Interval Type 2- Fuzzy Rule based System Approach for Selection of Alternatives using TOPSIS

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Abstract: The paper considers fuzzy rule based system for multi criteria group decision making problem. A novel version of TOPSIS method using interval type 2 fuzzy rule based system approach is proposed with the objective of improving the type 2 TOPSIS ability to deal with ambiguity through the combination of the mathematical process involved in the type 2 TOPSIS with the expert empirical knowledge. On the other hand, a hybrid analysis of decision making process that requires the use of human sensitivity to reflect influence degree of decision maker can be expressed by a fuzzy rule base. To ensure practicality and effectiveness of proposed method, stock selection problem is studied. The ranking based on proposed method is validated comparatively using Kendall's Tau rank correlation. Based on the result, the proposed method outperforms the established non-rule based version of type 2 TOPSIS in term of ranking performance.

1 INTRODUCTION

Multi criteria decision making has received great attention recently in optimization problems (Shidpour et al. 2013)(Awasthi et al. 2011) and (Şengül et al. 2015). This is due to the fact that the ability of decision makers in providing result that is consistent with actual situation remains as major concern in decision making environment. Conventional Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) was originally developed in (Hwang, C.L.Yoon 1981). Later, the conventional to T1 TOPSIS was enhanced to provide additional flexibility in represent the uncertainty (Chen 2000). A decade later, a fuzzy rule based version of T1-TOPSIS was develop, which provide a basis for automatic generation of a rule base to assist the analysis of decision making problem (Santos and Camargo 2010).

In 2010, a TOPSIS method based on interval Type 2 (T2) fuzzy set was introduced which demonstrate additional degree of freedom to represent the uncertainty and the fuzziness of the real world problems. In order to improve the ability in dealing and presenting vagueness of information in established non rule based T2 TOPSIS method(Chen and Lee 2010), T2 - Fuzzy Rule Based TOPSIS (T2 FRBS TOPSIS) is introduce in this paper, which has capability in providing a useful way to handle MCDM problems in a more flexible and intelligent manner but also presenting expert's knowledge accurately and significantly.

The paper is organized as follows. Section 2 briefly reviews the concept of interval T2 fuzzy set. The proposed method is systematically explained in section 3. In section 4, stock selection problem is explained using the proposed method. Next, ranking performance is assessed and analysis of result is discussed in section 5. Finally, the conclusion is drawn.

2 BASIC CONCEPT

In the following, we briefly review some basic definitions of fuzzy sets from (Chen, 2000) and (Chen and Lee 2010). These basic definitions and

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notations are used throughout the paper unless stated otherwise.

Definition 1: Fuzzy set

A fuzzy set $\tilde{A}$ is defined on a universe X may be given as:

$$\tilde{A} = \{(x, \mu_{\tilde{A}_i}(x)) \mid x \in X\}$$

Where $\mu_{\tilde{A}_i}(x): X \rightarrow [0,1]$ is the membership function $\tilde{A}$. The membership value $\mu_{\tilde{A}_i}(x)$ describes the degree of belongingness of $x \in X$ in $\tilde{A}$. Throughout this paper, type-1 fuzzy number, and type-2 fuzzy number are presented in the form of trapezoidal fuzzy number. It is easy to deal with because it is piece wise linear. On the other hand, the good coverage of trapezoidal fuzzy number is a good compromise between efficiency and effectiveness.

Definition 2: Type-2 Fuzzy Number

A trapezoidal interval type-2 fuzzy set $\tilde{A}$ can be represented by

$$\tilde{A} = \left(\tilde{A}_i^U, \tilde{A}_i^L\right) = \begin{pmatrix} \left(a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U; H_1\left(\tilde{A}_i^U\right), H_2\left(\tilde{A}_i^U\right)\right), \\ \left(a_{i1}^L, a_{i2}^L, a_{i3}^L, a_{i4}^L; H_1\left(\tilde{A}_i^L\right), H_2\left(\tilde{A}_i^L\right)\right) \end{pmatrix}$$

as shown in Figure 1, where $\tilde{A}_i^U$ and $\tilde{A}_i^L$ are type-1 fuzzy sets, $a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U, a_{i1}^L, a_{i2}^L, a_{i3}^L$ and a_{i4}^L are the reference points of the interval type-2 fuzzy set $\tilde{A}_i, H_j(\tilde{A}_i^U)$ denotes the membership value of the element $a_{i(j+1)}^U$ in the upper trapezoidal membership function $\tilde{A}_i^U, 1 \leq j \leq 2, H_j(\tilde{A}_i^L)$ denotes the membership value of the lower trapezoidal membership function $\tilde{A}_i^U, 1 \leq j \leq 2$, and $1 \leq i \leq n$

$H_1(A_i^U) \in [0,1], H_2(A_i^U) \in [0,1], H_1(A_i^L) \in [0,1], H_2(A_i^L) \in [0,1],$

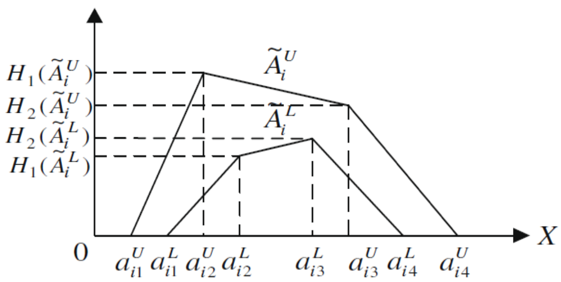

Figure 1: Type-2 Fuzzy Number (Chen and Lee 2010).

3 PROPOSED METHOD

In this section, the authors extend the T2-TOPSIS method from (Chen and Lee 2010) using fuzzy rule based approach for handling multi criteria decision making problem. The main purpose of modification is to extend the ability of fuzzy rules based approach into established method. Thus, the implementation by proposed method allow the empirical knowledge of the expert, represented by fuzzy rule, also be considered in the decision making process.

The use of the methods associated with the empirical knowledge of experts, allows a hybrid analysis of the decision making problems where the process of decision making requires the use of human sensitivity, which often can be expressed by a fuzzy rules base. Thus, the behavior of the system may have greater influence then the rule defined by the decision maker. The authors adopt the methods described in (Santos and Camargo 2010) for the knowledge of the influence degree of each decision maker.

In case in which one decision maker has more knowledge of the domain, optionally the opinion of this expert may have greater degree of importance than the other decision makers in the analysis of the problem. Thus, the proposed method can identify and aggregate the different opinions of decision makers with varying influence degrees to suggest the final solution. Figure 2 schematically demonstrations such a system, where C_1, C_2, C_3… C_m is the input, in this case criteria and Y represents the output/ alternative level. The rules for such a system are normally derived from expert knowledge.

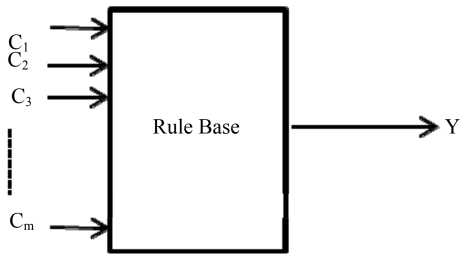

Figure 2: Fuzzy System.

Just as in established fuzzy TOPSIS method, Table 1 and Table 2 are used to represent the importance of criteria and the rating of the alternative. In order to deal with influence degree of decision maker in T2 fuzzy rule based approach, Table 3 introduce here, which implement the consequent part of the rules. The linguistic terms that represents the consequents of rules was named “Alternative Level” and is represented by fuzzy sets “Very bad”, “Bad”, “Regular”, “Good” and “Excellent”.

Table 1: Linguistic terms for the importance weight of each criterion.

Linguistic	Type 2 Fuzzy Number
Very Low (VL)	(0.00,0.00,0.00,0.10,1,1)(0.00,0.00,0.00,0.10,1,1)
Low (L)	(0.00,0.10,0.10,0.25,1,1)(0.00,0.10,0.10,0.25,1,1)
Medium Low (ML)	(0.15,0.30,0.30,0.45,1,1)(0.15,0.30,0.30,0.45,1,1)
Medium (M)	(0.35,0.50,0.50,0.65,1,1)(0.35,0.50,0.50,0.65,1,1)
Medium High (MH)	(0.55,0.70,0.70,0.85,1,1)(0.55,0.70,0.70,0.85,1,1)
High (H)	(0.80,0.90,0.90,1.00,1,1)(0.80,0.90,0.90,1.00,1,1)
Very High (VH)	(0.90,1.00,1.00,1.00,1,1)(0.90,1.00,1.00,1.00,1,1)

Table 2: Linguistic terms for rating of all alternative.

Linguistic	Trapezoidal Fuzzy Number
Very Poor (VP)	(0,0,0, 1,1) (0,0,0, 1,1)
Poor (P)	(0,1,1,3,1,1) (0,1,1,3,1,1)
Medium Poor (MP)	(1,3,3,5,1,1) (1,3,3,5,1,1)
Fair (F)	(3,5,5,7,1,1) (3,5,5,7,1,1)
Medium Good (MG)	(5,7,7,9,1,1) (5,7,7,9,1,1)
Good (G)	(7,9,9,10,1,1) (7,9,9,10,1,1)
Very Good (VG)	(9,10,10,10,1,1)(9,10,10,10,1,1)

Table 3: Linguistic term for alternative level.

Linguistic	Trapezoidal Fuzzy Number
Very Bad(VB)	(0.00,0.00,0.00,0.25,1,1)(0.00,0.00,0.00,0.25,1,1)
Bad (B)	(0.00,0.25,0.25,0.50,1,1)(0.00,0.25,0.25,0.50,1,1)
Regular (R)	(0.25,0.50,0.50,0.75,1,1)(0.25,0.50,0.50,0.75,1,1)
Good (G)	(0.50,0.75,0.75,1,1,1) (0.50, 0.75, 0.75, 1,1,1)
Very Good (VG)	(0.75,1.00,1.00,1.00,1,1) (0.75,1.00,1.00,1.00,1,1)

The following algorithm is conducted to get the ranking of alternatives, whereby Step 1-5 are taken from (Chen & Lee 2010), whereas Step 6 to Step 8 are introduced in this paper.

T2- FRBS TOPSIS algorithm

Instead of calculating the average decision matrix as the previous TOPSIS methods(Mohamad and Jamil 2012),(Kelemenis et al. 2011). Here, the opinion of each decision maker evaluated independently. Assume that there are m alternatives $A_1, A_2, \cdots, A_m$ and assume that there are n criteria $C_1, C_2, \cdots, C_n, C_{n+1}$. Where C_{n+1} represent the influence level of each decision maker. Let there are k decision makers $DM_1, DM_2, \cdots, DM_k$ then will have k decision matrix.

Step 1: Construct Fuzzy Decision Matrix, (D_K) and Fuzzy Weight of Alternative (W_K) as shown in Eq. (1).

$$(D_K) = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \text{and} \quad W_K = [w_1 \quad w_2 \quad \cdots \quad w_n] \tag{1}$$

where x_{ij} and w_i are interval T2 fuzzy set based from Table 1 and Table 2 respectively. Its represent the rating and the important weights of the K^{th} decision maker of alternative A_i with respect to criterion C_j $(j = 1, \cdots, n)$ respectively.

Step 2: Weighted fuzzy decision matrix (V_K)

The weighted fuzzy decision matrix (V_K) is shown in Eq. (2).

$$V_K = \lfloor v_{ij} \rfloor_{m \times n} \quad \text{for } i = 1, \cdots, m \text{ and } j = 1, \cdots, n \tag{2}$$

where $v_{ij} = x_{ij}(\cdot)w_{ij}$ is an multiplication of interval T2 fuzzy set.

Step 3: Construct the ranking weighted decision matrix

Calculate the ranking value $Rank_K(A_i)$ (Lee and Chen 2008), in order to find ranking value, the maximum number s of edges in the upper membership function v^U_{ij} and the lower membership function v^L_{ij} of interval T2 fuzzy set v_{ij} are defined, where $1 \le i \le n$ and $1 \le j \le m$. If s is odd number and $s \ge 3$, then $r = s + 1$. If s is even number and $s \ge 4$, then $r = s$. The $Rank(A_i)$ of interval T2 fuzzy set is shown in Eq. (3).

$$\begin{aligned} Rank(A_i) = & \sum_{j\in\{U,L\}} M_1(A^U_i) + \sum_{j\in\{U,L\}} M_2(A^j_i) + \cdots + \sum_{j\in\{U,L\}} M_{r-1}(A^j_i) \\ & - \frac{1}{r}\left(\sum_{j\in\{U,L\}} S_1(A^j_i) + \sum_{j\in\{U,L\}} S_2(A^j_i) + \cdots + \sum_{j\in\{U,L\}} S_r(A^j_i) \right) \\ & + \sum_{j\in\{U,L\}} H_1(A^j_i) + \sum_{j\in\{U,L\}} H_2(A^j_i) + \cdots + \sum_{j\in\{U,L\}} H_{r-2}(A^j_i) \end{aligned} \tag{3}$$

where $M_p(A^j_i)$ denotes the average of the elements a^j_{ip} and $a^j_{i(p+1)}$

$$M_p(A^j_i) = \left(a^j_{ip} + a^j_{i(p+1)}\right)\Big/2 \quad \text{for } 1 \le p \le r-1$$

$$S_q(A^j_i) = \sqrt{\frac{1}{2}\sum_{k=q}^{q+1}\left(a^j_{ik} - \frac{1}{2}\sum_{k=q}^{q+1} a^j_{ik}\right)^2} \quad \text{for } 1 \le p \le r-1$$

The $S_r(A^j_i)$ denotes the standard deviation of the elements $a^j_{i1}, a^j_{i2}, \cdots, a^j_{ir}$

$$S_r(A^j_i) = \sqrt{\frac{1}{r}\sum_{k=1}^{r}\left(a^j_{ik} - \frac{1}{r}\sum_{k=1}^{r} a^j_{ik}\right)^2}$$

The $H_p\left(A_i^j\right)$ denotes the membership value of the element $a_{i(p+1)}^j$, $1 \le p \le r-2$, $j \in \{U, L\}$ and r is even number.

Step 4: The fuzzy positive ideal solution $\left(A^+\right)$ and the fuzzy negative ideal solution $\left(A^-\right)$ as shown in Eq. (4).

$$A^+ = \left(v_1^+, v_2^+, \cdots, v_n^+\right)$$
$$\text{and} \quad A^- = \left(v_1^-, v_2^-, \cdots, v_n^-\right) \tag{4}$$

where

$$v_i^+ = \begin{cases} \max\limits_{1 \le j \le n} \left\{Rank\left(v_{ij}\right)\right\}, x_i \in B \\ \min\limits_{1 \le j \le n} \left\{Rank\left(v_{ij}\right)\right\}, x_i \in C \end{cases}$$

and

$$v_i^- = \begin{cases} \min\limits_{1 \le j \le n} \left\{Rank\left(v_{ij}\right)\right\}, x_i \in B \\ \max\limits_{1 \le j \le n} \left\{Rank\left(v_{ij}\right)\right\}, x_i \in C \end{cases}$$

where B denotes the set of benefit attribute and C denotes the set of cost attribute and $1 \le i \le n$

To calculate the distance $d^+(A_i)$ between each alternative A_i and the fuzzy positive ideal solution A^+ is shown in Eq. (5).

$$d^+(A_i) = \sqrt{\sum_{i=1}^{m}\left(Rank\left(v_{ij}\right) - v_i^+\right)^2} \text{ for } 1 \le j \le n \tag{5}$$

Calculate the distance $d^-(A_i)$ between each alternative A_i and the fuzzy negative ideal solution A^-, as shown in Eq. (6).

$$d^-(A_i) = \sqrt{\sum_{i=1}^{m}\left(Rank\left(v_{ij}\right) - v_i^-\right)^2} \tag{6}$$

$$\text{for } 1 \le j \le n$$

Step 5: The closeness coefficient $\left(CC_i\right)$

Calculate the relative degree of closeness $\left(CC_i\right)$ of A_i calculated as shown in Eq. (7)

$$CC_i = \frac{d_i^-}{d_i^+ + d_i^-} \text{ for } i = 1, \cdots, m \tag{7}$$

Step 6: The influence Closeness coefficient of each alternative

The influence degree of each decision maker has been defined at this point, noting that experts with more experience have a greater degree of influence than the expert with less experience.

$$\text{Let } \sigma_K = \theta_i \Big/ \sum_{i=1}^{K} \theta_i \tag{8}$$

$$\text{for } i = 1, \cdots, m$$

Where σ_K represent normalized influence degree for K^{th} decision maker. θ_i is the importance degree between 0 (unimportant) and 10 (very importance) of decision maker. Then

$$ICC_i = \sigma_K * CC_i \tag{9}$$

And it is necessary to normalize the ICC_i $\left(NICC_i\right)$ to ensure that the ICC_i value varies between 0 to 1.

$$NICC_i = ICC_i \Big/ \max_i ICC_i \tag{10}$$

Step 7: The matrix of antecedent $\left(\Lambda\right)$ and the matrix of consequent $\left(\chi\right)$

A matrix of antecedents is defined as in Eq. (11)

$$\Lambda = \begin{bmatrix} X_{11} & X_{12} & \cdots & X_{1n} \\ X_{21} & X_{22} & \cdots & X_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ X_{m1} & X_{m2} & \cdots & X_{mn} \end{bmatrix} \tag{11}$$

where X_{ij} is a linguistic terms representing decision maker opinion of each alternative with respect to the criteria.

Once $NICC_i$ for each alternative defined by each decision maker is obtained, it is used to determine the consequents of alternative rules according to the fuzzy set with higher membership in Table 3. Then a matrix of consequents is define in Eq. (12)

$$\chi = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_m \end{bmatrix} \tag{12}$$

where Y_j is a linguistic terms based on Table 3 representing the output of the system based on Eq. (10) to find the value of $NICC_i$.

Hence, matrix of antecedent and matrix of consequent in Eq. (11) and (12) can be written as If-then rule as follow:

If C_1 is X_{11} and C_2 is X_{12} and ... and C_{1n} is X_{1n} then A_1 is Y_1
If C_1 is X_{21} and C_2 is X_{22} and ... and C_{2n} is X_{2n} then A_1 is Y_2

$\vdots \qquad \vdots \qquad \vdots$

If C_1 is X_{m1} and C_2 is X_{m2} and ... and C_{mn} is X_{mn} then A_1 is Y_m

Step 8: The final score (Γ) for each alternative is given as shown in Eq. (13).

$$\Gamma = \lambda * \Omega \tag{13}$$

where λ is a crisp value of aggregate membership function of the output in Eq. (12) defined as shown in Eq. (14)

$$\lambda = \sum_{i=1}^{K} \alpha_{ij} \Big/ K \tag{14}$$

where $\alpha_{ij} \in Y_j$ is maximum membership degree of the output. In order to obtain a better representation in the ranking made by T2- FRBS TOPSIS, It is importance to have influence multiplier when the alternatives have same ranking position. This show exactly how each alternative is different even a small difference. The following general formula to calculate influence multiplier (Ω) uses a marginal closeness coefficient that has maximum membership degree as shown in Eq. (15).

$$\Omega = \sum_{i=1}^{K} NICC_i \Big/ K \tag{15}$$

Therefore, from the value of Γ, the ranking order of all alternatives can be determined. The best alternative has higher value of Γ.

4 IMPLEMENTATION OF METHOD

In this case study a Stock selection problem is considered in which the evaluation was done by three decision makers. These financial experts included finance lecturer (DM1), fund manager (DM2) and PhD finance student (DM3). They evaluated 25 stocks (S1-S25) listed on Main Board in Kuala Lumpur Stock Exchange (KLSE) and then make investment recommendations according to financial ratio considered.

The most importance ratio considered in investment is Market Value of Firm (C1) defined as Market value of firm-to-earnings before amortization, interest and taxes ratio. This ratio is one of the most frequently used financial indicators and the lower this ratio is better. Return on Equity (C2) used to examine how much the company earns on the investment of its shareholders. Portfolio managers examine ROE very carefully and used it when deciding whether to buy or sell. The higher the ratio is better. ROE is usually measured as net income divide by stockholder. Dept/equity ratio (C3), this ratio belongs to long term solvency ratios that are intended to address the firm's long run ability to meet its obligations. So, it is assume by DMs that the lower the ratio the better. Current ratio (C4) is one of the ways to measure liquidity of company. It explains the ability of a business to meet its current obligations when fall due. Higher the ratio is better. Market value/net sales(C5) is market value ratios of particular interest to the investor are earnings per common share, the price-to-earnings ratio, market value-to book value ratio, earning-to-price ratio. The lower the ratio is the better. Price/earnings ratio (C6) measure the ratio of market price of each share of common stock to the earnings per share, the lower this ratio is better.

In the case study, the alternative of decision makers to be rank and to be weighted according to the above mention ratios are 25 stocks listed in KLSE .In this study, Microsoft Excel was used to calculate all the calculation involved in the evaluating the ranking of stocks and the weight of each criterion. The DMs use the linguistic weighting terms in Table 1 to assess the importance of the criteria, and make use information in Table 2 to give rating for each alternative. All linguistic terms can be expressed as type 2 trapezoidal fuzzy numbers as shown in Table 1, 2 and 3. The T2- FRBS TOPSIS algorithm introduced in Section 3 is now illustrated for the case study of stock selection problem.

Step 1: Based on information given by experts and applying Eq. (1), the decision matrix for each alternative can be constructed.

The important of criteria and the rating of each are obtained from questionnaire.

Step 2: Construct a Weighted Decision Matrix (V_K)

Based on Eq. (2), the normalized weighted decision matrices can be determined, shown as follows:

$w_1 = (0.9,1.0,1.0,1.0,1.0,1.0)(0.9,1.0,1.0,1.0,1.0,1.0)$

$x_{11} = (9,10,10,10,1.0,1.0)(9,10,10,10,1.0,1.0)$

Then

$$v_{11} = ((0.9\times 9),(1.0\times 10),(1.0\times 10),(1.0\times 10),1.0,1.0)$$
$$((0.9\times 9),(1.0\times 10),(1.0\times 10),(1.0\times 10),1.0,1.0)$$
$$v_{11} = (8.1,10,10,10,1.0,1.0)(8.1,10,10,10,1.0,1.0)$$

Using formula stated to construct the normalized fuzzy decision matrix.

The $\max_i C_{1j} = 10$,

$$v_{11} = \left(\frac{8.1}{10},\frac{10}{10},\frac{10}{10},\frac{10}{10},1.0,1.0\right)\left(\frac{8.1}{10},\frac{10}{10},\frac{10}{10},\frac{10}{10},1.0,1.0\right)$$
$$v_{11} = (0.81,1.0,1.0,1.0,1.0,1.0)(0.81,1.0,1.0,1.0,1.0,1.0)$$

The normalization method is to preserve the property that the ranges of normalized trapezoidal fuzzy number belong to $[0,1]$.

Step 3: Construct the ranking weighted decision matrix

Based on Eq.(3) the ranking values $Rank_K(A_i)$ of the trapezoidal type 2 fuzzy number v_{ij} can be calculated, illustrate for S1 as follows

$$Rank(v_{11}) = M_1(v_{11}^U) + M_1(v_{11}^L) + M_2(v_{11}^U) + M_2(v_{11}^L) + M_3(v_{11}^U)$$
$$+ M_3(v_{11}^L) - \frac{1}{4}\begin{pmatrix} S_1(v_{11}^U) + S_1(v_{11}^L) + S_2(v_{11}^U) + S_2(v_{11}^L) + S_3(v_{11}^U) \\ + S_3(v_{11}^L) + S_4(v_{11}^U) + S_4(v_{11}^L) \end{pmatrix}$$
$$+ H_1(v_{11}^U) + H_1(v_{11}^L) + H_2(v_{11}^U) + H_2(v_{11}^L)$$
$$Rank(v_{11}) = 0.905 + 0.905 + 1.0 + 1.0 + 1.0 + 1.0$$
$$-\frac{1}{4}(0.095 + 0 + 0 + 0.0823 + 0.095 + 0 + 0 + 0.0823)$$
$$+1+1+1+1$$
$$Rank(v_{11}) = 9.7214$$

Rank values for S1 are
$\{9.7214,7.9931,8.6119,6.8219,5.4877\}$
with respect to 6 criteria respectively.

Similarly, the rank value for each alternative can be obtained.

Step 4: The fuzzy positive-ideal solution (A^*) and fuzzy negative-ideal solution (A^-)

Based on Eq. (4), fuzzy positive ideal solution and fuzzy negative ideal solution are determined as follows:

$$A^+ = \{9.7214,8.7525,9.5526,7.5375,9.5526,6.3229\}$$
$$A^- = \{0.0533,2.8955,1.1805,2.1593,3.5259,1.4233\}$$

Using Eq.(5) to calculate the distance $d^+(A_i)$ between each alternative and the ideal solution A^+

$$d^+(A_1) =$$
$$= \sqrt{(9.72.14 - 9.7214)^2 + \cdots + (5.4877 - 6.3229)^2}^{\,2}$$
$$= 1.8858$$

The distance between fuzzy negative ideal solution and S1 calculated based on Eq.(6) as follows:

$$d^-(A_1) =$$
$$= \sqrt{(9.72.14 - 0.0533)^2 + \cdots + (5.4877 - 1.4233)^2}$$
$$= 15.4534$$

Step 5: Using Eq.(7), the relative degree of closeness (CC_i) of each alternative A_i with respect to the fuzzy positive ideal solution A^+.The calculation for CC_1 shown as follows:

$$CC_1 = \frac{d_i^-}{d_i^* + d_i^-}$$
$$= \frac{15.4534}{1.8858 + 15.4534}$$
$$= 0.8912$$

Follow the same procedure to calculate CC_i for each alternative.

In the next step shows how the new criteria C_{n+1} involved in the evaluation of T2- FRBS TOPSIS.

Step 6: The influence closeness coefficient (ICC_i) of each alternative

Firstly the influence degree (σ_K) of each decision maker must be determined using Eq.(8) based on their experience on the field. In this case study, C_7 in Table 4 represent the importance degree of decision maker. DMs evaluate themselves by giving value 0 to 10, for uninfluential and very influential respectively. For instance influence degree of DM_1 is calculated as follows:

$$\sigma_1 = \frac{8}{8+10+7}$$
$$= 0.32$$

Following by Eq. (9) to get the influenced closeness coefficient for A_1

$$ICC_1 = 0.32 \times 0.8912$$
$$= 0.2852$$

Next, the influenced closeness coefficients need to be normalized prior to matching the coefficient to the linguistic terms in Table 3. As an example in this case study, assuming the maximum value of $ICC_i = 0.2852$ out of 25 stocks then the normalized influenced closeness degree calculated as follows:

$$NICC_1 = \frac{0.2852}{0.2852}$$
$$= 1$$

Step 7: The matrix of antecedent (Λ) and the matrix of consequent (χ)

Each decision maker has t matrix of antecedent and consequent separately.

$$\text{If } \begin{matrix} R_1 \\ R_2 \\ \vdots \\ R_m \end{matrix} \begin{bmatrix} X_{11} & X_{12} & \cdots & X_{1n} \\ X_{21} & X_{22} & \cdots & X_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ X_{m1} & X_{m2} & \cdots & X_{mn} \end{bmatrix} \text{ then } \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_m \end{bmatrix}$$

The rules have the following format:

If X_{11} is VG and X_{12} is VG and X_{13} is VG and X_{14} is VG and X_{15} is VG and X_{16} is VG Then Y_1 is VG.

Now, the value of $NICC_i$ can be match to the linguistic terms for alternative in Table 3. For instance, $NICC_1 = 1$

Then Y_1 belong to interval type 2 fuzzy set VG in Table 3.

Step 8: The final score (Γ) for each alternative.

Based on Eq. (13) the value of final score has been calculated. Assuming S1 has three rules R1, R2 R3 from three decision makers. For example final score for S1 is shown below:

R1: If C1 is VG and C2 is VG and C3 is G and C4 is VG and C5 is G and C6 is G Then S1 is VG

R2: If C1 is VG and C2 is VG and C3 is F and C4 is MG and C5 is MG and C6 is F Then S1 is VG

R3: If C1 is G and C2 is VG and C3 is VG and C4 is VG and C5 is G and C6 is G Then S1 is VG

To calculate the value of λ as in Eq. (14)

Let the output of each rule for S1 are as follows

R1: VG =(0.80,0.9,0.9,1.0,1.0,1.0) (0.80,0.9,0.9,1.0,1.0,1.0)

R2: VG = (0.80,0.9,0.9,1.0,1.0,1.0) (0.80,0.9,0.9,1.0,1.0,1.0)

R3: VG = (0.80,0.9,0.9,1.0,1.0,1.0) (0.80,0.9,0.9,1.0,1.0,1.0)

Then, λ is calculated as

$$\lambda = \frac{0.9 + 0.9 + 0.9}{3}$$
$$= 0.9$$

Furthermore, the Ω is calculated using Eq. (15) as follows

From Step 6, by assuming ICC of each rule for S1 are

R1: 1.0 R2: 1.0 R3: 1.0

The value of Ω defined as

$$\Omega = \frac{1.0 + 1.0 + 1.0}{3}$$
$$= 1.0$$

Lastly, the final score (Γ) can be derived as Eq. 13

$$\Gamma = \lambda * \Omega$$
$$= 0.9 \times 1.0$$
$$= 0.9$$

Therefore, from the value of Γ, the ranking order of all alternative can be determine. The best alternative has higher value of Γ. Hence the ranking based on proposed method (PM) can be seen in Table 4.

5 ANALYSIS OF RESULTS

For the validation purposes, the authors considered the ranking based on established T2 TOPSIS (non-rule based approach) and actual price change (return on investment). The rankings are compared descriptively using Kendall'Tau rank correlation (τ)(Adler 1957). The advantages of Kendall tau correlation are its easy algebraic structure and intuitively simple interpretation. In general, the coefficient of tau shows the degree of concordances between two columns of ranking data. The Tau Coefficient can be determine by

$$\tau = \frac{\sum G_{ij} - \sum J_{ij}}{\sum G_{ij} + \sum J_{ij}}$$

where G_{ij} and J_{ij} represent concordance pair and discordances pair, respectively. In particular, concordance pair interprets the number of observed ranks below a particular rank which are larger than that particular rank, whereas discordance is the number of observed ranks below a particular rank which are smaller than that particular rank. To test the significant of the rank, the statistical z-score can be define by following (Dibley & Trowbridge 1987).

$$z = \frac{3 * \tau * \sqrt{n(n-1)}}{\sqrt{2(2n+5)}}$$

Obviously, statistical z-score shows how far that data is from the mean. The distance from the mean is measured in term of standard deviation. The bigger the z- score value, the more significant the ranking to the actual ranking. Thus, based on the analysis of Kendall'Tau Correlation in Table 5, it's observed that the z-score value of FRBS T2 TOPSIS is higher than, which is outperform and more significant to the actual ranking comparison to T2-TOPSIS.

Table 4: Ranking based investment return, established T2 TOPSIS and proposed method (T2-FRBS TOPSIS).

Stock	Ranking		
	Actual	T2-TOPSIS	T2-FRBS TOPSIS (PM)
S3	1	24	24
S1	2	1	1
S15	3	15	10
S2	4	16	13
S16	5	9	8
S23	6	3	4
S21	7	11	11
S11	8	10	7
S14	9	13	15
S25	10	6	19
S6	11	19	17
S18	12	14	12
S12	13	4	3
S24	14	22	25
S19	15	23	22
S20	16	5	5
S7	17	2	2
S17	18	17	18
S5	19	21	21
S22	20	20	20
S4	21	7	6
S10	22	12	14
S9	23	25	23
S8	24	8	9
S13	25	18	16

Table 5: Assessing the ranking performance based on Kendal's tau correlation.

STOCKS	T2-TOPSIS		T2- FRBS TOPSIS (PM)	
	G_{ij}	J_{ij}	G_{ij}	J_{ij}
S3	1	23	1	23
S1	23	0	23	0
S15	9	13	14	8
S2	8	13	11	10
S16	13	7	14	6
S23	18	1	17	2
S21	11	7	12	6
S11	11	6	13	4
S14	9	7	9	7
S25	12	3	5	10
S6	5	9	6	8
S18	7	6	8	5
S12	11	1	11	1
S24	2	9	0	11
S19	1	9	1	9
S20	8	1	8	1
S7	8	0	8	0
S17	4	3	3	4

Table 5: Assessing the ranking performance based on Kendal's tau correlation (cont.).

STOCKS	T2-TOPSIS		T2- FRBS TOPSIS (PM)	
	G_{ij}	J_{ij}	G_{ij}	J_{ij}
S5	1	5	1	5
S22	1	4	1	4
S4	4	0	4	0
S10	2	1	2	1
S9	0	2	0	2
S8	1	0	1	0
S13				
Summation	170	130	173	127
τ		0.1333		0.1533
z		0.9342		1.0743
Kendal Tau Coefficient		**0.8238**		**0.8577**

6 CONCLUSION

In this paper, a novel variation of TOPSIS method via extending established T2-TOPSIS method (Chen and Lee 2010) by attached the ability of fuzzy rule based system approach in solving the multi criteria decision making problems. The ranking based on proposed method is validated comparatively using Kendall tau correlation. The results shows proposed method (PM) outperform the established non rule based version of type 2 TOPSIS in term of ranking performance. The proposed method not only provides a useful way to handle MCDM problems in a more flexible and intelligent manner also presents expert knowledge more accurately. In this paper, the authors have successfully extended established T2-TOPSIS using fuzzy system. The next objective is to implement T2-TOPSIS using fuzzy networks, which is new type of fuzzy system by aiming to improve significantly the transparency of the TOPSIS method.

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