\tilde{A}_1^L)=((a_{11}^U,a_{12}^U,a_{13}^U,a_{14}^U;H_1(\tilde{A}_1^U),H_2(\tilde{A}_1^U)),(a_{11}^L,$ $a_{12}^L,a_{14}^L;H_1(\tilde{A}_1^L),H_2(\tilde{A}_1^L)))$ and $\tilde{\tilde{A}}_2=(\tilde{A}_2^U,\tilde{A}_2^L)=((a_{21}^U,$ $a_{22}^U,a_{23}^L,a_{24}^L;H_1(\tilde{A}_2^L),a_{23}^U,a_{24}^U;H_1(\tilde{A}_2^U),H_2(\tilde{A}_2^U)),(a_{21}^L,a_{22}^L$ $H_2(\tilde{A}_2^L)))$ is defined as follows (Lee and Chen, 2008):

$$\begin{aligned}\tilde{\tilde{A}}_1\times\tilde{\tilde{A}}_2&=(\tilde{A}_1^U,\tilde{A}_1^L)\times(\tilde{A}_2^U,\tilde{A}_2^L)\\&=((a_{11}^U\times a_{21}^U,a_{12}^U\times a_{22}^U,a_{13}^U\times a_{23}^U,a_{14}^U\times a_{24}^U;\\&\min(H_1(\tilde{A}_1^U),(H_1(\tilde{A}_2^U)),\min(H_2(\tilde{A}_1^U),(H_2(\tilde{A}_2^U)),\\&=((a_{11}^L\times a_{21}^L,a_{12}^L\times a_{22}^L,a_{13}^L\times a_{23}^L,a_{14}^L\times a_{24}^L;\\&\min(H_1(\tilde{A}_1^L),(H_1(\tilde{A}_2^L)),\min(H_2(\tilde{A}_1^L),(H_2(\tilde{A}_2^L)))\end{aligned} \tag{4}$$

Definition .6: The arithmetic operation between the trapezoidal IT2 FSs $\tilde{\tilde{A}}_1=(\tilde{A}_1^U,\tilde{A}_1^L)=((a_{11}^U,a_{12}^U,a_{13}^U,$ $a_{14}^U;H_1(\tilde{A}_1^U),H_2(\tilde{A}_1^U)),(a_{11}^L,a_{12}^L,a_{14}^L;H_1(\tilde{A}_1^L),H_2(\tilde{A}_1^L)))$ and the crisp constant value *k* is defined as follows (Lee and Chen, 2008):

$$\begin{aligned}&(k\times a_{11}^L,k\times a_{12}^L,k\times a_{13}^L,k\times a_{14}^L;H_1(\tilde{A}_1^L),(H_2(\tilde{A}_2^L))),\\&\frac{\tilde{\tilde{A}}_1}{k}=((\frac{1}{k}\times a_{11}^U,\frac{1}{k}\times a_{12}^U,\frac{1}{k}\times a_{13}^U,\frac{1}{k}\times a_{14}^U;H_1(\tilde{A}_1^U),(H_2(\tilde{A}_2^U)),\\&(\frac{1}{k}\times a_{11}^L,\frac{1}{k}\times a_{12}^L,\frac{1}{k}\times a_{13}^L,\frac{1}{k}\times a_{14}^L;H_1(\tilde{A}_1^L),(H_2(\tilde{A}_2^L)))\end{aligned} \tag{5}$$

where $k>0$.

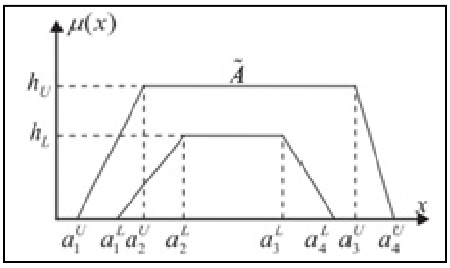

Figure 1: The upper trapezoidal membership function $\tilde{A}_i^U$ and lower trapezoidal membership function $\tilde{A}_i^L$ of IT2 FSs.

2.2 Bayesian Logistic Regression

The principal of Bayesian inference for logistic regression analyses follows the typical pattern for Bayesian analysis (Joseph, 2015):

1. Write down the likelihood function of the data
2. Form a prior distribution over all unidentified parameters
3. Find posterior distribution using Bayes theorem over all parameters

Likelihood function: the likelihood contribution from the i^{th} subject is binomial

$$likelihood_i=\pi(x_i)^{y_i}(1-\pi(x_i))^{(1-y_i)} \tag{6}$$

where $\pi(x_i)$ represents the probability of the event for subject *i*, which has covariate vector x_i and y_i specifies the liver-disorder $y_i=1$, or liver-normal $y_i=2$ of the event for the subject. Logistic regression is denoted as

$$\pi(x)=\frac{e^{\beta_0+\beta_1X_i+\ldots+\beta_pX_p}}{1+e^{\beta_0+\beta_1X_i+\ldots+\beta_pX_p}} \tag{7}$$

So the likelihood from the i^{th} subject is

$$\begin{aligned}likelihood_i&=\left(\frac{e^{\beta_0+\beta_1X_{i1}+\ldots+\beta_pX_{ip}}}{1+e^{\beta_0+\beta_1X_i1+\ldots+\beta_pX_{ip}}}\right)^{y_i}\left(1-\frac{e^{\beta_0+\beta_1X_{i1}+\ldots+\beta_pX_{ip}}}{1+e^{\beta_0+\beta_1X_{i1}+\ldots+\beta_pX_{ip}}}\right)^{(1-y_i)}\\likelihood&=\prod_{i=1}^{n}\left[\left(\frac{e^{\beta_0+\beta_1X_{i1}+\ldots+\beta_pX_{ip}}}{1+e^{\beta_0+\beta_1X_i+\ldots+\beta_pX_{ip}}}\right)^{y_i}\left(1-\frac{e^{\beta_0+\beta_1X_{i1}+\ldots+\beta_pX_{ip}}}{1+e^{\beta_0+\beta_1X_i1+\ldots+\beta_pX_{ip}}}\right)^{(1-y_i)}\right]\end{aligned} \tag{8}$$

Prior distribution: in general, any prior distribution can be used, depending on the available prior information.

$$\beta_j \sim Normal(c_j, \sigma_j^2) \tag{9}$$

The most common choice for *c* is zero, and σ is usually chosen to be large enough to be considered as non-informative in the range from $\sigma = 10$ to $\sigma = 100$.

Posterior distribution via Bayes theorem: the posterior distribution is divided by multiplying the prior distribution over all parameter by the full likelihood function, so that

$$Posterior = \prod_{i=1}^{n}\left[\left(\frac{e^{\beta_0+\beta_1 X_{i1}+...+\beta_p X_{ip}}}{1+e^{\beta_0+\beta_1 X_i 1+...+\beta_p X_{ip}}}\right)^{y_i}\left(1-\frac{e^{\beta_0+\beta_1 X_{i1}+...+\beta_p X_{ip}}}{1+e^{\beta_0+\beta_1 X_i 1+...+\beta_p X_{ip}}}\right)^{(1-y_i)}\right]$$

$$\times \prod_{j=0}^{p}\frac{1}{\sqrt{2\pi\sigma_j}}\exp\left\{-\frac{1}{2}\left(\frac{\beta_j - c_j}{\sigma_j}\right)^2\right\} \tag{10}$$

The latter part of the above expression being recognised as normal distribution for the β parameters. For liver-disorder classification problem, $p(y=1|\beta_p x_p)$, will be an estimate of the probability that the *p*th document belongs to the category. The decision of whether to assign the category can be based on comparing the probability estimate with a threshold or by computing which decision gives optimal expected utility.

2.3 Interval-Valued Possibility Mean Value

The concept of interval-valued possibility mean value are divided into two parts which are lower and upper possibility mean value. The lower $\underline{M}(\overset{\approx}{A})$ and upper $\overline{M}(\overset{\approx}{A})$ possibility mean value for interval type-2 fuzzy sets are denoted as follow (Gong et al., 2015):

$$\underline{M}(\overset{\approx}{A}) = \int_0^{h_U}\alpha\left(a_1^U + \frac{a_2^U - a_1^U}{h_u}\alpha\right)d\alpha + \int_0^{h_L}\beta\left(a_1^L + \frac{a_2^L - a_1^L}{h_L}\beta\right)d\beta \tag{11}$$

$$\overline{M}(\overset{\approx}{A}) = \int_0^{h_U}\alpha\left(a_4^U + \frac{a_3^U - a_4^U}{h_u}\alpha\right)d\alpha + \int_0^{h_L}\beta\left(a_4^L + \frac{a_3^L - a_4^L}{h_L}\beta\right)d\beta \tag{12}$$

For crisp value, we can compute by using the average of lower and upper possibility mean value above that denoted as follows

$$M(\overset{\approx}{A}) = \frac{\underline{M} + \overline{M}}{2} \tag{13}$$

In this paper, the numerical analysis for proposed methodology is compared with interval-valued possibility mean value that proposed by Gong et al. (2015).

3 PROPOSED METHOD

As noted in the introduction, the useful of interval type-2 fuzzy sets nowadays are widely applied in many research areas in dealing with uncertainty in data analysis which consistent with human intuition. Most of researchers attempt to eliminate the need of human intuition in data analysis processes. Human intuition is strictly can't be eliminated because it can lead us towards uncertainty problems.

This study simplify the concept of attributes to $\mu_{\overset{\approx}{A}} \in [0,1]$ for fuzzy events. The values of attributes correspond to interval type-2 fuzzy sets. This study proposed a new centroid defuzzification methodology for Bayesian logistic regression algorithm. The methodology consist of two stages here namely:

A. Stage one

The development of an extension of the Vectorial Centroid defuzzification [29] for interval type-2 fuzzy sets.

B. Stage two

The implementation of Vectorial Centroid in Bayesian logistic regression.

Full description for both stages are as follow:

A. Stage one

Let consider by $\overset{\approx}{A}_i = (\widetilde{A}_i^U, \widetilde{A}_i^L) = ((a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U; H_1(\widetilde{A}_i^U), \widetilde{H}_2(\widetilde{A}_i^L)), (a_{i1}^L, a_{i2}^L, a_{i3}^L, a_{i4}^L; H_1(\widetilde{A}_i^L), H_2(\widetilde{A}_i^L)))$ as the interval type-2 fuzzy sets. The complete method process of Vectorial Centroid is signified as follow

Step 1: Find the centroids of the three parts of α, β and γ in interval type-2 fuzzy set representation as shown in Figure 2.

$$\alpha_{\overline{\alpha},\underline{\alpha}}(x,y) = \left(\frac{1}{3}(a_1^U + \frac{1}{2}a_2^U) + \frac{1}{3}(a_1^L + \frac{1}{2}a_2^L), \frac{1}{6}(h^U + h^L)\right) \tag{14}$$

$$\beta_{\overline{\beta},\underline{\beta}}(x,y) = \left(\frac{1}{4}(a_2^U + a_3^U + a_2^L + a_3^L), \frac{1}{4}(h^U + h^L)\right) \tag{15}$$

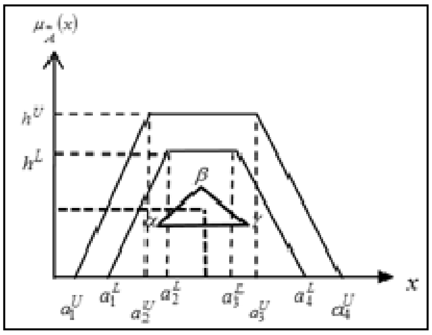

Figure. 2: Vectorial Centroid plane representation.

$$\gamma_{\overline{\gamma},\underline{\gamma}}(x,y)=\left(\frac{1}{3}(a_3^U+\frac{1}{2}a_4^U)+\frac{1}{3}(a_3^L+\frac{1}{2}a_4^L),\frac{1}{6}(h^U+h^L)\right) \tag{16}$$

ep 2: Connect all vertices centroids points of α, β and γ each other, where it will create another triangular plane inside of trapezoid plane.

Step 3: The centroid index of Vectorial Centroid of $(\tilde{x},\tilde{y})$ with vertices α, β and γ can be calculated as

$$VC_{\underset{A}{\approx}}(\tilde{x},\tilde{y})=\left(\frac{\alpha_{\overline{\alpha},\underline{\alpha}}(x,y)+\beta_{\overline{\beta},\underline{\beta}}(x,y)+\gamma_{\overline{\gamma},\underline{\gamma}}(x,y)}{3},\right.$$
$$\left.\beta_{\overline{\beta},\underline{\beta}}(x,y)+\left[\frac{2}{3}\left(\frac{\alpha_{\overline{\alpha},\underline{\alpha}}(x,y)+\gamma_{\overline{\gamma},\underline{\gamma}}(x,y)}{2}-\beta_{\overline{\beta},\underline{\beta}}(x,y)\right)\right]\right) \tag{17}$$

Vectorial Centroid can be summarised as

$$VC_{\underset{A}{\approx}}(\tilde{x},\tilde{y})=\left(\frac{1}{9}\left[a_1^U+a_1^L+\frac{5}{4}(a_2^U+a_2^L)+\frac{7}{4}(a_3^U+a_3^L)+\right.\right.$$
$$\left.\left.\frac{1}{2}(a_4^U+a_4^L)\right],\frac{11}{36}(h^U+h^L)\right) \tag{18}$$

where

α : the centroid coordinate of first triangle plane

β : the centroid coordinate of rectangle plane

γ : the centroid coordinate of second triangle plane

$(\tilde{x},\tilde{y})$: the centroid coordinate of fuzzy number $\overset{\approx}{A}$

Centroid index of Vectorial Centroid can be generated using Euclidean distance by Cheng (1998):

$$R(\overset{\approx}{A})=\sqrt{\tilde{x}^2+\tilde{y}^2} \tag{19}$$

B. Stage two

Integrating fuzzy sets with Bayesian logistic regression in fuzzy states of nature, where if there is fuzzy dataset, defuzzification process is needed in converting into crisp values where at the same time the fuzzy nature is not lost. Reinterpretation of degree $\mu_{\underset{A}{\approx}} \in [0,1]$ using Vectorial Centroid to the $P(y=1|\beta_p X_p)$ is developed as follows:

Step 1: Lift the reintergration of the fuzzy values membership function using trapezoidal interval type-2 fuzzy sets. Vectorial Centroid formulation are applied for trapezoidal interval type-2 fuzzy set rule formula. The $\mu_{\underset{A}{\approx}}$ represents as $\overset{\approx}{A}_1=(\tilde{A}_1^U,\tilde{A}_1^L)=((a_{11}^U,a_{12}^U,a_{13}^U,a_{14}^U;H_1(\tilde{A}_1^U),H_2(\tilde{A}_1^U)),(a_{11}^L, a_{12}^L,a_{14}^L;H_1(\tilde{A}_1^L),H_2(\tilde{A}_1^L)))$ in calculation to avoid cluttering. Suppose that $\mu_{\beta_i x_i}$ are fuzzy events for attribute alkaline phosphatase, aspartate aminotransferase, gamma-glutamyl transpeptidase and alamine aminotransferase in BUPA liver-disorder dataset.

Step 2: The centroid index of Vectorial Centroid, $R(\overset{\approx}{A})$ is inserted into Bayesian logistic regression rule as

$$R(\overset{\approx}{A})=\sqrt{\tilde{x}^2+\tilde{y}^2}=\mu(\overset{\approx}{A})$$

The computational process of likelihood and posterior distribution of fuzzy Bayesian logistic regression using Vectorial Centroid are denoted as

$$likelihood=\prod_{i=1}^{n}\left[\left(\frac{e^{\beta_0+\beta_1\mu(\overset{\approx}{A})_{i1}+...+\beta_p\mu(\overset{\approx}{A})_{ip}}}{1+e^{\beta_0+\beta_1\mu(\overset{\approx}{A})_{i1}+...+\beta_p\mu(\overset{\approx}{A})_{ip}}}\right)^{y_i}\left(1-\frac{e^{\beta_0+\beta_1\mu(\overset{\approx}{A})_{i1}+...+\beta_p\mu(\overset{\approx}{A})_{ip}}}{1+e^{\beta_0+\beta_1\mu(\overset{\approx}{A})_{i1}+...+\beta_p\mu(\overset{\approx}{A})_{ip}}}\right)^{(1-y_i)}\right] \tag{20}$$

$$Posterior=\prod_{i=1}^{n}\left[\left(\frac{e^{\beta_0+\beta_1\mu(\overset{\approx}{A})_{i1}+...+\beta_p\mu(\overset{\approx}{A})_{ip}}}{1+e^{\beta_0+\beta_1\mu(\overset{\approx}{A})_{i1}+...+\beta_p\mu(\overset{\approx}{A})_{ip}}}\right)^{y_i}\left(1-\frac{e^{\beta_0+\beta_1\mu(\overset{\approx}{A})_{i1}+...+\beta_p\mu(\overset{\approx}{A})_{ip}}}{1+e^{\beta_0+\beta_1\mu(\overset{\approx}{A})_{i1}+...+\beta_p\mu(\overset{\approx}{A})_{ip}}}\right)^{(1-y_i)}\right]$$
$$\times\prod_{j=0}^{p}\frac{1}{\sqrt{2\pi\sigma_j}}\exp\left\{-\frac{1}{2}\left(\frac{\beta_j-c_j}{\sigma_j}\right)^2\right\} \tag{21}$$

4 EXPERIMENTAL SETTINGS

In this section, we describe the required parameters to conduct the experiments. The experiment is conducted using 10-fold cross validation on BUPA liver-disorder dataset from UCI machine learning repository (Forsyth, 2015) is used where donated by

BUPA Medical Research Ltd. This liver-disorder classification dataset has 345 examples, 7 attributes and binary classes for dependent attribute. The first 5 attributes are measurements taken by blood tests that are thought to be sensitive to liver-disorders and might arise from excessive alcohol consumption. The sixth attribute is a sort of selector attribute where the subjects are single male individuals. The seventh attribute shows a selector on the dataset which being used to split into two categories that indicating the class identity. The attributes include:

a. Mean corpuscular volume,
b. Alkaline phosphatase,
c. Aspartate aminotransferase,
d. Gamma-glutamyl transpeptidase,
e. Alamine aminotransferase,
f. Number if half-pint equivalents of alcoholic beverage drunk per day, and
g. Output attributes either liver disorder or liver normal

Among all the people, there are 145 belonging to the liver-disorder group and 200 belonging to the liver-normal group. These attributes are selected with the aid of experts. The original dataset are fuzzified randomly in interval type-2 fuzzy sets form in operating centroid methods. Below depicts the example of interval type-2 fuzzy sets are used in this research study:

Example 1: If the trapezoidal interval type-2 fuzzy set $\overset{\approx}{A}_i = (\tilde{A}_i^U, \tilde{A}_i^L) = ((15.35, 16.68, 18.06, 20.51; 1),$ $(16, 17, 18, 19; 0.9))$, then the centre points are computed using proposed (Vectorial Centroid) and established method (Interval-valued possibility mean value) formulation respectively as follows:

Vectorial Centroid:
$VC(x) = 17.3678$ and $VC(y) = 0.58056$

Index Vectorial Centroid, $VC(\overset{\approx}{R}) = 17.3775$

Interval-Valued Possibility Mean Value:

$$M(\overset{\approx}{A}) = \left[M_*(\overset{\approx}{A}), M^*(\overset{\approx}{A}) \right] = [14.8683, 16.8633]$$

Crisp possibility mean value, $M(\overset{\approx}{A}) = 15.8658$

5 SIMULATION RESULTS

This section illustrates the validation process that are divided into two parts which are theoretically and empirically. Therefore, the theoretical of Vectorial Centroid validation process are as follow:

A. Stage one

The relevant properties considered for justifying the applicability of centroid for interval type-2 fuzzy sets, where they depend on the practicality within the area of research however, they are not considered as complete. Therefore, without loss of generality, the relevant properties of the centroid are as follow:

Let $\overset{\approx}{A}$ and $\overset{\approx}{B}$ are be trapezoidal and triangular interval type-2 fuzzy sets respectively, while $MC_{\overset{\approx}{A}}(\tilde{x}, \tilde{y})$ and $MC_{\overset{\approx}{B}}(\tilde{x}, \tilde{y})$ be centroid points for $\overset{\approx}{A}$ and $\overset{\approx}{B}$ respectively. Centroid index of Vectorial Centroid, (R) shows the crisp value of centroid point that is denoted as $R(\overset{\approx}{A}) = \sqrt{\tilde{x}^2 + \tilde{y}^2}$.

Property 1: *If $\overset{\approx}{A}$ and $\overset{\approx}{B}$ are embedded and symmetry, then* $R(\overset{\approx}{A}) > R(\overset{\approx}{B})$ **.**

Proof:

Since $\overset{\approx}{A}$ and $\overset{\approx}{B}$ are embedded and symmetry, hence from equation (19) we have $\sqrt{\tilde{x}_{\overset{\approx}{A}}^{\ 2} + \tilde{y}_{\overset{\approx}{A}}^{\ 2}} > \sqrt{\tilde{x}_{\overset{\approx}{B}}^{\ 2} + \tilde{y}_{\overset{\approx}{B}}^{\ 2}}$.

Therefore, $R(\overset{\approx}{A}) > R(\overset{\approx}{B})$.

Property 2: *If $\overset{\approx}{A}$ and $\overset{\approx}{B}$ are embedded with $(h^U, h^L)_{\overset{\approx}{A}} > (h^U, h^L)_{\overset{\approx}{B}}$, then* $R(\tilde{A}) > R(\tilde{B})$**.**

Proof:

Since $\overset{\approx}{A}$ and $\overset{\approx}{B}$ are embedded and *with* $(h^U, h^L)_{\overset{\approx}{A}} (h^U, h^L)_{\overset{\approx}{B}}$, hence we know that $\tilde{y}_{\tilde{A}} > \tilde{y}_{\tilde{B}}$ **.**

Then, from equation (19) we have $\sqrt{\tilde{x}_{\overset{\approx}{A}}^{\ 2} + \tilde{y}_{\overset{\approx}{A}}^{\ 2}} > \sqrt{\tilde{x}_{\overset{\approx}{B}}^{\ 2} + \tilde{y}_{\overset{\approx}{B}}^{\ 2}}$. Therefore, $R(\overset{\approx}{A}) > R(\overset{\approx}{B})$ **.**

Property 3: *If $\overset{\approx}{A}$ is singleton fuzzy number, then* $R(\overset{\approx}{A}) = \sqrt{\tilde{x}_{\overset{\approx}{A}}^{\ 2} + \tilde{y}_{\overset{\approx}{A}}^{\ 2}}$.

Proof:

For any crisp (real) interval type-2 fuzzy set, we know that $a_1^U = a_2^U = a_3^U = a_4^U = a_1^L = a_2^L = a_3^L = a_4^L = \tilde{x}_{\overset{\approx}{A}}$ which are equivalent to equation (18).

Therefore, $R(\tilde{\tilde{A}}) = \sqrt{\tilde{x}_{\tilde{\tilde{A}}}^{\,2} + \tilde{y}_{\tilde{\tilde{A}}}^{\,2}}$.

Property 4: *If $\tilde{\tilde{A}}$ is any symmetrical or asymmetrical interval type-2 fuzzy number, then* $a_1^U < R(\tilde{\tilde{A}}) < a_4^U$

Proof:

Since any symmetrical or asymmetrical interval type-2 fuzzy set has $a_1^U \le a_2^U \le a_3^U \le a_4^U$, hence $a_1^U < MC_{\tilde{\tilde{A}}}(\tilde{x},\tilde{y}) < a_4^U$ *. Therefore,* $a_1^U < R(\tilde{\tilde{A}}) < a_4^U$.

B Stage two

In this stage, the empirical validation is implemented where the BUPA liver-disorder data set is used in conducting Bayesian Logistic Regression

Note that this study is considered all type of possible interval type-2 fuzzy sets for attributes randomly as figures follow:

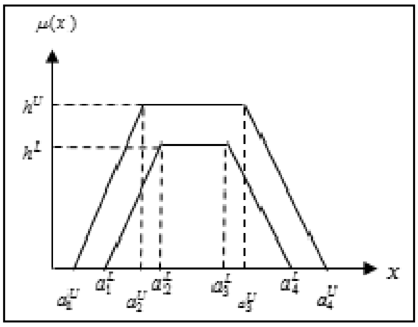

Figure 3: Trapezoidal Non-Normal Symmetry.

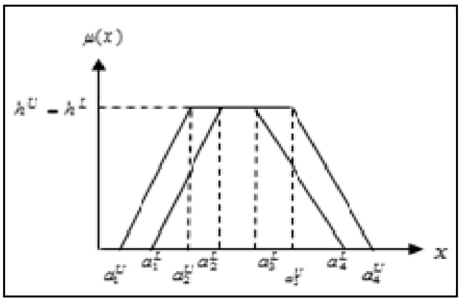

Figure 4: Trapezoidal Normal Symmetry.

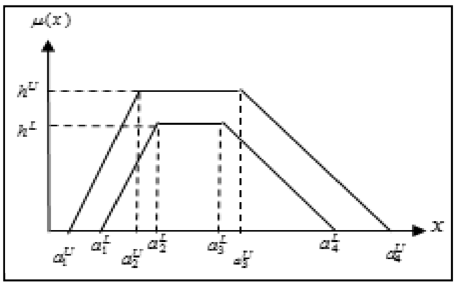

Figure 5: Trapezoidal Non-Normal Asymmetry.

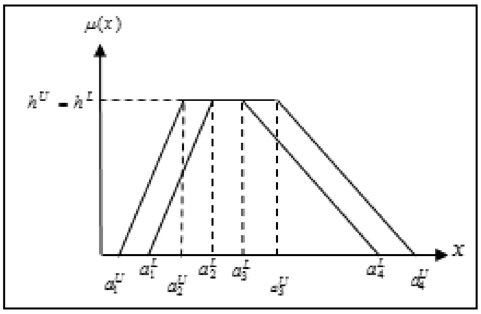

Fig. 6: Trapezoidal Normal Asymmetry.

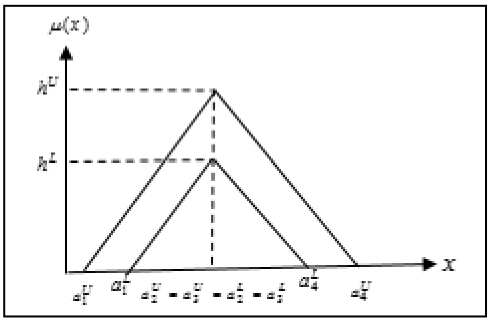

Figure 7: Triangular Non-Normal Symmetry.

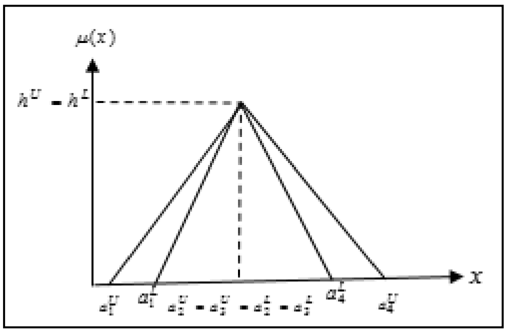

Figure 8: Triangular Normal Symmetry.

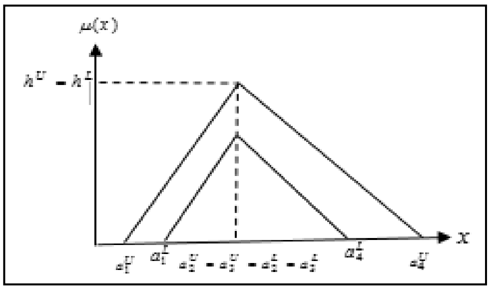

Figure 9: Triangular Non-Normal Asymmetry.

Table I presents a comparative results between classical Bayesian logistic regression (BLR-Classic), Bayesian logistic regression using possibility mean value (Gong et al., 2015) method (BLR-PMV), and Bayesian logistic regression using proposed Vectorial Centroid (BLR-VC). The comparison results are

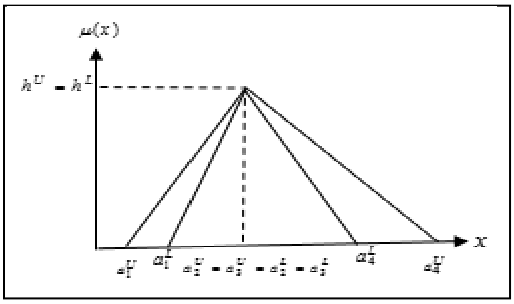

Figure 10: Triangular Normal Asymmetry.

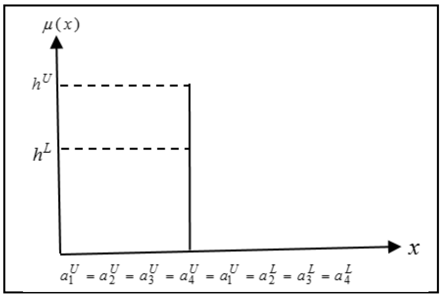

Figure 11: Singleton Non-Normal.

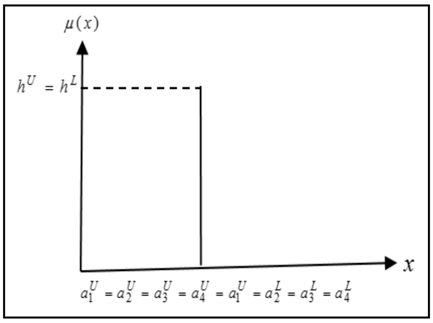

Figure 12: Singleton Normal.

Table 1: Accuracy, precision, sensitivity, specificity, kappa statistic and errors results.

Method	BLR-Classic	BLR-PMV	BLR-VC
Accuracy	*67.2464%*	58.5507%	68.1159%
Precision	*17.67%*	*1.4%*	*30.34%*
Sensitivity	*82%*	*66.67%*	*83.02%*
Specificity	*64.75%*	*58.41%*	*65.41%*
Kappa Statistic	*0.2613*	0.0203	0.2832
Errors: ***MAE*** ***RMSE*** ***RAE*** ***RRSE***	 *0.3275* *0.5723* *67.2025%* *115.9404%*	 *0.4145* *0.6438* *85.0438%* *130.4259%*	 *0.3188* *0.5647* *65.4183%* *114.391%*

Centroid (BLR-VC). The comparison results are based on accuracy precision, sensitivity, specificity, kappa statistic, and some error terms which are Mean Absolute Error (MAE), Root Mean Square Error (RMSE), Relative Absolute Error (RAE) and Root Relative Square Error (RRSE).

The accuracy and precision of a measurement system plays significant role in quantifying the actual measure value. It is commonly used as metric for evaluation of machine learning systems. The precision is dependent of accuracy where the model can be very precise but inaccurate. The higher the value of accuracy and precision, the better classification prediction is made. In this research study, Table 1 shows the classification accuracy results that show the correctness of a model classifies the dataset in each class. Below show the formulation of accuracy and precision:

$$Accuracy: \frac{TotalPositive + TotalNegative}{Positive + Negative} \tag{22}$$

$$\Pr ecision: \frac{TotalPositive}{TotalPositive + FalseNegative} \tag{23}$$

The classification accuracy results of BLR-Classic, BLR-PMV and BLR-VC are 67.2464%, 58.5508% and 68.1159% respectively. It shows that the proposed methodology is significantly more accurate compared to others. The highest precision in this case study is BLR-VC with 30.34%, followed by BLR-Classic with 17.67% and BLR-PMV with 1.4%. Precision discusses the closeness of two or more measurements to each other.

The sensitivity test refers to the ability of the test to correctly identify those observers with positive predictive value. A high sensitivity is clearly imperative where the test is used to identify the correct class. But, specificity test is inversely proportional to sensitivity where it has the ability of the test to correctly identify those observers with negative predictive value (Lalkhen and McCluskey, 2015). Below are formulation to calculate sensitivity and specificity:

$$Sensitivity: \frac{TotalPositive}{TotalPositive + FalseNegative} \tag{24}$$

$$Specificity: \frac{TotalNegative}{FalsePositive + TotalNegative} \tag{25}$$

The proposed method, BLR-VC produces the highest sensitivity and specificity value with 83.02% and 65.41% respectively. The results for BLR-PMV shows the lowest results for sensitivity and specificity with 66.67% and 58.41% respectively. It depicts that the goodness of prediction of both tests for BLR-PMV is lesser than BLR-Classic and BLR-VC.

Kappa statistic technique is used to measure the agreement of two classifiers and estimate the probability of two classifiers agree simply by chance (Jeong et al., 2010). Known as chance-corrected measure of agreement between classification and the true classes, it is an evaluation metric which is based on the difference between the actual agreement in the error matrix and the chance agreement. The values for Kappa range from 0 to 1 and the higher the value of kappa statistic, the stronger the strength of agreement between two classifiers by chance.

$$KappaStat, k = \frac{p_o - p_e}{1 - p_e} \tag{26}$$

where p_o is relative observed agreement among raters, and p_e is the hypothetical probability of chance agreement.

Referring Table 1, BLR-VC shows the highest value of kappa statistic with 0.2832 followed by BLR-Classic and BLR-PMV with 0.2613 and 0.0203 respectively.

The last part in Table I depicts the errors for the experiment carried out. The errors are computed by using Mean Absolute Error (MAE), Root Mean Square Error (RMSE), Relative Absolute Error (RAE) and Root Relative Square Error (RRSE). All the statistic errors compare true values to theirs estimates, but do it in a slightly different way. Below depict the formulation in calculating MAE, RMSE, RAE and RRSE:

$$MAE = \frac{1}{N}\sum_{i=1}^{N}\left|\hat{\theta}_i - \theta_i\right| \tag{27}$$

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\hat{\theta}_i - \theta_i)^2} \tag{28}$$

$$RAE = \frac{\sum_{i=1}^{N}\left|\hat{\theta}_i - \theta_i\right|}{\sum_{i=1}^{N}\left|\bar{\theta}_i - \theta_i\right|} \tag{29}$$

$$RRSE = \sqrt{\frac{\sum_{i=1}^{N}(\hat{\theta}_i - \theta_i)^2}{\sum_{i=1}^{N}(\bar{\theta}_i - \theta_i)^2}} \tag{30}$$

These error terms show how disperse away the estimated values from the true value of θ. MAE and RMSE calculate the average difference between those two values. In RAE and RRSE, we divide those differences by the variation of θ where they have a scale from 0 to 1, then we would multiply those value by 100 to get the similarity in 0-100 scale. In this case study, the proposed methodology, BLR-MC performs better results in error terms where all of these errors are less than BLR-Classic and BLR-PMV.

6 CONCLUSION

This study has brought out an extension based Vectorial Centroid (Ku Khalif and Gegov, 2015) for interval type-2 fuzzy sets with Bayesian logistic regression. Bayesian logistic regression algorithm that takes into account the need of fuzzy events in attributes. This work suggests Vectorial Centroid defuzzification on interval type-2 fuzzy sets method for Bayesian logistic regression which consist of two stages which are: The development of Vectorial Centroid defuzzification method for interval type-2 fuzzy sets: and the implementation of Vectorial Centroid in Bayesian logistic regression. For the first stage, the development of new centroid method can cater all the possible cases of interval type-2 fuzzy sets precisely that matching for human intuition. The implementation in Bayesian logistic regression using proposed methodology on stage two is easily capable constructed and handled in data analysis when dealing with fuzzy data sets.

Several limitations may exist in this research study. First, the proposed classification model for interval type-2 fuzzy numbers was developed and tested on BUPA liver-disorder dataset from WEKA software. The useful of interval type-2 fuzzy sets are randomly applied. Second, the scope of this research study is focused to be automated diagnosis liver-disorder. Still, more experimental work should be enthusiastic to obtain a medical classification model with a better ability of generalization under fuzzy environment. The proposed Vectorial Centroid only applied in one machine learning which is Bayesian logistic regression. It should be applied and compared with more machine learning systems in the future work that would make research much more convincing.

Furthermore, this study can be valuable alternatively in the set of existing Bayesian logistic regression algorithms for numerous problems in machine learning such as inference, classification, clustering, regression and so forth. There are four relevant properties for centroid development are constructed and well proved in theoretical validation, where corresponding with all possible interval type-2 fuzzy sets representation. Several tests for validation have been done and the results have been studied in-depth using BUPA liver-disorder classification dataset from UCI machine learning repository. The validation results show the proposed research study more effective in dealing with fuzzy events empirically. Finally, it can be concluded that the main focus of this research study can be proceeded in order to make some contributions by considering real case study drawn for diverse fields crossing ecology, health, genetics, finance and so forth.

REFERENCES

Carlsson, C., Fuller, R. 2001. *On possibilistic mean value and variance of fuzzy numbers*, Fuzzy Sets and Systems, vol. 122.

Chen, C.C.M., Schwender, H., Keith, J., Nunkesser, R., Mengersen, K., Macrossan, P. 2011. *Method of identifying SNP interactions: A review on variations of logic regression, random forest and Bayesian logistic regression*, IEEE/ ACM Transactions on Computational Biology and Bioinformatics, vol. 8. a

Cheng, C.H. 1998. *A new approach for ranking fuzzy numbers by distance method*, Fuzzy Sets and Systems, vol. 95.

Choy, S.L. 2013. *Priors: Silent or active partners of Bayesian inference?,* Case Studies in Bayesian Statistical Modelling and Analysis, John Wiley & Sons Ltd, Sussex.

Deng, H. 2013. *Comparing and ranking fuzzy numbers using ideal solutions*, Applied Mathematics Modelling, vol. 38.

Forsyth, R.S. 2015. *Liver disorder data set*, UCI Machine Learning Repository: Internet Source: https://archive.ics.uci.edu/ml/datasets/Liver+Disorders [Feb. 1, 2015].

Gong, Y., Hu, N., Zhang, J., Liu. G., Deng. J. 2015. *Multi-attribute group decision making method based on geometric Bonferroni mean operator of trapezoidal interval type-2 fuzzy numbers*, Computer and Industrial Engineering, vol. 81.

Hullermeier, E. 2011. *Fuzzy sets in machine learning and data mining*, Applied Soft Computing, vol. 11.

Jeong, D., Kang, D., Won, S. 2010. *Feature selection for steel defects classification*, International Conference on Control, Automation and Systems.

Joseph. 2015. Bayesian inference for logistic regression parameters. Internet Source: http://www.medicine.mcgill.ca/epidemiology/joseph/courses/EPIB-621/bayeslogit.pdf [March, 15, 2015].

Karnik, N.N., Mendel, J.M. 2001. *Centroid of type-2 fuzzy set*, Information Sciences, vol. 132.

Klir, G., Yuan, B. 1995. *Fuzzy sets and fuzzy logic: Theory and applications,* Prentice Hall, Upper Saddle River.

Ku Khalif, K.M.N., Gegov, A. 2015. Generalised fuzzy Bayesian network with adaptive Vectorial Centroid, 16th World Congress of the International Fuzzy Systems Association (IFSA) and 9th Conference of European Society for Fuzzy Logic and Technology (EUSFLAT).

Lalkhen, A.G., McCluskey, A. 2015. Clinical test sensitivity and specificity, Internet Source: http://ceaccp.oxfordjournals.org/content/8/6/221.full [March, 16, 2015].

Lee, L.W., Chen, S.M. 2008. *Fuzzy multiple attributes group decision-making based on the extension of TOPSIS method and interval type-2 fuzzy sets*, 7th IEEE International Conference on Machine Learning and Cybernatics.

Liu. F. 2008. *An efficient centroid type-reduction strategy for general type-2 fuzzy logic system*, Information Sciences, vol. 178.

Mendel, J.M. 2001. *Uncertain rule-based fuzzy logic systems: Introduction and new directions*, Prentice-Hall, Upper Saddle River, New Jersey.

Mendel, J.M., John, R.I., Liu, F.L. 2006. *Interval type-2 fuzzy logical systems made simple*, IEEE Transactions on Fuzzy Systems, vol. 14.

Mendel, J.M., Wu, H.W. 2006. *Type-2 fuzzistic for symmetric interval type-2 fuzzy sets: Part 1, forward problems*, IEEE Transactions on Fuzzy Systems, vol. 14.

Mogharreban, N., Dilalla, L.F. 2006. *Comparison of defuzzification techniques for analysis of non-interval data*, Fuzzy Information Processing Society.

Tang, Y., Pan, H., Xu, Y. 2002. *Fuzzy naïve Bayes classifier based on fuzzy clustering*, Systems, Man and Cybernatics, IEEE International Conference, vol. 5.

Wagner, C., Hagras, H. 2010. *Uncertainty and type-2 fuzzy sets systems*, IEEE UK Workshop on Computational Intelligent (UKCI).

Wallsten, T.S., Budescu, D.V. 1995. *A review of human linguistic probability processing general principles and empirical evidences,* The Knowledge Engineering Review, vol.10 (a).

Yager, R.R., Filev, D.P. 1994. *Essential of fuzzy modelling and control*, Wiley, New York.

Zadeh, L.A. 1965. *Fuzzy sets*, Information and Control, vol. 8.

Zadeh, L.A., 1975. *The concept of a linguistic variable and its application to approximate reasoning, Information Sciences*, vol. 8.

Zimmermann, H.J. 2000. *An application – oriented view of modelling uncertainty*, European Journal of Operational Research, vol 122.