

Choosing Suitable Similarity Measures to Compare Intuitionistic Fuzzy Sets that Represent Experience-Based Evaluation Sets

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Abstract: Which similarity measures can be used to compare two Atanassov's intuitionistic fuzzy sets (IFSs) that respectively represent two experience-based evaluation sets? To find an answer to this question, several similarity measures were tested in comparisons between pairs of IFSs that result from simulations of different experience-based evaluation processes. In such a simulation, a support vector learning algorithm was used to learn how a human editor categorizes newswire stories under a specific scenario and, then, the resulting knowledge was used to evaluate the level to which other newswire stories fit into each of the learned categories. This paper presents our findings about how each of the chosen similarity measures reflected the perceived similarity among the simulated experience-based evaluation sets.

1 INTRODUCTION

If you ask about comic books suitable for 7-year-old kids, a coworker who does not like slang expressions might judge 'Popeye the Sailor' as a *quite unsuitable* comic book, whereas a coworker who learned eating spinach due to "they are the source of Popeye's super strength" might judge it as a *totally suitable* one. We deem the evaluations resulting from this kind of judgments to be *experience-based evaluations*, which mainly depend on what each person has experienced or understood about a particular concept (e.g., 'comic books suitable for 7-year-old kids').

Imagine that your sister is looking for a proper comic book for your 7-year-old nephew. If you want to know which of your coworkers could choose a comic book on behalf of your sister, you might be interested in measuring the level to which the evaluations given by each coworker are similar to your sister's evaluations. A problem in such similarity comparisons is that those experience-based evaluations are fairly subjective and a "pseudo-matching" between them is possible, i.e., the evaluations could match even though the evaluators have distinct understandings of the evaluated concept (Loor and De Tré, 2014).

Considering that an experience-based evaluation

could be imprecise and marked by hesitation, in (Loor and De Tré, 2014) the authors proposed modeling it as an element of an intuitionistic fuzzy set, or IFS for short (Atanassov, 1986; Atanassov, 2012). However, the authors pointed out that, to compare two IFSs that represent experience-based evaluation sets, the similarity measures based on a metric distance approach such as the studied in (Szmíd and Kacprzyk, 2013; Szmíd, 2014) might not be applicable to the case because of their implicit assumption about symmetry and transitivity, which does not reflect judgments of similarity observed from a psychological perspective (Tversky, 1977).

To study empirically which of those similarity measures can be used to compare such IFSs, which is the main purpose of this paper, we tested those similarity measures in comparisons between pairs of IFSs resulting from simulations of experience-based evaluation processes. Our motivation for this study is to complement the existing theoretical work within the context of IFSs to find suitable methods that allow us to compare experience-based evaluation sets given from persons that might have different learning experiences.

To simulate an experience-based evaluation process, we first made use of a learning algorithm that uses support vector machines (Vapnik, 1995; Vapnik

and Vapnik, 1998) to learn how a human editor categorizes newswire stories under a given scenario. We then made use of the previous knowledge to evaluate the level to which other stories fit into one of the learned categories and, thus, we obtained the simulated experience-based evaluation sets. Each of the established learning scenarios included a training collection that contains a certain proportion of opposite examples in relation to the original data, which consist of manually categorized newswire stories —by opposite example is meant that, e.g., if a story is assigned to a particular category in the original training collection, the story will not be assigned to the category in the training collection related to the current scenario.

An interesting aspect about testing the similarity measures in that way is that we can observe how they reflect the perceived similarity between two experience-based evaluation sets given from dissimilar learning scenarios. For instance, we could test a similarity measure to observe how it reflects the perceived similarity between the IFSs given by two persons who use training collections having examples that are totally opposite to each other —here, one can anticipate that the resulting level of similarity will be the lowest.

The remainder of this work is structured as follows: Section 2 presents the IFS concept as well as the similarity measures that were tested; Section 3 describes how the simulated experience-based evaluation sets were obtained; Section 4 describes the test procedure that was carried out for each of the chosen similarity measures; Section 5 presents the results and our findings during the testing process; and Section 6 concludes the paper.

2 PRELIMINARIES

This section presents a brief introduction to the IFS concept and shows how an IFS is used to model an experience-based evaluation set. Additionally, it presents some of the existing similarity measures for IFSs and introduces the formal notation that has been used throughout the paper.

2.1 IFS Concept

In (Atanassov, 1986; Atanassov, 2012), an *intuitionistic fuzzy set*, IFS for short, was proposed as an extension of a *fuzzy set* (Zadeh, 1965) and was defined as follows:

Definition 1 ((Atanassov, 1986; Atanassov, 2012)). *Consider an object x in the universe of discourse X*

and a set $A \subseteq X$. An intuitionistic fuzzy set is a collection

$$A^* = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid (x \in X) \wedge (0 \leq \mu_A(x) + \nu_A(x) \leq 1) \}, \quad (1)$$

such that the functions $\mu_A : X \mapsto [0, 1]$ and $\nu_A : X \mapsto [0, 1]$ define the degree of membership and the degree of non-membership of $x \in X$ to the set A respectively.

In addition, the equation

$$h_A(x) = 1 - \mu_A(x) - \nu_A(x) \quad (2)$$

was proposed in (Atanassov, 1986) to represent the lack of knowledge (or hesitation) about the membership or non-membership of x to the set A .

2.1.1 Modeling Experience-based Evaluations

An IFS can be used to model an experience-based evaluation set (Loor and De Tré, 2014). For instance, $X = \{ \text{'Popeye the Sailor'}, \text{'The Avengers'} \}$ could represent the ‘comic books’ that you asked your coworkers to evaluate for, and A could represent a set of the ‘comic books suitable for 7-year-old kids’. If so, the IFS $A^* = \{ \langle \text{'Popeye the Sailor'}, 0, 0.8 \rangle, \langle \text{'The Avengers'}, 0.5, 0.3 \rangle \}$ might represent the evaluations given by one of your coworkers.

2.1.2 IFS Notation

Even though Definition 1 and the previous example show the difference between the IFS A^* and the set A , as it was suggested in (Atanassov, 1986) we shall hereafter use A instead of A^* as a notation for an IFS.

2.2 Similarity Measures for IFSs

Let A and B be two IFSs in $X = \{x_1, \dots, x_n\}$, a similarity measure S is usually defined as a mapping $S : X^2 \mapsto [0, 1]$ such that $S(A, B)$ denotes the level to which A is similar to B with 0 and 1 representing the lowest and the highest levels respectively.

Recalling the difference between an IFS P^* and a set P in Definition 1, the IFSs A and B in $S(A, B)$ correspond to

$$P_{@A}^* = \{ \langle x, \mu_{P_{@A}}(x), \nu_{P_{@A}}(x) \rangle \mid (x \in X) \wedge (0 \leq \mu_{P_{@A}}(x) + \nu_{P_{@A}}(x) \leq 1) \},$$

and

$$P_{@B}^* = \{ \langle x, \mu_{P_{@B}}(x), \nu_{P_{@B}}(x) \rangle \mid (x \in X) \wedge (0 \leq \mu_{P_{@B}}(x) + \nu_{P_{@B}}(x) \leq 1) \},$$

respectively, where $P_{@A}$ and $P_{@B}$ represent the individual understanding of P as seen from the perspectives of the evaluators who provide the IFSs A and B . This means that, in the context of experience-based evaluations, $S(A, B)$ measures the similarity between IFSs A and B with regards to individual understandings of a *common set* P . For instance, if P represents a collection of ‘comic books suitable for 7-year-old kids,’ $S(A, B)$ will measure the similarity between two experience-based evaluation sets taking into account the individual understandings of P that the providers of IFSs A and B might have.

The above clarification is needed because we identify two approaches in the formulation of similarity measures for IFSs: a *symmetric* (or metric distance) approach, which considers that $S(A, B) = S(B, A)$ always holds; and a *directional* approach, which considers that $S(A, B) = S(B, A)$ only holds in situations in which the evaluators who provide the IFSs A and B have the same understandings of the common set behind these IFSs.

2.2.1 Symmetric Similarity Measures

Among others, the following symmetric similarity measures for IFSs have been studied:

$$S_{H3D}(A, B) = 1 - \frac{1}{2n} \sum_{i=1}^n (|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |h_A(x_i) - h_B(x_i)|) \quad (3)$$

and

$$S_{H2D}(A, B) = 1 - \frac{1}{2n} \sum_{i=1}^n (|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|), \quad (4)$$

which are based on Hamming distance (Szmidi and Kacprzyk, 2000);

$$S_{E3D}(A, B) = 1 - \left(\frac{1}{2n} \sum_{i=1}^n ((\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2 + (h_A(x_i) - h_B(x_i))^2) \right)^{\frac{1}{2}} \quad (5)$$

and

$$S_{E2D}(A, B) = 1 - \left(\frac{1}{2n} \sum_{i=1}^n ((\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2) \right)^{\frac{1}{2}}, \quad (6)$$

which are based on Euclidean distance (Szmidi and Kacprzyk, 2000); and

$$S_{COS}(A, B) = \frac{1}{n} \sum_{i=1}^n (\mu_A(x_i) \mu_B(x_i) + \nu_A(x_i) \nu_B(x_i) + h_A(x_i) h_B(x_i)) / \left((\mu_A(x_i)^2 + \nu_A(x_i)^2 + h_A(x_i)^2)^{\frac{1}{2}} \left(\mu_B(x_i)^2 + \nu_B(x_i)^2 + h_B(x_i)^2 \right)^{\frac{1}{2}} \right), \quad (7)$$

which is based on Bhattacharyas’s distance (Szmidi and Kacprzyk, 2013).

2.2.2 Directional Similarity Measures

The following two directional similarity measures for IFSs have been studied:

$$S^\alpha(A, B) = 1 - \frac{1}{n} \sum_{i=1}^n |dif^\alpha(\mathbf{a}_i, \mathbf{b}_i)|, \quad (8)$$

where $\alpha \in [0, 1]$ is called *hesitation splitter*,

$$\mathbf{a}_i = \begin{pmatrix} \mu_A(x_i) + \alpha h_A(x_i) \\ \nu_A(x_i) + (1 - \alpha) h_A(x_i) \end{pmatrix}$$

and

$$\mathbf{b}_i = \begin{pmatrix} \mu_B(x_i) + \alpha \pi_B(x_i) \\ \nu_B(x_i) + (1 - \alpha) \pi_B(x_i) \end{pmatrix}$$

are vector interpretations of the IFS-elements in IFSs A and B related to x_i (Loor and De Tré, 2014), and

$$dif^\alpha(\mathbf{a}_i, \mathbf{b}_i) = (\mu_A(x_i) - \mu_B(x_i)) + \alpha (h_A(x_i) - h_B(x_i)) \quad (9)$$

is the *spot difference* between the IFS-elements corresponding to x_i in A and B respectively (Loor and De Tré, 2014); and

$$S_{@A}^\alpha(A, B) = \Delta_{@A} \cdot S^\alpha(A, B), \quad (10)$$

which is an extension of (8) based on the weight $\Delta_{@A} \in [0, 1]$ of a *connotation-differential print* (CDP) between A and B as seen from the perspective of the evaluator who provides A (Loor and De Tré, 2014). A CDP is defined as a sequence that represents any difference in the understandings of the common set behind IFSs A and B (Loor and De Tré, 2014). Since such a difference in understandings is deemed to be subjective, the assembling of a CDP will depend on either the perspective of who provides A or the perspective of who provides B (i.e., it is directional); so will do its weight (Loor and De Tré, 2014).

3 SIMULATION

As was mentioned in the introduction, the aim of this work is to study empirically which of the similarity measures presented in Section 2.2 can be used to compare experience-based evaluation sets represented by IFSs. Hence, in this section we describe both the learning and the evaluation processes that were used to obtain the IFSs that represent the *simulated* experience-based evaluation sets.

3.1 Learning Process

In this part we describe the data, scenarios and algorithm that were employed to simulate how a human editor categorizes newswire stories.

3.1.1 Learning Data

We made use of the Reuters Corpora Volume I (RCV1) (Rose et al., 2002), which is a collection of manually categorized newswire stories provided by Reuters, Ltd. Specifically, we made use of the corrected version RCV1.v2, which is available (and fully described) in (Lewis et al., 2004). This collection has 804414 newswire stories, each assigned to one or more (sub) categories within three main categories: *Topics*, *Regions* and *Industries*.

We made use of the 23149 newswire stories in the training file *lyrl2004_tokens_train.dat* to learn how to categorize newswire stories into one or more of the following categories from *Topics*: *ECAT*, *E11*, *E12*, *GSCI*, *GSPO*, *GTOUR*, *GVIO*, *CCAT*, *C12*, *C13*, *GCAI*, *G15*, *GDEF*, *GDIP*, *GDIS*, *GENT*, *GENV*, *GFAS*, *GHEA* and *GJOB*. The interested reader is referred to (Lewis et al., 2004) for a full description of these categories.

3.1.2 Learning Scenarios

We established the following scenarios to learn how to categorize newswire stories into each of the chosen categories:

- *R0*: All the stories in the training data preserve the assignation of the training category in its original state.
- *R20*, *R40*, *R60*, *R80*, *R100*: The assignation of the training category is opposite to its original state in the 20%, 40%, 60%, 80% and 100% of the stories in the training data respectively. The assignation of the training category in the remainder of the stories is preserved. The selection of the stories that do not preserve the original state is made through a simple random sampling.

For instance, consider the story with code 2286, which was assigned to the category *ECAT*. In the scenario *R20*, if the training category is *ECAT* and the story is selected to change its category, the story will be considered as a nonmember of *ECAT*.

3.1.3 Learning Algorithm

We made use of an algorithm based on *support vector machines*, or SVM for short (Vapnik, 1995; Vapnik and Vapnik, 1998), which have been successfully used in statistical learning theory. Specifically, we made use of the application of SVMs for the *text categorization problem* proposed in (Joachims, 1998), which has demonstrated superior results to deal with such a problem (Lewis et al., 2004).

In the context of the text categorization problem, the words in a newswire story are the features that determine whether the story belongs or not to a category. This follows an intuition in which, according to his/her experience, a person focuses on the words in a document to decide whether it fits or not into a given category.

To use the SVM algorithm, each story must be modeled as a vector whose components are the words in the story. A story might contain words such as 'the', 'of' or 'at' that have a negligible impact on the categorization decision, or words such as 'learning', 'learned' or 'learn' that have a common stem. To simplify the vector representation, such words are usually filtered out and stemmed by using different algorithms. Hence, for the sake of reproducibility of the simulation, we made use of the stories in the training file *lyrl2004_tokens_train.dat* (Lewis et al., 2004), which already have reduced and stemmed words. For example, the story with code 2320 has the following words: *tuesday*, *stock*, *york*, *seat*, *seat*, *nys*, *level*, *million*, *million*, *sold*, *sold*, *current*, *off*, *exchang*, *exchang*, *bid*, *prev*, *sale*, *mln*.

Since the impact of the words on the categorization decision could be different, a weight should be assigned to each word. Thus, to compute the (initial) weight of a word in a story (or document), as it was suggested in (Lewis et al., 2004), we applied the equation

$$weight(f,x) = (1 + \ln n(f,x)) \ln(|X_0|/n(f,X_0)), \quad (11)$$

which is a kind of tf-idf weighting given in (Buckley et al., 1994) where X_0 is the training collection (i.e., the collection of stories in *lyrl2004_tokens_train.dat*), $x \in X_0$ is a story, f is a word in x , $n(f,x)$ is the number of occurrences of f in x , $n(f,X_0)$ is the number of stories in X_0 that contain f , and $|X_0|$ is the number of stories in X_0 (i.e., $|X_0| = 23149$). For ex-

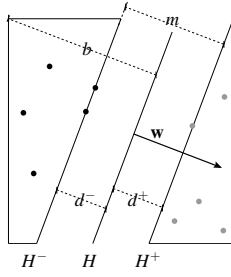


Figure 1: Idea behind the SVM algorithm.

ample, the weight of the word *exchang* in the story with code 2320 is given by $weight(exchang, 2320) = (1 + \ln 3) \ln(23149/2485) = 4.6834$.

After computing the weight of each word in a story with code i , say x_i , we represented x_i as a vector $\mathbf{x}_i = \beta_{i,1}\hat{\mathbf{f}}_1 + \dots + \beta_{i,|F|}\hat{\mathbf{f}}_{|F|}$ such that:

- F is a dictionary having all the distinct words in the training collection X_0 ;
- $|F|$ is the number of words in F (for the chosen training collection, $|F| = 47152$);
- $\hat{\mathbf{f}}_k$ is a unit vector that represents an axis related to a word $f_k \in F$ (i.e., $\hat{\mathbf{f}}_k$ belongs to a multi-dimensional feature space in which each dimension corresponds to a word $f_k \in F$); and
- $\beta_{i,k} = weight(f_k, x_i)$ is the weight of f_k in x_i (if f_k is not present in the story, $\beta_{i,k}$ will be fixed to 0).

Since the stories may have different number of words, each $\beta_{i,k}$ in \mathbf{x}_i was divided by $\|\mathbf{x}_i\| = \sqrt{\mathbf{x}_i \cdot \mathbf{x}_i}$, i.e., \mathbf{x}_i was transformed to a unit vector (Lewis et al., 2004).

Idea behind the SVM algorithm: So far we have described how each story x_i in the training collection X_0 was represented by a vector \mathbf{x}_i . To describe how we made use of those vectors (and the resulting ones later on), in what follows we briefly explain the idea behind the SVM algorithm (see (Burges, 1998) for a tutorial about SVM).

In Figure 1 the vectors corresponding to stories that fit into a given category (i.e., positive examples) are depicted with gray-circle heads, while the vectors that do not fit into the category (i.e., negative examples) are depicted with black-circle heads. The hyperplane H separates the positive from the negative examples—here H is defined by $\mathbf{w} \cdot \mathbf{x} + b = 0$, where \mathbf{w} is a vector perpendicular to H , \mathbf{x} is a point lying on H , and b is the perpendicular distance between H and the origin. The hyperplane H^+ is parallel to H and contains the closest positive example to it. The hyperplane H^- is also parallel to H and contains the closest negative example to it. The margin $m = d^+ + d^-$ between H^+ and H^- is the largest. The *support vectors* are the vectors whose heads lay either on H^- or H^+ .

To find the hyperplane H that maximizes the margin between H^+ and H^- the following quadratic programming problem should be solved

$$\Lambda = \sum_{i=1}^n \lambda_i - \frac{1}{2} \sum_{i=1, j=1}^n \lambda_i \lambda_j y_i y_j \mathbf{x}_i \cdot \mathbf{x}_j, \quad (12)$$

where \mathbf{x}_i and \mathbf{x}_j are the vectors corresponding to stories in the training collection, y_i (or y_j) denotes whether the \mathbf{x}_i (or \mathbf{x}_j) fits ($y_i = 1$) or not ($y_i = -1$) into the category, $\lambda_i, \lambda_j \geq 0$, and n is the number of stories in the training collection. The solution is given by both

$$\mathbf{w} = \sum_{k=1}^n \lambda_k y_k \mathbf{x}_k \quad (13)$$

and

$$b = y_k - \mathbf{w} \cdot \mathbf{x}_k, \quad (14)$$

for any \mathbf{x}_k such that $\lambda_k > 0$.

To compute both (13) and (14), we made use of the package *SVMLight Version V6.02* (Joachims, 1999). We issued the command “*svm_learn.exe -c 1 svm-TrainingFile svmModelFile*”, where *svmTrainingFile* is an input file that contains the training vectors for a category under a given scenario, and *svmModelFile* is an output file that contains the solution (or model) of the scenario-category learning process. Using the 6 scenarios and 20 categories described above, we obtained 120 *scenario-category* models during this learning process—hereafter a model will be referred to using the nomenclature *scenario-category*.

3.2 Evaluation Process

Consider a collection of newswire stories X . To evaluate the level to which a newswire story $x \in X$ fits into a category, say *ECAT*, under a given scenario, say *R20*, we use the *R20-ECAT* model, which represents the experience (or knowledge) acquired after the previous learning process. After evaluating all the newswire stories in X , we obtain an evaluation set for X . This evaluation set corresponds to the *simulated* experience-based evaluation set given by a person who learned the concept *ECAT* using the training data specified in the scenario *R20*.

The data and the process that were utilized to generate such *simulated* experience-based evaluation sets are described below.

3.2.1 Evaluation Data

We made use of the first 12500 newswire stories in each of the following files from RCV1.v2 (Lewis et al., 2004):

- *lyrl2004_tokens_test_pt0.dat*,

- *lyrl2004_tokens_test_pt1.dat*,
- *lyrl2004_tokens_test_pt2.dat* and
- *lyrl2004_tokens_test_pt3.dat*.

With these 50000 stories, we built 1000 50-story collections.

3.2.2 Obtaining an IFS as a Result of an Evaluation Process

Let X_k be one of the 50-story collections that constitute the evaluation data. To evaluate the level to which a story $x_i \in X_k$ fits into a category, say C , under a given (learning) scenario, say LS , we made use of the LS - C model resulting from the previous learning process to obtain an IFS-element $\langle x_i, \mu_C(x_i), \nu_C(x_i) \rangle$ as follows.

First, we represented x_i as a vector $\mathbf{x}_i = \beta_{i,1}\hat{\mathbf{f}}_1 + \dots + \beta_{i,|F|}\hat{\mathbf{f}}_{|F|}$ according to the procedure described in the previous section, where X_0 corresponds to the training collection in the scenario S .

Then, we made use of $\mathbf{w} = \omega_1\hat{\mathbf{f}}_1 + \dots + \omega_{|F|}\hat{\mathbf{f}}_{|F|}$ and b in the S - C model to figure out $\mu_C(x_i)$ and $\nu_C(x_i)$ by means of the equations

$$\mu_C(x_i) = \check{\mu}_C(x_i)/\sigma \quad (15)$$

and

$$\nu_C(x_i) = \check{\nu}_C(x_i)/\sigma \quad (16)$$

respectively, where

$$\check{\mu}_C(x_i) = \begin{cases} \frac{(\sum_{j=1}^{|F|} \beta_{i,j}\omega_j) + |b|}{\|\mathbf{x}_i\| \|\mathbf{w}\|} & : (\beta_{i,j}\omega_j > 0) \wedge (b < 0); \\ \frac{\sum_{j=1}^{|F|} \beta_{i,j}\omega_j}{\|\mathbf{x}_i\| \|\mathbf{w}\|} & : (\beta_{i,j}\omega_j > 0) \wedge (b \geq 0); \\ 0 & : \text{otherwise}; \end{cases} \quad (17)$$

$$\check{\nu}_C(x_i) = \begin{cases} \frac{(\sum_{j=1}^{|F|} |\beta_{i,j}\omega_j|) + b}{\|\mathbf{x}_i\| \|\mathbf{w}\|} & : (\beta_{i,j}\omega_j < 0) \wedge (b > 0) \\ \frac{\sum_{j=1}^{|F|} |\beta_{i,j}\omega_j|}{\|\mathbf{x}_i\| \|\mathbf{w}\|} & : (\beta_{i,j}\omega_j < 0) \wedge (b \leq 0); \\ 0 & : \text{otherwise}; \end{cases} \quad (18)$$

and

$$\sigma = \max(1, \check{\mu}_C(x_i) + \check{\nu}_C(x_i)), \forall x_i \in X_k. \quad (19)$$

Finally, after computing all the IFS-elements for each $x_i \in X_k$, we obtained an IFS that represents the simulated experience-based evaluations for the stories in X_k according to what was learned (or experienced) about the category C under the scenario LS .

Since we built 1000 50-story collections, we obtained 1000 IFSs for each scenario-category model. We made use of the notation $C_{@LS}(X_k)$ to denote an IFS that represents the simulated experience-based evaluations for the stories in X_k according to what was learned about category C under a scenario LS . For

Table 1: IFSs that represent the simulated experience-based evaluations for the stories in each $X_k \in \{X_1, \dots, X_{1000}\}$ according to what was learned about category $E11$ under the scenarios $R0$, $R20$, $R40$, $R60$ and $R100$ respectively.

$E11$	X_1	\dots	X_{1000}
$R0$	$E11_{@R0}(X_1)$	\dots	$E11_{@R0}(X_{1000})$
$R20$	$E11_{@R20}(X_1)$	\dots	$E11_{@R20}(X_{1000})$
$R40$	$E11_{@R40}(X_1)$	\dots	$E11_{@R40}(X_{1000})$
$R60$	$E11_{@R60}(X_1)$	\dots	$E11_{@R60}(X_{1000})$
$R80$	$E11_{@R80}(X_1)$	\dots	$E11_{@R80}(X_{1000})$
$R100$	$E11_{@R100}(X_1)$	\dots	$E11_{@R100}(X_{1000})$

example, Table 1 shows the IFSs that represent the simulated experience-based evaluations for the stories in each $X_k \in \{X_1, \dots, X_{1000}\}$ according to what was learned about category $E11$ under the scenarios $R0$, $R20$, $R40$, $R60$ and $R100$ respectively.

Considering that we chose 20 categories and built 6 scenarios during the learning phase, we obtained a total of 120000 IFSs during this phase.

4 TESTING

In this section we describe how the similarity measures presented in Section 2.2 were tested with the IFSs that represent simulated experience-based evaluation sets.

4.1 A point of Reference for the Perceived Similarity

Consider a scenario-category model LS - C represented by both \mathbf{w} and b according to the equations (13) and (14) respectively (see Section 3.1.3). Consider then a story $x_i \in X_k$ represented by \mathbf{x}_i , where X_k is one of the 50-story collections in the evaluation data (see Section 3.2.2). Consider finally a collection $Y_k = \{y_i | (y_i = \mathbf{w} \cdot \mathbf{x}_i + b)\}$ such that y_i is the *SVM-based evaluation* of story $x_i \in X_k$ fitting into the category C under the scenario LS . In this context, the decision about the fittingness of the story x_i into the category C under the scenario LS will depend on y_i : when $y_i > 0$, the decision will be “ x_i fits into C ,” when $y_i < 0$, the decision will be “ x_i does not fit into C ,” and when $y_i = 0$, no decision will be taken. A visual interpretation of this decision process is observable in Figure 1: when $y_i > 0$ the head of the vector \mathbf{x}_i corresponding to story x_i will be on the H^+ -side, i.e., it will have a gray-circle head; when $y_i < 0$ the head of \mathbf{x}_i will be on the H^- -side, i.e., it will have a black-circle head; and when $y_i = 0$ the head of \mathbf{x}_i will be on H (see (Lewis et al., 2004) for more details about the influence of this decision process in the text categorization problem).

Now consider the collections $Y_{k@L1}$ and $Y_{k@L2}$ having SVM-based evaluations under scenarios $L1$ and $L2$ respectively. Consider also $y_{i@L1} \in Y_{k@L1}$ and $y_{i@L2} \in Y_{k@L2}$. In this situation, when

$$\begin{aligned} & ((y_{i@L1} < 0 \wedge y_{i@L2} < 0) \\ & \vee (y_{i@L1} > 0 \wedge y_{i@L2} > 0) \\ & \vee (y_{i@L1} = 0 \wedge y_{i@L2} = 0)) \end{aligned}$$

is true, an *agreement on decision* about the fittingness of story x_i between the evaluations given under scenarios $L1$ and $L2$ occurs.

We made use of the agreements on decisions between $Y_{k@L1}$ and $Y_{k@L2}$ to obtain an *agreement-on-decision ratio*, AoD for short, which is expressed by

$$AoD(Y_{k@L1}, Y_{k@L2}) = n/N, \quad (20)$$

where n represents the number of agreements on decision between $Y_{k@L1}$ and $Y_{k@L2}$, and N represents the number of stories in X_k . Since the AoD ratio denotes how similar the decisions are, we deemed it to be an indicator of the perceived similarity between the evaluations given by two persons that learned (or experienced) C under $L1$ and $L2$ respectively.

4.2 Testing Procedure and Settings

As was mentioned in the Introduction, an experience-based evaluation mainly depends on what an evaluator has experienced or learned about a particular concept. Thus, one could expect that the level of similarity between the evaluation sets given by two evaluators who learned a concept under the same (learning) scenario will be greater than or equal to the level of similarity between the evaluation sets given by two evaluators who learned the same concept under different scenarios. For instance, consider three evaluators: P , Q and R . While P and Q learned about category $E11$ under the same scenario $R0$, R learned so under the scenario $R80$. Consider also that the IFSs $E11_{@R0}^P(X_k)$, $E11_{@R0}^Q(X_k)$ and $E11_{@R80}^R(X_k)$ represent the experience-based evaluation sets about the fittingness of the stories in the 50-story collection X_k into category $E11$ given by P , Q and R respectively. In this context, one could expect that the similarity between $E11_{@R0}^P(X_k)$ and $E11_{@R0}^Q(X_k)$ will be greater than the similarity between $E11_{@R0}^P(X_k)$ and $E11_{@R80}^R(X_k)$.

We made use of the intuition given above to test the similarity measures presented in Section 3.2.2. Since we chose the AoD ratio as an indicator of the perceived similarity, we first tested it to observe how the agreement on decisions between two SVM-based evaluation sets is affected according to their respective learning scenarios. We then tested the similarity measures, some of them with different configurations.

4.2.1 Testing the Agreement-on-decision Ratio

Again, one could expect that the AoD ratio between two SVM-based evaluation sets resulting from the same scenario will be greater than the AoD ratio between two SVM-based evaluation sets resulting from distinct scenarios. Thus, we considered the question: is there sufficient evidence in the evaluation data to suggest that the mean AoD ratio is different after altering a given percentage of the training data? To answer this, for each category and for each 50-story collection, we obtained the AoD ratio between the SVM-based evaluation set given under scenario $R0$ (i.e., $R0$ is a *referent* scenario) and each of the SVM-based evaluation sets given under the scenarios $R0, R20, R40, R60, R80$ and $R100$ respectively. Algorithm 1 shows the steps to obtain the AoD ratios.

Algorithm 1 : Obtaining AoD ratios.

Require: *ChosenCategories* {see Section 3.1.1}
Require: *LearningScenarios* {see Section 3.1.2}
Require: *50storyCollections* {see Section 3.2.1}
Require: *SVMVals* {see Section 4.1}

- 1: $Z \leftarrow \emptyset$ {resulting ratios}
- 2: **for all** $C \in \text{ChosenCategories}$ **do**
- 3: **for all** $X_k \in \text{50storyCollections}$ **do**
- 4: $Y_{k@R0} \leftarrow \text{SVMVals}[X_k][R0][C]$
- 5: **for all** $LS \in \text{LearningScenarios}$ **do**
- 6: $Y_{k@LS} \leftarrow \text{SVMVals}[X_k][LS][C]$
- 7: $r \leftarrow \text{AoD}(Y_{k@R0}, Y_{k@LS})$
- 8: $Z[C][LS][X_k] \leftarrow r$
- 9: **end for**
- 10: **end for**
- 11: **end for**
- 12: **return** Z

4.2.2 Testing the Similarity Measures

First, we applied the following labels and settings to the similarity measures:

- for (3) and (4), $H3D$ and $H2D$ respectively;
- for (5) and (6), $E3D$ and $E2D$ respectively;
- for (7), COS ;
- for (8), $VB-\alpha$, with $\alpha = 0, 1$;
- for (10), $XVB-\alpha-w$, with $\alpha = 0, 0.5, 1$ and $\Delta_{@A} = \text{weightCDP}(A, B, w)$, with $w = 0.05, 0.1, 0.2$ as explained below.

To compute $\Delta_{@A}$ in the settings of (10), we made use of $\text{weightCDP}(A, B, w)$, where A and B are the IFSs in the comparison, and $w \in [0, 1]$ is a value that allows us to obtain a CDP (see Section 3.2.2) between A and

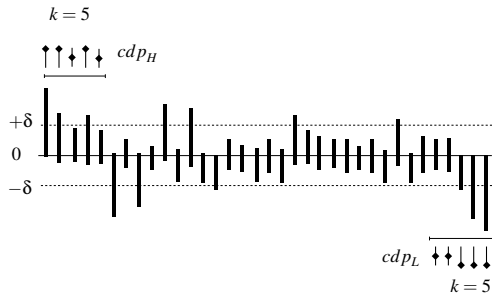


Figure 2: Obtaining a CDP and its weight. The bars represent the spot differences between the elements of IFSs A and B . The CDPs for the k -highest and the k -lowest IFS-elements according to A 's perspective are denoted by cdp_H and cdp_L respectively.

B according to the wide of the average gap between the membership and non-membership values as seen from the perspective of who provides A . The method *weightCDP* involves the following steps:

1. Obtain $\delta \in [0, 1]$ for IFS A by means of

$$\delta = \frac{w}{n} \sum_{i=1}^n (\mu_A(x_i) + \nu_A(x_i)). \quad (21)$$

2. Compute the spot differences among the IFS-elements in A and B using (9).
3. Order the IFS-elements in A by descending membership values and then by ascending non-membership values.
4. Fix $k = 0.1n$ (i.e., $k = 5$) and obtain the connotation-differential markers (i.e., \uparrow , $\hat{\uparrow}$ and \downarrow (Loor and De Tré, 2014)) for the k -highest and the k -lowest IFS-elements in the arranged IFS A (see Figure 2). For a spot difference s , the marker will be: \uparrow when $|s| \leq \delta$; $\hat{\uparrow}$ when $s > \delta$; and \downarrow when $s < -\delta$.
5. Build the CDPs cdp_H and cdp_L with the markers corresponding to k -highest and the k -lowest IFS-elements respectively (see Figure 2).
6. Fix $w[\uparrow] = 1$, $w[\hat{\uparrow}] = 0.01$ and $w[\downarrow] = 0.01$, and compute $\Delta_{@A}$ by means of

$$\Delta_{@A} = \max \left(\frac{1}{k} \sum_{m \in cdp_H} w[m], \frac{1}{k} \sum_{m \in cdp_L} w[m] \right) \quad (22)$$

Then, as was done with the agreement-on-decision ratio, for each category and for each 50-story collection we obtained the level of similarity between the IFS given under scenario $R0$ and each of the IFSs given under the scenarios $R0, R20, R40, R60, R80$ and $R100$ respectively by means of each of the established similarity measures. Algorithm 2 shows the steps to obtain the levels of similarity.

Algorithm 2 : Testing similarity measures.

Require: *SimMeasures* {see Sections 2.2 and 4.2.2}
Require: *ChosenCategories* {see Section 3.1.1}
Require: *LearningScenarios* {see Section 3.1.2}
Require: *50storyCollections* {see Section 3.2.1}
Require: *IFSEvals* {see Section 3.2.2}

- 1: $Z \leftarrow \emptyset$ {resulting levels}
- 2: **for all** $C \in \text{ChosenCategories}$ **do**
- 3: **for all** $X_k \in \text{50storyCollections}$ **do**
- 4: $C_{@R0}(X_k) \leftarrow \text{IFSEvals}[X_k][R0][C]$
- 5: **for all** $LS \in \text{LearningScenarios}$ **do**
- 6: $C_{@LS}(X_k) \leftarrow \text{IFSEvals}[X_k][LS][C]$
- 7: **for all** $S \in \text{SimMeasures}$ **do**
- 8: $l \leftarrow S(C_{@R0}(X_k), C_{@LS}(X_k))$
- 9: $Z[C][LS][X_k][S] \leftarrow l$
- 10: **end for**
- 11: **end for**
- 12: **end for**
- 13: **end for**
- 14: **return** Z

5 RESULTS AND DISCUSSION

This section presents the results after following the test conditions described in the previous section.

5.1 Agreement-on-decision Ratio as an Indicator of the Perceived Similarity

To answer the question *is there sufficient evidence in the evaluation data to suggest that the mean AoD ratio is different after altering a given percentage of the training data?*, we first made use of the collection resulting of Algorithm 1 to compute the averages of the AoD ratios per scenario-category. We then ran the t-test for the null hypothesis “*the average of the AoD ratio is the same after altering the $r\%$ of the training data*” in contrast to the alternative one “*the average of the AoD ratio is different after altering the $r\%$ of the training data*” according to r given in each scenario (see Table 2).

The results in Table 2 show that, for the scenarios $R20, R40, R60$ and $R100$, the t-values were statistically significant ($p < 0.05$). Consequently, we can say that there is sufficient evidence in the evaluation data to suggest that the average of the AoD ratio is different after altering the 20%, 40%, 60% or 100% of the training data.

Recalling that we deemed the AoD ratio to be an indicator of the perceived similarity, we can confidently expect that it will be affected by the different learning scenarios established in the simulation. This

Table 2: Averages of the AoD ratios per scenario-category, and t-test for the null hypothesis “the average of the AoD ratio is the same after altering the $r\%$ of the training data” according to r given in each scenario (e.g., $r = 20$ in scenario $R20$), where $R0$ ($r = 0$) is the referent scenario.

Category	R20	R40	R60	R80	R100
C12	0.7292	0.5900	0.4757	0.2897	0.0001
C13	0.7385	0.6091	0.4281	0.2766	0.0002
CCAT	0.9372	0.7505	0.2711	0.0663	0.0001
E11	0.6431	0.5740	0.4722	0.3519	0.0003
E12	0.7156	0.5792	0.4796	0.3080	0.0001
ECAT	0.8187	0.6273	0.4052	0.1853	0.0002
G15	0.7186	0.5954	0.4781	0.3039	0.0002
GCAT	0.9314	0.7472	0.2690	0.0661	0
GDEF	0.6515	0.5717	0.4668	0.3602	0.0002
GDIP	0.7433	0.5990	0.4587	0.2672	0.0002
GDIS	0.7229	0.5951	0.4729	0.3022	0.0002
GENT	0.7066	0.5796	0.4802	0.3209	0.0002
GENV	0.6941	0.6009	0.4812	0.3248	0.0004
GFA5	0.6763	0.5787	0.4882	0.3457	0.0002
GHEA	0.7016	0.5850	0.4636	0.3451	0.0003
GJOB	0.7359	0.5883	0.4542	0.2886	0.0003
GSCI	0.6899	0.5854	0.4844	0.3424	0.0004
GSPO	0.8208	0.6508	0.4130	0.1962	0
GTOUR	0.5383	0.5197	0.4866	0.4663	0.0005
GVIO	0.7551	0.6368	0.4749	0.2796	0.0002
Mean	0.7334	0.6082	0.4452	0.2844	0.0002
stdDev	0.0913	0.0551	0.0643	0.0951	0.0001
N	20	20	20	20	20
df	19	19	19	19	19
t-value	13.06	31.80	38.59	33.67	34025.85
p-value	0	0	0	0	0

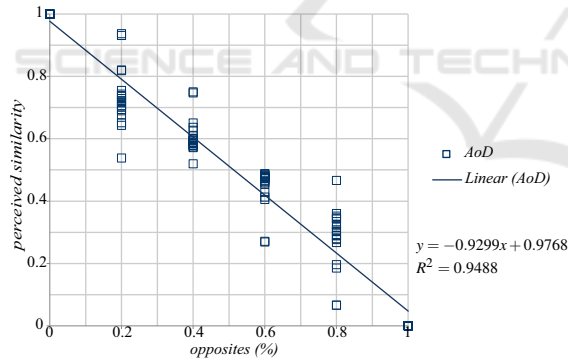


Figure 3: Bivariate plot between the averages of the AoD ratios and the percentage of opposites included in the learning scenarios. The relationship is represented by means of a linear model and described by the statistic R (Pearson Product Moment Correlation).

can be observed in the bivariate plot depicted in Figure 3, which shows a strongly negative (or inverse) relationship ($R = -0.9741$) between the averages of the AoD ratios and the percentage of opposites included in the learning scenarios.

5.2 How Each Similarity Measure Reflects the Perceived Similarity

To observe how each of the configurations of similarity measures given in Section 4.2.2 reflects the perceived similarity between the simulated IFSs, we first made use of the collection resulting of Algorithm 2 to compute the averages of the levels of similarity per scenario-category. Then, we obtained linear models for the relationships between each one of those averages and the percentage of opposites considered in each scenario. After that, each of the resulting models was contrasted with the linear model corresponding to the AoD ratio. As an indicator of how well a similarity measure reflects the perceived similarity, we computed a *manifest* index, which is defined by

$$m = (a_{SM}/a_{AoD})(b_{SM}/b_{AoD})(R_{SM}^2/R_{AoD}^2), \quad (23)$$

where a_{SM} and a_{AoD} are the slopes, b_{SM} and b_{AoD} are the intercepts, and R_{SM}^2 and R_{AoD}^2 are the R -statistics in the linear models corresponding to the similarity measure SM and the AoD ratio respectively. For readability, we shall use hereafter *SM-vs.-OP* to denote the relationship between the averages of the levels (of similarity) resulting from the (configuration of) similarity measure SM and the percentage of opposites *OP*.

Table 3: Linear models and m -indices for each *SM-vs.-OP* representing the relationship between the averages levels that result from the (configuration of) similarity measure SM and the percentage of opposites *OP*.

<i>SM-vs.-OP</i> (linear model: $y = ax + b$)				
SM	slope (a)	intercept (b)	R^2	m -index
H2D	-0.0139	0.9939	0.4128	0.0066
H3D	-0.0138	0.9852	0.1442	0.0023
E2D	-0.0171	0.9920	0.4189	0.0082
E3D	-0.0167	0.9853	0.2034	0.0039
COS	-0.0004	0.9831	0	0
VB-0	-0.0133	0.9955	0.4527	0.0070
VB-1	-0.0144	0.9922	0.3170	0.0053
XVB-0-0.05	-0.7318	0.7831	0.6871	0.4569
XVB-0.5-0.05	-0.6738	0.6999	0.5974	0.3269
XVB-1-0.05	-0.6388	0.6185	0.4666	0.2139
XVB-0-0.1	-0.6587	0.9307	0.6560	0.4666
XVB-0.5-0.1	-0.6240	0.8358	0.6878	0.4162
XVB-1-0.1	-0.5727	0.6978	0.4805	0.2228
XVB-0-0.2	-0.4218	1.0241	0.4575	0.2293
XVB-0.5-0.2	-0.4657	1.0029	0.5944	0.3221
XVB-1-0.2	-0.4335	0.8321	0.4414	0.1847
AoD	-0.9299	0.9768	0.9488	1

The results in Table 3 show that, in contrast to what happens with the AoD ratio, the averages of the levels of *H2D*, *H3D*, *E2D*, *E3D*, *COS*, *VB-0* and *VB-1* are hardly affected by the variation of the percentage of opposites (see in Figure 4 the broad dif-

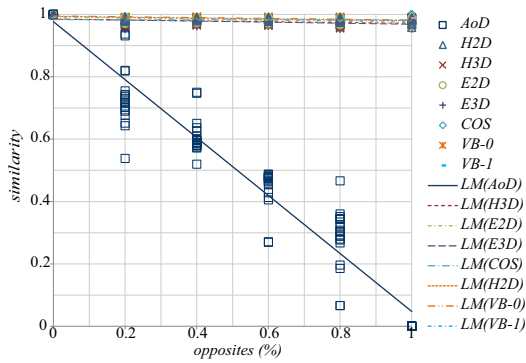


Figure 4: Bivariate plots $H2D$ -vs.- OP , $H3D$ -vs.- OP , $E2D$ -vs.- OP , $E3D$ -vs.- OP , COS -vs.- OP , $VB-0$ -vs.- OP and $VB-1$ -vs.- OP in contrast to AoD -vs.- OP .

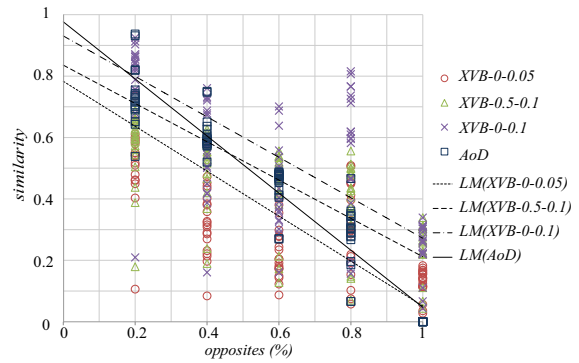


Figure 5: Bivariate plots $XVB-0-0.05$ -vs.- OP , $XVB-0-0.1$ -vs.- OP and $XVB-0.5-0.1$ -vs.- OP in contrast to AoD -vs.- OP .

ference among the slopes of the linear models corresponding to these similarity measures and the slope of the linear model corresponding to the AoD ratio). By way of illustration, if we use the resulting model for COS (i.e., $y = -0.0004x + 0.9831$) to compute the level to which the average of evaluations given under the scenarios $R0$ and $R100$ are similar, we will obtain $y = 0.9827$ as a result —since $R100$ contains the 100% of opposite training examples in relation to $R0$, we fix $x = 1$ to make this computation. Notice that this result, which is reflected by the lowest manifest index (i.e., $m = 0$), differs markedly from the result obtained for AoD (i.e., $y = 0.0469$). This means that, e.g., if one of these (configurations of) similarity measures is used in a clustering process to group evaluations given by people with different knowledge (or understanding) about a particular category, say $E11$, the evaluations given by two persons having contradictory understandings of $E11$ will probably (and badly) be put into the same group.

With respect to the averages of the levels of the configurations related to (10), the results in Table 3 show that three of them, namely $XVB-0-0.05$, $XVB-0-0.1$ and $XVB-0.5-0.1$, are fairly affected by the variation of the percentage of opposites (see Figure 5). Notice that the correlations for $XVB-0-0.05$, $XVB-0-0.1$ and $XVB-0.5-0.1$ (i.e., $R = -0.8289$, $R = -0.8099$ and $R = -0.8294$ respectively) denote fairly strong negative relationships that are roughly comparable with the strongly negative relationship ($R = -0.9741$) in AoD -vs.- OP . To illustrate this, if the resulting model for $XVS-0-0.1$ (i.e., $y = -0.6587x + 0.6560$) is used in the above example (i.e., with $x = 1$), we will obtain $y = -0.0027$ as a result, which is fairly close to the result obtained for AoD (i.e., $y = 0.0469$).

Since (10) is based on the weight of a CDP and the computation of this weight was based on the w -parameter in our testing procedure (see Section 4.2.2),

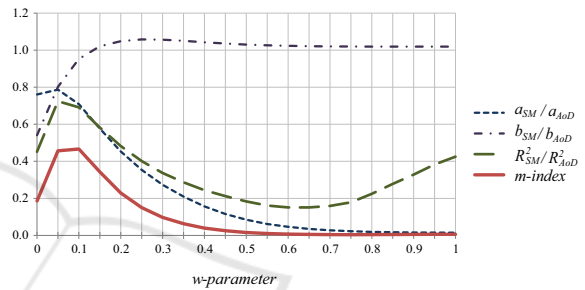


Figure 6: Influence of the w -parameter on the quality of the m -index for $XVB-0-w$.

we performed additional tests to observe the influence of this parameter on the quality of the results of this similarity measure. In such additional tests, we configured (10) with $\alpha = 0$ and $w = 0.05, 0.1, 0.15, \dots, 1$ and used the same nomenclature (i.e., $XVB-\alpha-w$) to label each configuration. Figure 6 shows how the m -index corresponding to the linear model for each $XVB-0-w$ -vs.- OP relationship is affected by the w -parameter. Notice that the peak m -index is reached at $w = 0.1$ and is projected to decline after that point. Recalling from Section 4.2.2, the w -parameter determines the *wide* of the average gap between the membership and non-membership values, which is then used to build a CDP for the IFSs in the similarity comparison as seen from the perspective of the person who provides the referent IFS. This means that, in this scenario, a spot difference with a magnitude less than or equal to the 10% of the average gap between the membership and non-membership values (see Sections 2.2.2 and 4.2.2) will roughly reflect a similar understanding (or knowledge) of the evaluate concept. This result seems to support the idea behind a CDP, which suggests that “a difference in understanding of a concept could be marked by a difference in one or more evaluations” (Loor and De Tré, 2014).

5.3 Discussion

The results suggest that only some of the configurations of the similarity measure (10), namely $XVB-0-0.05$, $XVB-0-0.1$ and $XVB-0.5-0.1$, reflect adequately the perceived similarity between the simulated IFSs. This means that, e.g., if you and a new coworker have been individually asked to evaluate the level to which several stories can be published in $E11$ and an experience-based clustering system has been used with (10) to group similar evaluations, your coworker's evaluations and yours will be adequately grouped. That is, if your coworker's understanding about $E11$ is very similar to your understanding, the evaluations of you two will fairly put into the same group; otherwise, they will be put into different groups.

A possible explanation for those results might be that, by means of the $\Delta_{@A}$, the similarity measure (10) takes into account what is understood as a qualitative difference between two IFS-elements from the perspective of the evaluator who provides the IFS A . This situation is observable when the average gap between the membership and non-membership components of the IFS-elements in A is taken into account to compute $\Delta_{@A} = \text{weightCDP}(A, B, w)$ (see Equation (21)). Since such an average gap could be very narrow in the simulated IFSs (i.e., the average hesitation margin could be closer to the highest value), it might not be taken into account by the other (configurations of) similarity measures. In other words, the similarity measure (10) seems to have the added advantage of weighting a CDP between the IFSs involved in a similarity comparison. If so, someone might ask: why not using the weight of a CDP to extend the other similarity measures and, thus, improve their results? An answer might be: yes, it is an option but just keep in mind that a CDP is *directional*, which contrasts with the *symmetric* approach assumed in some similarity measures (see Section 2.2).

Another possible explanation for the results might be that a gap between the membership and non-membership components is contextually related to the categorization decision (see Sections 3.2.2 and 4.1), which is deemed to be a point of reference for the perceived similarity through the agreement on decision ratio. Hence, a similarity measure such as (10) that takes into account the aforesaid gap could reflect more adequately the similarity perceived from the perspective of who makes the categorization decision.

Even though these results are based on simulated IFSs that use a manually categorized newswire stories, they need to be interpreted with caution because of the dependency of the IFSs with the learning algo-

rithm and the (text categorization) context that were chosen for the simulations. Consequently, conducting simulations with other learning algorithms and experiments with real evaluators is recommended and subject to further study.

6 CONCLUSIONS

An *experience-based evaluation* is deemed to be a judgment that depends on what each person has experienced or understood about a particular concept or topic. Considering that such an evaluation could be imprecise and marked by hesitation, in (Loor and De Tré, 2014) the authors proposed modeling it as an element of an intuitionistic fuzzy set, or IFS for short (Atanassov, 1986; Atanassov, 2012). This means that, from a theoretical point of view, all the existing similarity measures for IFSs could be used to compare two experience-based evaluation sets.

To study empirically which similarity measures for IFSs can actually be used to compare such IFSs, in this paper we tested some of the existing similarity measures in comparisons between pairs of IFSs that result from simulations of experience-based evaluation processes. In such simulations we made use of a learning algorithm that uses support vector machines (Vapnik, 1995; Vapnik and Vapnik, 1998) to learn how a human editor categorizes newswire stories and, then, we made use of the resulting knowledge to evaluate other stories.

The simulations were conducted under different learning scenarios to observe how the chosen similarity measures reflect the perceived similarity between two IFSs that might be given by persons with different background. A ratio that denotes how similar the decisions are was deemed to be an indicator of the perceived similarity.

The results suggest that the similarity measure proposed in (Loor and De Tré, 2014), which takes into account what is understood as a qualitative difference between two IFS-elements by means of a *connotation differential print*, could reflect more adequately the perceived similarity. Consequently, this similarity measure could potentially be used in a process such as clustering or filtering of experience-based evaluations given by people with different background, in which a proper similarity comparison is needed —e.g., clustering of evaluations given by residents about routes in their city that are suitable for kids riding a bicycle.

However, the results need to be interpreted with caution due to the dependency of the simulated IFSs with the learning algorithm and the context that were chosen for the simulations. Thus, conducting simula-

tions in other contexts with other learning algorithms as well as conducting experiments with real evaluators is recommended and subject to further study.

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