# Time Series Forecasting using Clustering with Periodic Pattern

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Abstract: Time series forecasting have attracted a great deal of attention from various research communities. One of the method which improves accuracy of forecasting is time series clustering. The contribution of this work is a new method of clustering which relies on finding periodic pattern by splitting the time series into two subsequences (clusters) with lower potential error of prediction then whole series. Having such subsequences we predict their values separately with methods customized to the specificities of the subsequences and then merge results according to the pattern and obtain prediction of original time series. In order to check efficiency of our approach we perform analysis of various artificial data sets. We also present a real data set for which application of our approach gives more then 300% improvement in accuracy of prediction. We show that in artificially created series we obtain even more pronounced accuracy improvement. Additionally our approach can be use to noise filtering. In our work we consider noise of a periodic repetitive pattern and we present simulation where we find correct series from data where 50% of elements is random noise.

## 1 INTRODUCTION

Time series forecasting is rich and dynamically growing science field and its methods applied in numerous areas such as medicine, economics, finance, engineering and many other crucial fields [(Huanmei Wu, 2005),(Zhang, 2007),(Zhang, 2003),(Tong, 1983)]. Currently there are many popular and well developed methods of time series forecasting such as ARIMA models, Neural Networks or Fuzzy Cognitive Maps [(S. Makridakis, 1997) (J. Han, 2003) (Song and Miao, 2010) ]. Clustering is process of grouping into one clusters *"by some natural criterion of similarity"* (Duda and Hart, 1973). This vague definition is one of the reason why there are so many different clustering algorithms (Estivill-Castro, 2002). Although different clustering methods group elements according to completely different criterions of similarity there always has to be mathematically defined similarity measurement metric. Every algorithm using this metric groups together elements which are closer to each other then those in other clusters. Classical example of time series clustering's usage is classification based on ECG of a particular patient into cluster of normal or dysfunctional ECG. Other type of time series clustering is presented in partition methods such SAX algorithm (Jessica Lin, 2007). Goal of that type of algorithms is discretization of numerical data which shows some features of and compress data at the same time. However using knowledge gained by clustering into time series forecasting is very limited. This results from the simple fact that even if we are able to group elements into clusters with specific forecasting properties we do not know to which clusters future elements would belong to.

We would like to bypass this problem and present usage of time series clustering for time series forecasting. Our assumption is that there exist such periodic pattern in time series based on which we are able to create subsequence with much lower potential error of prediction then whole series. Elements which are not included in chosen subsequence are grouped in second subsequence. Due to the periodicity of the pattern we can assume to which cluster future elements should belong to. Because of that we are able to predict values of every subsequence separately and then merge them according to periodic pattern to get prediction of original series X. Main problem with that idea is that number of possible periodic patterns increase exponentially according to time series length. This means that in practise evaluating potential error for every periodic pattern is impossible but using our approach we can find proposal of best pattern in reachable time.

This paper is organized as fallows. Section 2 reviews related work. The proposed approach is described with in detail in section 3. Simulations of different series are presented in section 5. In section



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4 we estimate complexity of our approach overhead according to time series length. The last section 6 concludes the paper.

# 2 RELATED WORKS

In book "Data Mining: Concepts and Techniques, Morgan Kaufmann." (J. Han, 2001) are discussed five major categories of clustering: partitioning methods (for example k-mean algorithm (MacQueen, 1967) ), hierarchical methods (for example Chameleon algorithm (G. Karypis, 1999)), density based methods (for example DBSCAN algorithm (M. Ester, 1996) ), grid-based methods (for example STING algorithm (W. Wang, 1997)) and model-based methods (for example AutoClass algorithm (P. Cheeseman, 1996)). Main property for all of these categories is grouping into one cluster elements from one interval in contrast to our approach which groups into one clusters elements scattered across the whole time series. Other type of clustering is Hybrid Dimensionality Reduction and Extended Hybrid Dimensionality Reduction [(Moon S, 2012) (S. Uma, 2012)]. These method consists of clustering of all elements with specific type of value. Algorithm can group into one clusters elements scatter across a whole time series hover it does not suggest pattern. Because of that we are not able to assume to which cluster future elements should belong to, which is a significant difference between our approach and the methods described above.

# 3 THE PROPOSED APPROACH

Main idea of our approach is to find a pattern $S$ which is periodic binary vector and according to it splitting time series $X$ into two subsequences (clusters) $X^1$ and its complement $X^0$. After that we use prediction methods on $X^1$ subsequence and $X^0$ separately. Then we merge results according to $S$ pattern and get prediction of original time series. As measurement of error we use mean square error (MSE).

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(y-\hat{y})^2 \quad (1)$$

Where $n$ is number of all predicted values, $y$ is real value and $\hat{y}$ is predicted value. MSE can be treated as similarity measure according to which we group elements into clusters. Because every element belongs to exactly one subsequence we can say that our approach use strict partitioning clustering. In order to describe our approach with more details we split algorithm to simple functions and describe them separately.

## 3.1 Create Corresponding Subsequence

We have time series $X = (x_1, x_2, ..., x_n)$ and binary vector $S = (b_1, b_2, ..., b_n)$. We create subseries $X^1 = (x_1^1, x_2^1, ..., x_j^1)$ which contains all elements $x_i$ such that corresponding $b$ is equal to 1. Analogously we create subseries $X^0$ which contains all elements $x_i$ such that corresponding $b$ is equal to 0. For example:
$X = (x_1, x_2, ..., x_n)$, $S = (1,1,0,0,1,1,0,0...0,0,1)$
$X^1 = (x_1, x_2, x_5, x_6, .., x_n)$, $X^0 = (x_3, x_4 ... x_{n-2}, x_{n-1})$

## 3.2 Extend Binary Vector

The first step needed to extend binary vector is creation of vector dictionary. A pseudo-code definition of creating vector dictionary is given in Table 1.
*create_dictionary*: gets binary vector $S$ which we would like to extend. Initially we set variable $d$ to 1 and create empty dictionary set. We create vector which is interval of length $d+1$ starting from first position. We check if dictionary contains vector which is coincides our vector on first $d$ position but differs on last $d+1$ position. If such vector occurs that means that our dictionary words are too short to predict unequivocally binary vector $S$ so we clear dictionary, increase $d$ by 1 and repeat whole process from the first position. Else if dictionary does not contain the vector we add it to the dictionary. Then we increase interval starting position by 1 and repeat the whole process till end of the interval do not exceed end of the $S$ vector. The function returns the dictionary when end of the interval exceeds end of the $S$ vector .

When we have the dictionary we can start extending $S$ vector. A pseudo-code definition of extending binary vector is given in Table 2.
*extend_binary_vector*: gets binary vector $S$ which we would like to extend and expected length value. Thanks to function *create_dictionary* we have dictionary for binary vector $S$. Let $d$ be length of every vector in that dictionary and $n$ be the length of $S$ vector. In dictionary we try to find such vector which on every position but the last is equal to $S(b_{n-d+2}...b_n)$. If there is no such vector we extend $S$ by random binary number. Otherwise we extend $S$ by last value of the vector found in dictionary. We repeat this process till $S$ length reach expected new length.

## 3.3 Find Proposition of Best Pattern S

We have to find such binary vector which:

1. Splits vector $X$ into subsequences $X^1$ and its complement $X^0$ such that MSE of that subsequences would be lower then MSE of time series $X$.

```
FUNCTION create_dictionary(S)
d=1
dict = []
i=1
WHILE i<=(size(S)-d)
  window = S(i:i+d)
  IF ismember(window(1:end-1),dict(1:end-1)
  &&  !ismember(window,dict) )
    size_of_window=size_of_window+1
    i=1
    dict = []
  ELSE
    dict = dict.add_new_row(window)
  ENDIF
  i = i+1
ENDWHILE
RETURN dict
```

Figure 1: Pseudo-code of algorithm which creates vector dictionary for pattern $S$.

```
FUNCTION extend_binary_vector(S,new_length)
dict =  create_dictionary(S)
i = size(S)-length_of_row(dict)+1
WHILE length(S)<new_length
 small_win = S(i:i+length_of_row(dict)-1)
 index =
 index_of_element(small_win,dict(:,1:end-1) )
 IF index>0
   S.add(dict.elementAt(index).elementAt(end))
 ELSE
   S.add(randomly_0_or_1())
 ENDIF
 i = i+1
ENDWHILE
RETURN S
```

Figure 2: Pseudo-code of algorithm which extends binary vector to length $c$.

2. contains regularity such that it is possible to predict correctly new values of binary vector.

A pseudo-code definition of an algorithm for finding proposal of the best pattern binary vector is given in Table 3.

*find_best_subsequence*: gets time series and arbitrary chosen constants $c$ and multiplicity number $k$. Then we create all possible different binary vectors of length $c$ such that 1s are on at least $\lceil \frac{c}{2} \rceil$ positions. We save these vectors as rows of matrix $S$. This means that $S$ has $m$ rows where for odd $c$ we get $m = 2^{c-1}$ and for even $c$ we get $m = 2^{c-1} + \frac{1}{2}\binom{c}{c/2}$. For every $i-th$ row of $S$ matrix we create vector $X_i^1$ as it was described at Section 3.1. Every subsequence $X_i^1$ is splitted in such a way that 0.7 of that series is train set $X_{i_{train}}^1$ and 0.3 is test set $X_{i_{test}}^1$ where 0.7 and 0.3 are arbitrary chosen constants. Then by using arbitrary chosen prediction method we calculate MSE of prediction. It is worth noting that we can create $m$ processes and calculate $MSE$ for vectors $X_1^1, X_2^1, ..., X_m^1$ parallel. Parallel computing in practise can significantly decrease computational time. The number of possible $S$ subsequences increase exponentially according to c number. This is why it is the most time consuming part of the algorithm.

```
FUNCTION  find_best_subsequence(time_series,c,k)
S = cob(c)//cob returns all binary combinations
//of length c with 1 on at least c/2 positions
FOR i=1;i++;i<=number_of_rows(S)
 X1(i,:)=create_subseq(S(i,:),time_series)
 Xtrain=X1(1:0.7*size(X))
 Xtest =X1(0.7*size(X):end)
 MSE = chosen_prediction_method(Xtrain,Xtest)
 S(i,end+1) = MSE
ENDFOR
S = sort_ascendning_by_last_column(S)
S = S(1:ceiling(end/k),:)
WHILE c<size(time_series)
 c=k*c
 IF c>size(time_series)
  c = size(time_series)
 ENDIF
 FOR j=1;j++;j<number_of_rows(S)
 S(j,:)=extend_binary_vector(S(j,:),c)
 X(j,:)=create_subseq(S(j,:),time_series)
 Xtrain=X(1:0.7*size(X))
 Xtest=X(0.7*size(X):end)
 MSE=any_prediction_method(Xtrain,Xtest)
 S(j,end+1) = MSE
 ENDFOR
 S = sort_ascendning_by_last_column(S)
 S = S(1:ceiling(end/k),:)
ENDWHILE
//return S with lowest MSE
RETURN  S(1,:)
```

Figure 3: Pseudo-code of algorithm which finds proposition of best subsequence.

In this part of the algorithm we have set of pairs $(S_1, MSE_1), (S_2, MSE_2), \quad (S_3, MSE_3) ... (S_m, MSE_m)$. Then we reject all S rows but $\lceil \frac{1}{k} \rceil$ of the rows with the lowest MSE. We extend rows of $S$ using function *extend_binary_vector* (refer to pseudo-code in Table 2) to get binary vectors with length $k \cdot c$. Now we have set $S_1, S_2, ..., S_{\lceil \frac{1}{k} \rceil}$ where every row S has length $k \cdot c$. For every row $S_i$ we create vectors $X_i^1$ as it was described in section 3.1. We repeat process of calculating MSE, selection and extending $S$ rows length while its length does not exceed training set length. As the result we return row $S$ with corresponding lowest MSE.

### 3.4 Time Series Forecasting

In order to predict value of $x_{t+1}$ we predict value of $b_{t+1}$ in $S$ series (refer to pseudo-code in Table 2). Then if $b_{t+1} = 1$ we take prediction $x^1_{t+1}$ calculated on subsequence $X^1$ otherwise we choose prediction $x^0_{k+1}$ calculated on subsequence $X^0$, where $X^0$ is complementary subsequence $X^1$ to $X$.

## 4 COMPLEXITY OF PROPOSED APPROACH

Our goal is to prove that clustering with our approach has time complexity equal to $O(log^n_k MSE(n))$ where $MSE$ is arbitrary chosen prediction function, $k$ is constant multiplicity parameter and $n$ is length of series. We assume that prediction function has complexity not less than $O(n)$. Firstly we determine complexities of every part of the algorithm.

### 4.1 Time Complexity of the Algorithm Which Creates Corresponding Subsequence

Algorithm which creates vector $X^1$ and $X^0$ from orignal series using pattern $S$ is described in the section 3.1. Complexity of that algorithm is $O(n)$.

### 4.2 Time Complexity of the Algorithm Which Extends Binary Vector

Algorithm which extends binary vector (refer to pseudo-code in Table 2) contains two parts. Firstly we have to create vector dictionary which is able to extend binary sequence. We notice that maximal number of vectors in dictionary cannot be larger than $2^d$ where $d$ is the vector length. However, at the same time dictionary cannot contain more then $c$ elements where $c$ is the length of the vector on which we build dictionary. Due to that we can say that in every step dictionary length is not larger that $min(2^d, c)$. Moreover we know that algorithm will produce not more that $c$ such dictionaries. We know that number of operation is equal to

$$\sum_{i=1}^{c} min(2^d, c) * c < c^2 \tag{2}$$

so we can say that time complexity of that algorithm is $O(c^2)$. Another part of the algorithm is extending binary vector using created dictionary. What is important we create dictionary only once and then we use it during whole process of clustering. Finding proper vector in dictionary costs not more than $O(log_2 c)$. This is why extending binary vector by $n$ elements cost $O(nlog_2 c)$.

### 4.3 Time Complexity of the Algorithm Which Finds Best Pattern S

Algorithm which finds best pattern $S$ is described in subsection 3.3. We choose some arbitrary length of the first subsequence $c$ and multiplicity parameter $k$. We start with subsequence of length $c$ and then in every step we extend this subsequence $k$ times. Also we remove all $S$ proposals but $\lceil \frac{1}{k} \rceil$ with the lowest corresponding $MSE$. Number of operations can be approximated by:

$$\begin{aligned} & 2^{c-1} MSE(c) + 2^{c-1} c^2 + c log_2(c) \\ & + \lceil \frac{1}{k} \rceil 2^{c-1} MSE(kc) + kc log_2(c) \\ & + \lceil (\frac{1}{k})^2 \rceil 2^{c-1} MSE(k^2 c) + k^2 c log_2(c) \\ & + ... + \lceil (\frac{1}{k})^{log^{n/c}_k} \rceil 2^{c-1} MSE(n) \end{aligned} \tag{3}$$

which is equal to

$$2^{c-1} c^2 + \sum_{i=0}^{log^{n/c}_k} \lceil (\frac{1}{k})^i \rceil 2^{c-1} (MSE(k^i c) + k^i c log_2 c) \tag{4}$$

where $2^{c-1}$ is number of all proposals of $S$ proposed in the first step, $c^2$ is maximal cost for creation binary vector dictionary (refer to Section 4.2), $log^{n/c}_k$ is the maximal number of steps after which length of $S$ reaches $n$, $MSE(k^i c)$ is cost of approximation prediction error on every step for every $S$ proposal, $k^i c log_2 c$ is cost of extending binary vector $S$ $k$ times. Taking into account this equation we can say that number of operation in our approach is definitely smaller than

$$2^c c^2 + log^{n/c}_k (2^c)(MSE(n) + (nlog_2 c)) \tag{5}$$

After taking into consideration that complexity of MSE(n) is not less than $O(n)$ and omitting constants we can say that complexity of our approach according to $n$ is equal to:

$$O(log^n_k MSE(n)) \tag{6}$$

On the contrary, the time complexity according to $c$ is equal to:

$$O(2^c) \tag{7}$$

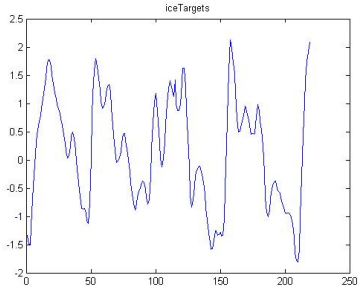



Figure 4: IceTargets series plot.

It is worth noticing that algorithm can be processed in parallel and consequently that time calculation in practise can decrease significantly.

# 5 SIMULATIONS

In order to check efficiency of our approach we made several simulations. In our simulations we used neural networks with hidden layer and delay equal to 2 (refer to diagram on Figure 5). As a neural network training method we used Levenberg-Marquardt backpropagation algorithm [(Marquardt, 1963)]. in comparative simulation we used neural networks with the same structure, training rate, training method and number of iteration as in our approach. The only difference was that neural networks in our approach were trained on subsequences chosen by our algorithm where neural network used in comparative simulation was trained on whole training set. In every simulation as a constant $c$ number we used 12 and as multiplicity parameter $k$ we used 2. In order to avoid random bias we repeated every simulation 10 times and used mean values. We also used IceTargets data which contains a time series of 219 scalar values representing measurements of global ice volume over the last 440,000 years (see Figure 4). Time series is available at (http://lib.stat.cmu.edu/datasets/, ) or in the standard Matlab library as $ice\_dataset$.

## 5.1 IceTargets with Random Noise

We modified *IceTargets* series by adding random numbers generated from uniform distribution on -1.81 to 2.12. Where -1.81 is minimum value from *IceTargets* series and 2.12 is maximum value from *IceTargets* series. Random number occurrence scheme is as fallow:
X = (rand(1),$IceTargets(1)$,rand(2),$IceTargets(2)$, rand(3), ... $IceTargets(219)$,rand(220))
Our approach finds vector $S = (0,1,0,1,...,1,0)$ which is correct pattern and splits time series according to it (refer to Table 1). Due to that neural networks separately predicts *IceTargets* series and random noise. Our approach has mean MSE equal to 0.62 where neural network trained on whole set gives MSE equal to 0.93. The results are presented in Table 2.

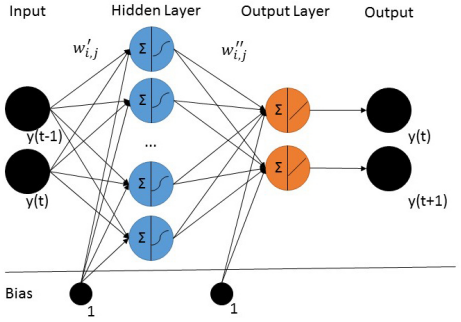



Figure 5: Diagram of neural network used in simulations.

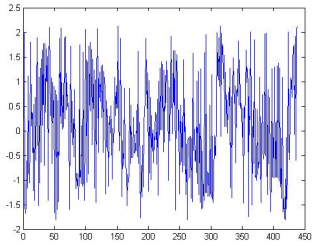
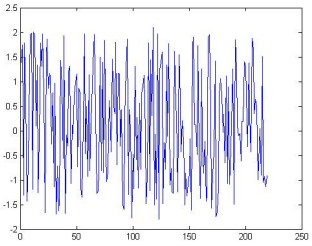
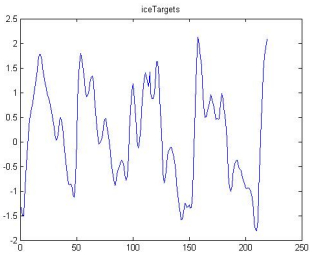
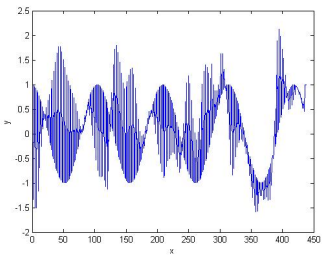
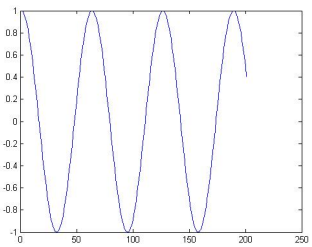
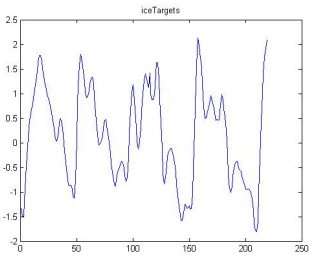
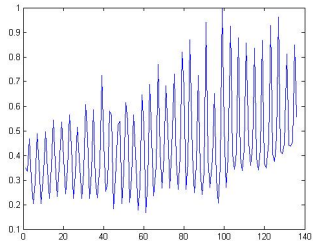
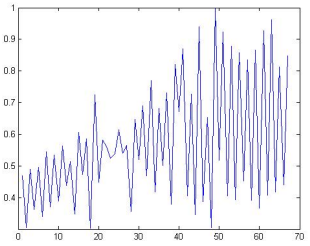
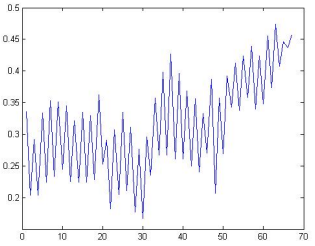
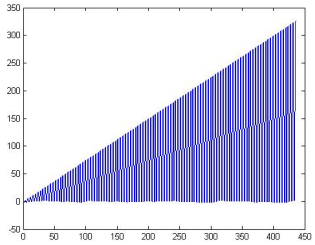
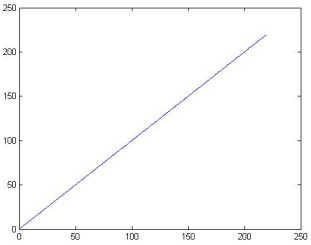
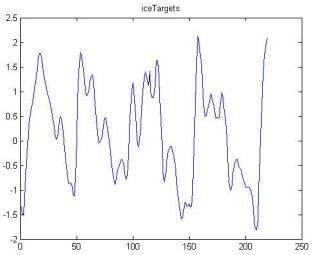
## 5.2 Cosinus with IceTargets

We created time series by merging cosinus and *IceTargets* time series using pattern:
X = (cos(0.1), $IceTargets(1)$ , cos(0.2), cos(0.3), $IceTargets(2)$, $IceTargets(3)$, cos(0.4), $IceTargets(4)$ , cos(0.5), cos(0.6), $IceTargets(5)$, $IceTargets(6)$ ...)
So pattern could be described by vector
$S = (10110010110010110010110 0...)$
Our approach finds correct pattern which splits time series into proper subsequences (refer to Table 1). Neural networks trained on subsequences give mean MSE equal to 0,0170 where neural network trained on whole training set gives MSE equal to 0,5144 (refer to Table 2).

## 5.3 Quarterly Australian Gross Farm Product

In this simulation we used real statistic data of Quarterly Australian Gross Farm Product $m 1989/90 prices. Time series is build from 135 data points represented values measured between September 1959 and March 1993. Data is available at (https://datamarket.com/data/set/22xn/quarterly-australian-gross-farm-product-m-198990-prices-sep-59-mar 93#!ds=22xn&display=line, ). The data was rescaled to 0-1 range. One of the proposed subsequence is presented on table 1. Average value of MSE of this time series forecasting calculated using our approach was equal to 0,007 when average value of MSE achieved by single neural network was equal to 0,0211 (refer to Table 2).

Table 1: Table which presents on different time series plots of subsequences $X^1$ and $X^2$ after clustering with our approach.

| Time series IceTargets with random noise | | |
|---|---|---|
| Pattern proposed by our approach: $S = (10101010101010101010101001...)$ | | |
| Time series | Subsequence X0 found by our approach | Subsequence X1 found by our approach |
|  |  |  |
| Time series Cosinus with IceTargets | | |
| Pattern proposed by our approach: $S = (10110010110010110010110 0...)$ | | |
| Time series | Subsequence X0 found by our approach | Subsequence X1 found by our approach |
|  |  |  |
| Time series Quarterly˙Australian˙Gross˙Farm | | |
| Pattern proposed by our approach: $S = (10110011001100110011001 00...)$ | | |
| Time series | Subsequence X0 found by our approach | Subsequence X1 found by our approach |
|  |  |  |
| Time series predicted with different methods | | |
| Pattern proposed by our approach: $S = (11001100110011001100110 0...)$ | | |
| Time series | Subsequence X0 found by our approach | Subsequence X1 found by our approach |
|  |  |  |

## 5.4 Series Predicted With Different Methods

In all previous simulations we used our approach to splitting time series into subsequences and then we predict their values with the same method - neural network. However, our approach gives a possibility to use completely different methods of prediction to each subsequence. Due to that we can choose different methods according to specific prediction proper-

Table 2: Comparisons with other methods for time series based on MSE.

| | *IceTargets* merged with noise | *IceTargets* merged with cos | *QuarterlyGrossFarmProduct* |
|---|---|---|---|
| Our approach | 0,62 | 0,0170 | 0,007 |
| Single Neural Network | 0,93 | 0,5144 | 0,0211 |
| Increase efficiency | 1,5 times | 30,25 times | 3,014 times |

ties of each subsequence and take advantage of both methods. To show that it is possible we merged two series with completely different prediction properties into one time series. We choose simple series which grows linearly according to time and statistic data *IceTargets* which expected value do not seems to change in time. We merged them with the pattern:
X = (1, 2, *IceTargets*(1), *IceTargets*(2), 3, 4, *IceTargets*(3), *IceTargets*(4), 5, 6, *IceTargets*(5), *IceTargets*(6) ...)
Pattern is described with a vector
$S = (11001100110011001100110 0...)$.
We use our approach which splits time series into two subsequences (Please see table 1). To predict X1 we use linear regression and to predict X0 we use neural network. Thanks to that we use advanteges of both methods and get MSE=0.0101. In case of using single neural network method we get MSE = 2535.45 and when using only single linear regression MSE = 30.35 (refer to Table 3). Our approach provides the prediction error over 250000 times smaller then using only neural network and 3000 times smaller then using only linear regression.

Table 3: Comparison of MSE calculated with different methods for the time series created by merging linear function and IceTarget.

| Method | Neural Network | Linear regression | Our approach |
|---|---|---|---|
| MSE | 2535.45 | 30.35 | 0,0101 |

# 6 CONCLUSIONS

In presented work, we proposed a novel method for time series forecasting. Our approach is based on splitting of the series into a subsequence and its complement what can result in much lower potential prediction error. Moreover, it allows application of different prediction methods to both subsequences and therefore to combine their benefits. The proposed approach is not associated with any specific time series forecasting method and can be applied as a generic solution in time series preprocessing. Moreover we show that our approach allows to noise filtering. In order to validate the efficiency of the introduced solution we conducted series of experiments. Obtained results proved that using our approach results in significant improvement of accuracy. Moreover we have proven that generated overhead asymptotically is logarithmic with respect to time series length. Low computation overhead caused by our approach suggests that it can be useful regardless of the time series length. Moreover algorithm can be processed parallel and therefore we can decrease time of computation by implementing it on multiple processors.

Our solution opens up broad prospects of further work. First of all our approach use strict partitioning clustering where every element belongs to exactly one cluster. Future research may design and examine our approach with overlapping clustering where single element may belong to many clusters. Efficiency of our approach with such modification should be investigated on real data. Another open question is influence of choice of maximal searched pattern period and minimal acceptable subseries length into our approach prediction efficiency. One of future area of research could be also design and implementation automated method of selecting different prediction methods to proposed subseries.

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