

# Multi-objective Optimization for Control and Process Operation

Helem Sabina Sánchez and Ramon Vilanova

*Departament de Telecomunicació i Enginyeria de Sistemes, Universitat Autònoma de Barcelona,  
08193 Bellaterra, Barcelona, Spain*

## 1 STAGE OF THE RESEARCH

It is well known, that satisfying the requirements and constraints required by a control engineering system in many cases is a difficult task to fulfilling for each of the objectives. Owing to this, this research aims to apply the Multi-objective Optimization Design (MOOD) procedure to PID controller tuning by means Multi-Objective Optimization based on deterministic algorithm. This procedure is focus on provide reasonable trade-off solution among the objectives in conflictive. Currently, we are working on contribution based on: an approach on the MOO process for PI controller, applying the methodology to Fractional-order PID controllers and doing a research stay at the University of Brescia (UNIBS), Italy.

## 2 OUTLINE OF OBJECTIVES

The following aims are defined for the development of this research:

- First Year:
  - Review of the state of art. Existing methods and approaches for the planning and the definition of a Multi-Objective Optimization (MOO) process and MOOD procedure. To have an idea of the work done and the work to be done.
  - Identifying methodologies and tools needed to solve optimization problems. Test different algorithms according to the preferences.
  - Limitations on the search domain and functional constraints linked to the operation of the system.
  - Select the methodology (algorithm and the decision making technique).
- Second Year:
  - Applications on controller tuning: PI, PID controllers (set-point step response).
  - Consider both operation modes (set-point and load disturbance).

– Collaborative work with others research group dedicate to Optimization.

• Currently:

- Contribution on MOO process (new methodology).
- Validation of the new methodology (benchmark, FOPID and others).

## 3 RESEARCH PROBLEM

This proposal seeks to develop a methodology to address issues of control and operation of processes by implementing a MOOD procedure. In a control system there are different measures and indexes that are made in order to measure their performance. Satisfying the specifications and constraints required is often a challenge. Sometimes, the improvement in performance of one of them is at the expense of worsening another. This kind of problems where the designer have to deal with the fulfillment of multiple objectives are known as Multi-Objective Problems (MOPs). Such problems can be addressed using a simultaneous optimization of all targets. This implies to seek for a Pareto optimal solution which the objectives have been improved as possible without giving anything in exchange. To guarantee the overall performance of a MOOD procedure the following steps are necessary: 1) definition of the MOP, 2) the MOO process to approximate the so-called Pareto set and 3) a Multi-criteria Decision Making (MCDM) is carried out in order to implement the most preferable solution from the set. The MOOD procedure brings to the designer the possibility to appreciate the trade-off of the objectives (conflictive) this characteristic can be useful for controller tuning.

## 4 STATE OF THE ART

The design of a PI-PID control system starts from a model of the process to be controlled and a set of re-

requirements to be satisfied. These requirements often enter into conflict making the task of finding the appropriate controller parameters not an easy task. It is on that basis that constrained optimization can enter into play by helping to delimit the tradeoff between possible conflicting requirements. Such requirements use to be the conflicting performance and robustness specifications (in the different forms that they can be established). Typical control system requirements include performance specifications on load disturbance attenuation, robustness, control input usage, set-point response and measurement noise. It is a fact that disturbance rejection is of primary interest in process control, where what really matters is steady-state regulate. On the other hand set-point changes are likely to occur. In such cases it is possible to tackle them by using an appropriate two-degree-of-freedom (2-DoF) architecture. Therefore, when introducing time response performance requirements we can confine ourselves to disturbance attenuation. In fact, as a feedback property, disturbance attenuation will enter into conflict with robustness as both are determined by the controller parameters that appear in the feedback loop. From a purely optimization problem point of view, all such requirements could be used to establish the set up of a Multiobjective Optimization Problem (MOP). In some cases, the high performance is not compatible with a robust controller for process variations. The controller design can be viewed as the search for the best compromise between all the specifications and thereby the idea of multiobjective optimization (MOO) can be an alternative to resolve this problem (Martínez et al., 2006). MOO provides the possibility of a better selection of the final solution as there is no part ignored in the search space. The final solutions (Pareto set), represents the whole space of the design variables and their projection in the space of objectives as the Pareto front.

#### 4.1 Multi-objective Optimization -MOO

A multi-objective optimization problem (MOP) can be handled by performing a simultaneous optimization of all objectives. This implies the existence of a set of solutions, where no one is better than the others, but differ in the degree of performance between the objectives (Miettinen, 1998). This set of solutions will offer a higher degree of flexibility at the decision making stage. The role of the designer is to select the most preferable solution according to his (her) needs and preferences for a particular situation. A MOP, without loss of generality (since a maximization problem can be converted to a minimization problem), can be stated as follows:

$$\min_{\theta \in \mathfrak{R}^n} \mathbf{J}(\theta) = [J_1(\theta), \dots, J_m(\theta)] \in \mathfrak{R}^m \quad (1)$$

Therefore a set of Pareto-optimal solutions is defined as the Pareto set  $\Theta_P$  and its projection into the objective space is known as the Pareto front  $\mathbf{J}_P$ . Where each solution in the Pareto front is said to be a non-dominated and Pareto-optimal solution. In general, it does not exist a unique solution because there is not a solution better than other in all the objectives.

MOO techniques search for a discrete approximation  $\Theta_P^*$  of the Pareto set  $\Theta_P$  capable of generate a good quality description of the Pareto front  $\mathbf{J}_P^*$ . In this way, the decision maker (or simply the designer) can analyze the set and select the most preferable solution.

A general framework is required to successfully incorporate the MOO approach into any engineering process. A multiobjective optimization engineering design (MOOD) methodology consists (at least) of three main steps (Reynoso-Meza et al., 2014b): the MOOP definition (objectives, decision variables and constraints), the MOO process (optimizer selection) and the decision-making (DM) stage (analysis and selection of the calculated solutions).

#### 4.2 Multiobjective Problem Definition

Requirements often include specification on load disturbance compensation, set-point following and robustness to process uncertainty.

Reasonably the most basic MOP statement for PID controller tuning goals could be represented as:

$$\theta_c = [K_p, T_i, T_d] \quad (2)$$

Where  $\theta_c$  are the parameters of the optimal PID controller.

$$\begin{aligned} \min \mathbf{J}(\theta_c) &= [J_1(\theta_c), J_2(\theta_c)] \quad (3) \\ J_1(\theta_c) &= \text{Performance}(\theta_c) \\ J_2(\theta_c) &= \text{Robustness}(\theta_c) \end{aligned}$$

Control performance  $J_1(\theta_c)$  can be characterized by the integrated absolute error

$$IAE = \int_0^{\infty} |e(t)| dt, \quad (4)$$

where  $e$  is the control error due to a unit step load disturbance. It is a good performance measure for control system with integral action.

In order to measure the smoothness of the control action we have the control signal total variation  $J_2(\theta_c)$  is defined as

$$TV_u = \sum_{k=1}^{\infty} u(k+1) - u(k) \quad (5)$$

The maximum sensitivity is an indication of the system robustness (relative stability):

$$M_s = \max_w \left| \frac{1}{1 + C_y(jw)P(jw)} \right| \quad (6)$$

where  $C_y$  is the feedback controller transfer function,  $P$  is the controlled process.

In some experiments we propose to use the integrated square error (ISE) as a measure of performance for the set-point and load disturbance step responses (as in (Arrieta et al., 2010)) and the maximum sensitivity (6) as a constraint of MOOP.

$$ISE_{sp} = \int_0^{\infty} e^2(t) dt, \quad d = 0 \quad (7)$$

is the integrated square error when a set-point step response is considered,

$$ISE_{ld} = \int_0^{\infty} e^2(t) dt, \quad r = 0 \quad (8)$$

## 5 METHODOLOGY

In this section a brief description about the algorithms used to calculate the Pareto front approximation and the MCDM technique implemented to select a trade-off point from the Pareto front as the final solution to the MOP. Both of them have shown to be useful for PI and PID controllers in (Sánchez and Vilanova, 2013a; Sánchez and Vilanova, 2013b; Sánchez and Vilanova, 2014; Sánchez et al., 2014; Reynoso-Meza et al., 2014a; Sánchez et al., 2013).

### 5.1 Normal Normalized Constraint for Multi-objective Optimization

This algorithm is used to determine the Pareto front and the set of optimal solutions. Using this algorithm, the optimization problem is separated into several constrained single optimization problems. After series optimizations, a set of evenly distributed Pareto solutions results. The NNC method incorporates a critical linear mapping of the design objectives. This mapping has the desirable property that the resulting performance of the method is entirely independent of the design objectives scales and in the ability to generate a well distributed set of Pareto points even in numerically demanding situations (Messac et al., 2003b). The NNC method is presented here to solve a bi-objectives problem, but it can be generalized to  $n$ -objectives. An extract of the algorithm is presented,

for more details about the method see (Messac et al., 2003a). The algorithm is available in Matlab Central.

|  |
|--|
| <ol style="list-style-type: none"> <li>1 Generate the anchor points <math>J _i(x)</math> for each objective;</li> <li>2 Calculate the Utopian Point and NADIR;</li> <li>3 Normalized the objective space;</li> <li>4 Generate the utopian hyperplane;</li> <li>5 Definition of the normalized increments;</li> <li>6 Generate the utopian lines;</li> <li>7 <b>while</b> <i>normalized increments do</i></li> <li>8   Optimize;</li> <li>9 <b>end</b></li> <li>10 Algorithm concludes. <math>J_P</math> is approximated by <math>J_P^* = A _G</math>;</li> </ol> |
|--|

Algorithm 1: NNC Algorithm outline.

### 5.2 Nash Solution-NS

All the points of the Pareto front are equally acceptable solutions. Thus, there is the need to choose one of such point as the final solution to the MOOP. This is the last part of the MOOD procedure, the decision-making stage. For this purpose we propose the Nash (NS) criteria, for which a graphical explanation can be seen in Figure 1. In order to understand this option, we introduce what can be called the disagreement point. If we think on both objectives independently, none of them would agree on this point as a common solution because it represents the worst situation. In addition, this selection can be improved with respect of both objectives. On that basis, the area of the rectangle defined by the points (NS, A, B) and the disagreement point provides a measure of the amount of solutions the NS point improves with respect to both objectives simultaneously. The NS is the solution that maximizes such area, this denomination comes from identifying this point as the Nash Solution on a bargaining game among both objectives (Aumann and Hart, 1994).

## 6 EXPECTED OUTCOME

This thesis contains a research line in MOO process, focuses on controller tuning applications. For this reason it is expecting the following contributions:

- Cover the theoretical background required for this thesis.
- Develop a methodology to apply for the controller tuning (algorithm and MCDM technique).

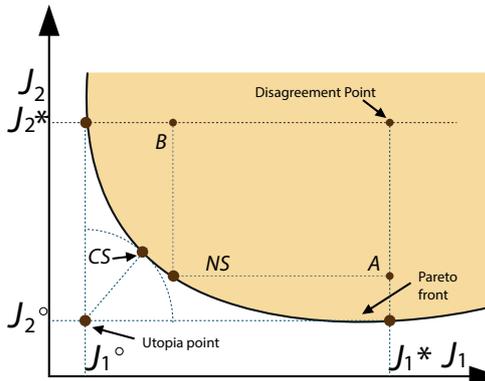


Figure 1: The Nash solution for a bi-objective case.

- Identify gaps that exist on the methodologies of the MOOD procedure.
- Applying the selected tools for the controller tuning: 1) Proportional-Integral (PI) controllers, 2) Proportional-Integral-Derivative (PID) controllers, 3) Fractional-Order Proportional-Integral-Derivative (FOPID) controllers.
- Implement the MOOD procedure in case of studies (benchmark).
- Develop proposals on tools to improve the usability and performance of the MOOD procedure.
- Nevertheless, during the develop of this thesis: collaborations with other research groups it will be carried out.
- Publishing the results.

This thesis is dedicated to find the methodology and techniques to address problems of control and operation of processes through the application of multi-objective optimization strategies. Some of the contributions are listed bellow.

### 6.1 Correlation Between $TV$ and $M_s$ (Sánchez and Vilanova, 2013a)

Figure (2) shows the pareto front that results for a process model with  $K = 1$  and  $\tau_o = 0.5$ . Anchor points and the location of a Ziegler-Nichols tuning are also shown. In this case, we do not have any constraint on the robustness. Just performance is considered. Instead, if we add as a robustness constraint, three usual values for  $M_s$ , for example  $M_s = \{1.4, 1.6, 1.8\}$ ; what we are doing is constraining the achievable system performance. This is shown in figure (3), where the new Pareto sets corresponding to each one of the robustness levels are jointly shown with the unconstrained one. As it is seen, we get a new pareto front,

considerably smaller than the previous one, constraining ourselves to a really small set of possible solutions.

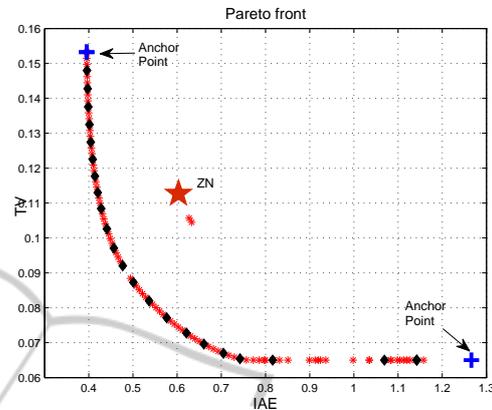


Figure 2: Pareto front for  $K = 1$  and  $\tau_o = 0.5$ .

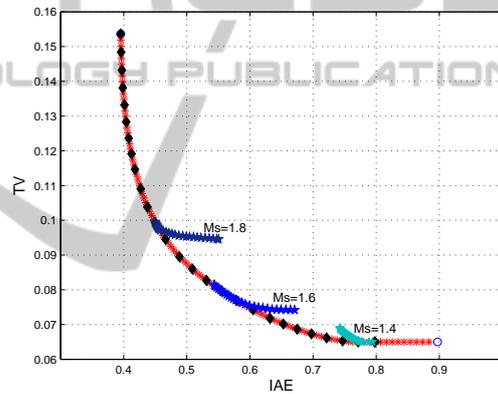


Figure 3: Comparison of Pareto fronts.

Therefore, there is a correlation between the value of  $M_s$  and the control input usage in terms of  $TV$ . Effectively, as pointed out in (Foley et al., 2005) there do exists a correlation. This is clearly shown in figure (4) for the example at hand. We can therefore directly associate the robustness to the  $TV$  performance index and think on that index not just as the input usage but also as a measure of the closed system robustness. From figure (4) the approximate relation is established  $M_s \approx 10.5TV + 0.77$ . This allows to think on the multi objective optimization problem simply as a tradeoff between  $TV$  and  $J_{IAE}$ , having in mind that the selection of the appropriate point form the pareto front will need to take into account the level of robustness we need (lower values for  $TV$ ). Effectively this simplifies the setup for the optimization problem and subsequent generation of the pareto front.

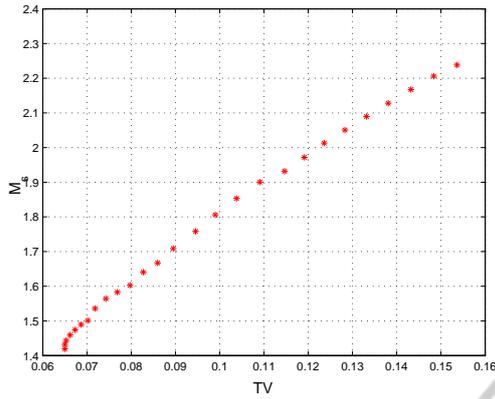
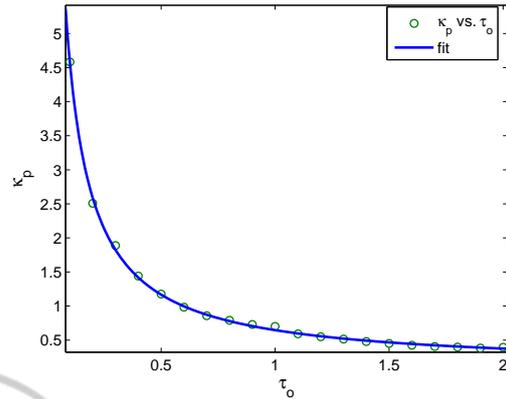
Figure 4:  $TV$  and  $M_s$  correlation,  $M_s \approx 10.5TV + 0.77$ .

Figure 5: Normalized controller gain.

## 6.2 Nash-based PI Tuning (Sánchez and Vilanova, 2013b)

The values of  $\tau_o$  are selected to take the FOPDT processes with small, medium and fairly long dead time into account, the values of the normalized dead-time ( $\tau_o$ ) are considered from 0.1 to 2. To obtain appropriate values of  $\kappa_p$  and  $\tau_i$  for a specified value of  $\tau_o$ , non-dominated solutions are determined using the NNC method. Finally, when we obtain the Pareto front one optimal solution is chosen known as Nash solution. The tuning formulae is as follows:

$$\kappa_p = \frac{a_0 * \tau_o + a_1}{\tau_o + b_0}, \quad (9)$$

$$\tau_i = \frac{a_0 * \tau_o + a_1}{\tau_o + b_0}, \quad (10)$$

Table 1: Controller constants for  $\kappa_p$  and  $\tau_i$ .

|            | $a_0$ | $a_1$ | $b_0$ |
|------------|-------|-------|-------|
| $\kappa_p$ | 0.098 | 0.565 | 0.026 |
| $\tau_i$   | 2.624 | 1.548 | 2.649 |

The constants in equations (9-10) are shown in Table 1. The fitting obtained using this formulae are also shown in figures (5-6).

## 6.3 Comparison Between Multi-objective Optimization Techniques (Sánchez and Vilanova, 2014)

Consider the four-order controlled benchmark process proposed in (Åström and Hägglund, 2000) and given by the transfer function

$$P_\alpha(s) = \frac{1}{\prod_{n=0}^3 (\alpha^n s + 1)} \quad (11)$$

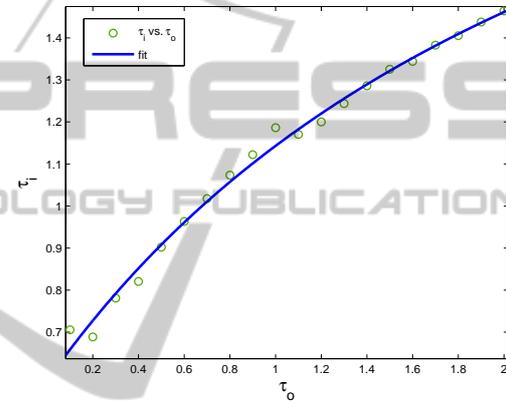


Figure 6: Normalized controller integral time.

with  $\alpha \in \{0.1, 0.5, 1.0\}$ . Using the three-point identification procedure *I23c* (Alfaro, 2006) FOPDT models have been obtained whose parameters are shown in Table 2. These models will be used for PI controller design.

Table 2: Example - FOPDT Models.

| $\alpha$ | $K$ | $T$   | $L$   | $\tau_o$ |
|----------|-----|-------|-------|----------|
| 0.50     | 1   | 1.247 | 0.691 | 0.554    |

- Case 1: The pareto front was obtained using the NNC and a reference point (Ziegler-Nichols as a initial point), using the ZN tuning rule ( $K_p = 1.62T_i = 2.30$ ), this solution is dominated by other solutions as we can see in figure (7)
- Case 2: The pareto front was calculated with the Multiobjective Differential Evolution Algorithm with Spherical Pruning (sp-MODE) using pertinence criteria. This concept refers to the ability to give a practical solution from the point of view of the designer. In this case, the Pareto front is bounded, if it is known a priori which solutions are looking for and are interesting for the

designer. It is a useful option when you know that in some cases, improving one objective does not justify the severe degradation in the other. The values ( $IAE = 0.9217$   $TV = 0.1319$ ) were used as the pertinence criteria. They were calculated with the parameters mentioned before. In figure (8) it is possible to see how the algorithm focus the search only in the range that contains the values of pertinence.

- Case 3: We use the sp-MODE but this time without of pertinence, see figure (9).

Table 3: Example - Controller Parameters.

| Methods                      | $\alpha = 0.5$ |      |
|------------------------------|----------------|------|
|                              | $Kp$           | $Ti$ |
| Case 1: NNC                  | 1.34           | 1.86 |
| Case 2: sp-MODE (pertinence) | 1.05           | 1.18 |
| Case 3: sp-MODE              | 0.84           | 1.37 |

For each one of these cases the Nash solution was calculated, therefore in Table 3 the parameters of the controller (PI) are shown and in Table 4 the values of performance and robustness of the system are displayed. Furthermore, figure (10) shows the achieved time response when facing a step load-disturbance.

Table 4: Example - Performance and Robustness.

|                | Case 1<br>NNC | Case 2<br>sp-MODE (pertinence) | Case 3:<br>sp-MODE |
|----------------|---------------|--------------------------------|--------------------|
| $\tau_{0=0.5}$ |               |                                |                    |
| $IAE$          | 0.9022        | 0.7662                         | 1.0601             |
| $TV$           | 0.1023        | 0.1118                         | 0.0856             |
| $M_s$          | 1.64          | 1.67                           | 1.44               |
| $Y_{max}$      | 0.3016        | 0.3167                         | 0.3446             |

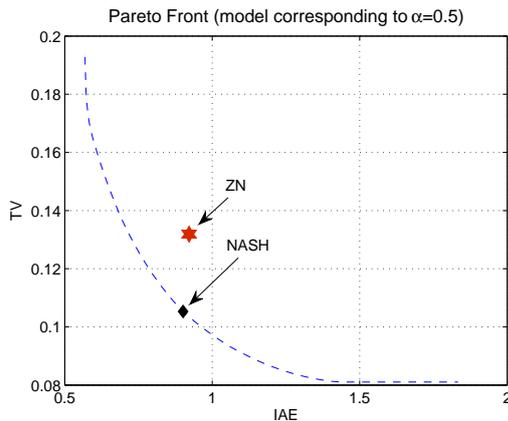


Figure 7: Pareto front for the process model corresponding to  $\alpha = 0.5$  (Case 1).

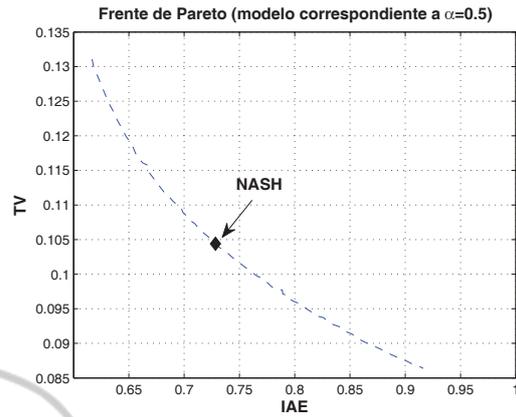


Figure 8: Pareto front for the process model corresponding to  $\alpha = 0.5$  (Case 2).

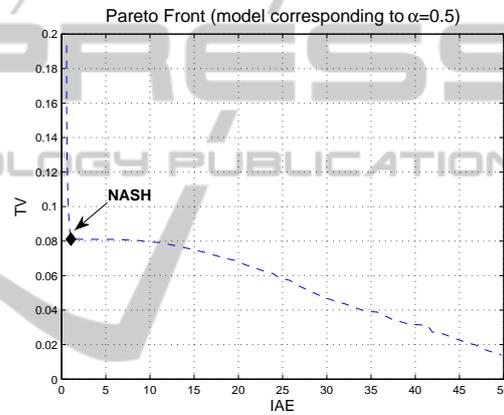


Figure 9: Pareto front for the process model corresponding to  $\alpha = 0.5$  (Case 3).

### 6.4 Equivalence and Optimality for PID Controllers (Sánchez et al., 2014)

Based on the initial work of ((Alfaro and Vilanova, 2012)) where the differet formulations for PID controllers are listed and conversion formulae is provided, the equivalence between them, is analyzed here.

The optimization was applied to the most general configuration  $PID_{2F}$  from  $\tau = [0.1 - 2.0]$ . Here we only present 3 representative cases of  $\tau = [0.6, 1.0, 2.0]$ , the Pareto front is shown in Figure 11. It can be seen that when the normalized dead time increases the IAE index as well.

Furthermore, the total variation (TV) is decreasing while also the robustness of the system is higher. In ((Sánchez and Vilanova, 2013a)) some experiments shows that there is correlations between both of them. This can be see in Table (5-6), where the values of performance and robustness are shown for each configuration and its respective  $\tau$ . In Figure (11) is also

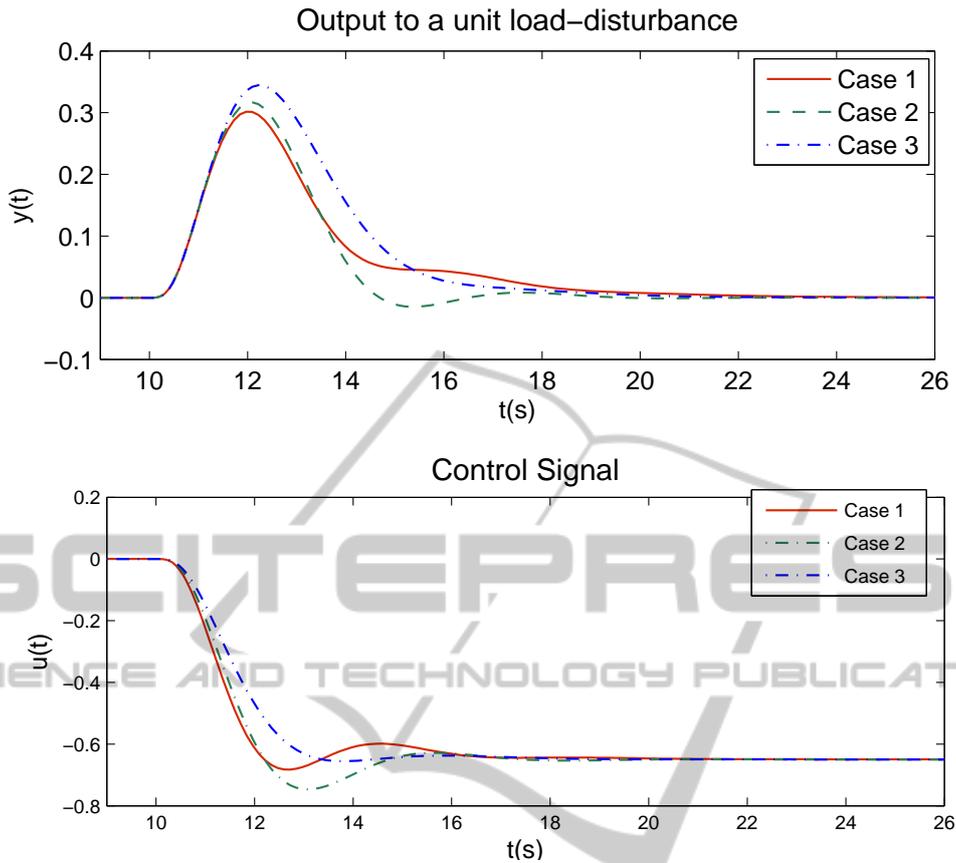


Figure 10: Time output and control responses for  $\alpha = 0.5$ .

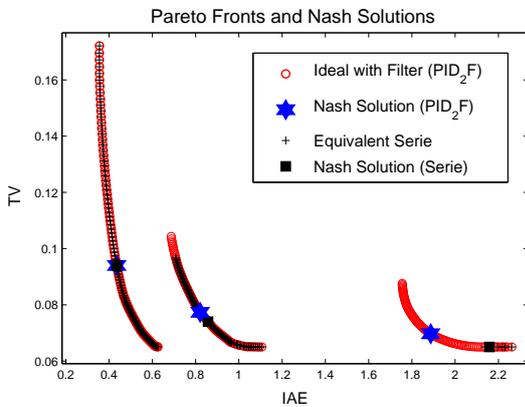


Figure 11: Pareto Front and Nash Solution.

Table 5: Performance and Robustness ( $PID_{2F}$  y  $PID_2$ ).

| $\tau_0$ | IAE    | TV     | $M_s$ |
|----------|--------|--------|-------|
| 0.6      | 0.4348 | 0.0940 | 1.69  |
| 1.0      | 0.8571 | 0.0740 | 1.51  |
| 2.0      | 2.1583 | 0.0650 | 1.42  |

shown the Series equivalent for  $PID_{2F}$  y  $PID_2$ . It is worth mentioning, that when it passed from

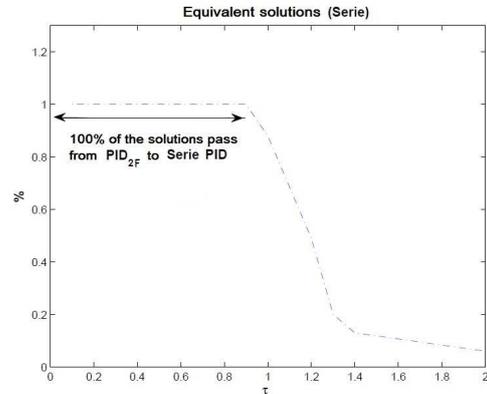


Figure 12: Equivalent solutions from a  $PID_{2F}$  to Serie  $PID$ .

$PID_{2F}$  configuration to the  $PID_2$ , all the solutions had the equivalent; owing to this both configurations have the same Nash solution. In Figure (12) we see that the equivalent solutions for the series configuration are less as  $\tau$  increases, in this cases the Nash solutions will not be the same for the Series  $PID$  see Table (6).

Table 6: Performance and Robustness (*Serie*).

| $\tau_0$ | <i>IAE</i> | <i>TV</i> | $M_s$ |
|----------|------------|-----------|-------|
| 0.6      | 0.4348     | 0.0940    | 1.69  |
| 1.0      | 0.8206     | 0.0773    | 1.53  |
| 2.0      | 1.8869     | 0.0697    | 1.48  |

### 6.5 Optimality Comparison of PID Implementations (Sánchez and Vilanova, 2014)

The Pareto front for each configuration was calculated with normalized dead times from 0.1 to 2.0. But in this paper only three representative cases (0.1, 0.75 and 1.75) are presented. The Pareto fronts are shown in figure 13.

The displacement of the Pareto fronts of each configuration shows the behavior of the PID algorithm from a general to a more constrained PID configuration. As it can be seen as the  $\tau_0$  increases, the Pareto fronts; Figure (13); are getting closer between each other but always dominated by the  $PID_{2F}$  configuration. The superiority of this configuration regarding the Standard one increases as  $\tau_0$  decreases. However both configurations show a clear superiority with respect to the series one. As shown in Table (7) the values of performance and robustness for each case are different, sometimes the 2DoF PID Standard and Ideal with filter have similar values compared to the Series configuration.

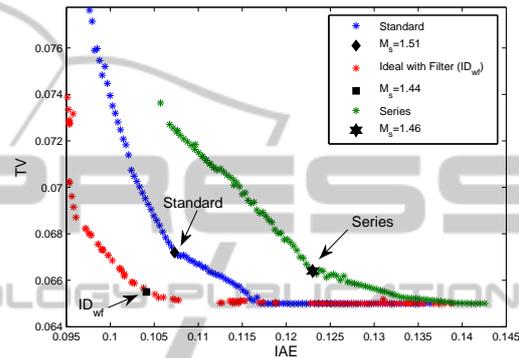
With the experiments performed in this work we realized that if we want a better functionality, it is not necessary to use the conversion formulae. For example see in Figure (13a), if we apply the equations to the 2DoF Series configuration in order to obtain a  $PID_{2F}$ , the results of the performance and robustness will be the same as the 2DoF Series (the new Pareto front calculated would be at the same position as the Series configuration). This means that instead of applying the conversion formulae we could tune the controller again in order to obtain a better performance, robustness and also less overshoot. Overall, this representation by the Pareto front allows us to see how important is to see the behavior of this indexes (*IAE*, *TV*,  $M_s$ ).

### 6.6 Reliability based Multiobjective Optimization Design Procedure for PI Controller Tuning (Reynoso-Meza et al., 2014a)

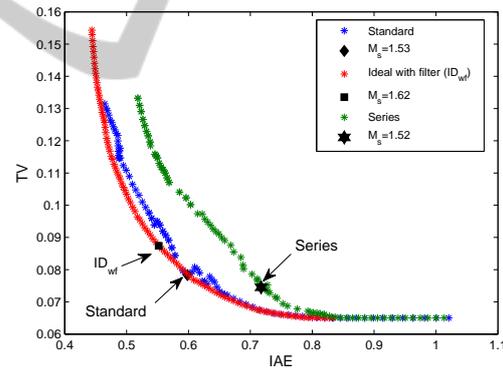
This was a collaborative work, we presented a MOOD procedure involving a reliability based MOOP state-

Table 7: Controller performance.

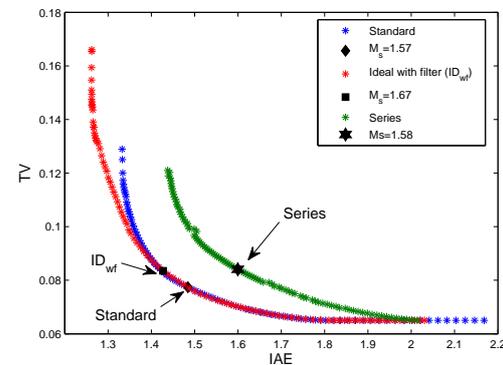
| $\tau_0$ | Controller        | <i>IAE</i> | <i>TV</i> | $M_s$ |
|----------|-------------------|------------|-----------|-------|
| 0.1      | Standard          | 1.3534     | 0.1725    | 1.51  |
|          | Series            | 1.4007     | 0.1679    | 1.46  |
|          | Ideal with Filter | 1.3365     | 0.1713    | 1.44  |
| 0.75     | Standard          | 2.7298     | 0.1994    | 1.53  |
|          | Series            | 3.2146     | 0.1774    | 1.52  |
|          | Ideal with Filter | 2.5964     | 0.2197    | 1.62  |
| 1.75     | Standard          | 5.3946     | 0.1876    | 1.57  |
|          | Series            | 5.9093     | 0.1913    | 1.58  |
|          | Ideal with Filter | 5.2701     | 0.1986    | 1.67  |



(a) Pareto Front  $\tau_0 = 0.1$



(b) Pareto Front  $\tau_0 = 0.75$



(c) Pareto Front  $\tau_0 = 1.75$

Figure 13: Pareto Front for different  $\tau_0$ .

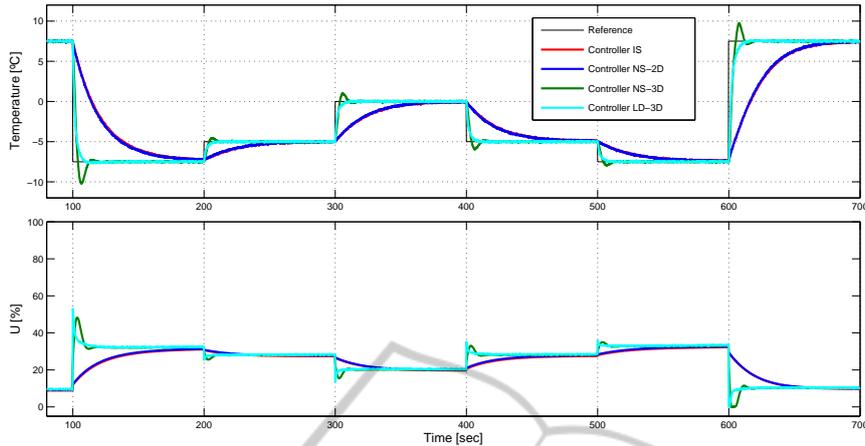


Figure 14: Performance comparison on the Peltier Cell.

ment for controller tuning. To improve the results in the MOO process a hybrid approach has been proposed to calculate the Pareto front approximation. Merging deterministic and evolutionary algorithms, the Normalized Normal Constraint (NNC) and the Multiobjective Differential Algorithm with Spherical Pruning (sp-MODE), respectively. A Peltier cell was chosen to evaluate the above, mentioned MOOD procedure. First we generate the preliminary bi-objective Pareto front  $J_p^*$  with the NNC algorithm. With such Pareto front, two solutions are selected for further evaluation: the initial solution employed in the optimization process and the Nash-based (IS<sup>1</sup> and NS-2D<sup>2</sup> respectively). In the execution using the sp-MODE and the Pareto front approximation from NNC algorithm, pertinency is included in the algorithm to bound the new objective using the starting solution of the NNC algorithm. Two solutions were selected: Nash-based and a solution selected by analyzing the Pareto front using Level diagrams (NS-3D and LD-3D respectively).

The performance of the selected controllers is shown in Figure (14) and in Table (8). Whilst performance of controllers IS and NS-2D are similar it is interesting to notice differences between controllers NS-2D and NS-3D. Both of them have been selected using the same Decision Making (DM) rule (Nash solution). Differences on the performance are due to the additional information used in the MOP statement minding degradation on IAE performance. Controller LD-3D selected from the LD visualization is consistent with the fact of improving IAE at expense of more control effort (TV). The results presented validate the procedure as useful for PI tuning of non-linear systems.

<sup>1</sup>Initial Solution<sup>2</sup>Nash Solution in 2D

Table 8: Controller performance on the Peltier cell case study.

| Controller | $K_p$  | $T_i$  | IAE   | TV    |
|------------|--------|--------|-------|-------|
| IS         | 0.19   | 2.74   | 992.4 | 86.0  |
| NS-2D      | 0.1898 | 2.6613 | 967.6 | 86.8  |
| NS-3D      | 0.5091 | 0.4057 | 154.6 | 263.9 |
| LD-3D      | 2.3735 | 3.2623 | 120.7 | 657.4 |

## 6.7 Nash Solution for Optimal Balance of Servo/Regulation Operation in PID Control (Sánchez et al., 2015)

A multi-objective optimization approach is proposed for the tuning of one degree-of-freedom proportional-integral-derivative controllers where both the trade-off between the servo and regulation operation modes and the trade-off between performance and robustness are considered. After having quantified the loss of performance that occurs if robustness is taken into account in the optimal design of the controller, a tuning rule is proposed based on the Nash solution, so that a balanced robust tuning is obtained simply starting from a first-order-plus-dead-time model of the (self-regulating) process.

It has been shown that, in this context, the robustness of the system can be a critical issue and therefore it has to be included explicitly in the optimization procedure. Tuning rules based on the Nash solutions have also been devised so that the methodology can be easily implemented in standard industrial controllers.

In this example we will illustrate how the implementation of the Nash tuning rules improves the robustness of the system and also maintains acceptable values of the performance with respect to both the operation modes. Consider the following process and the corresponding FOPDT approximation ((Zhuang

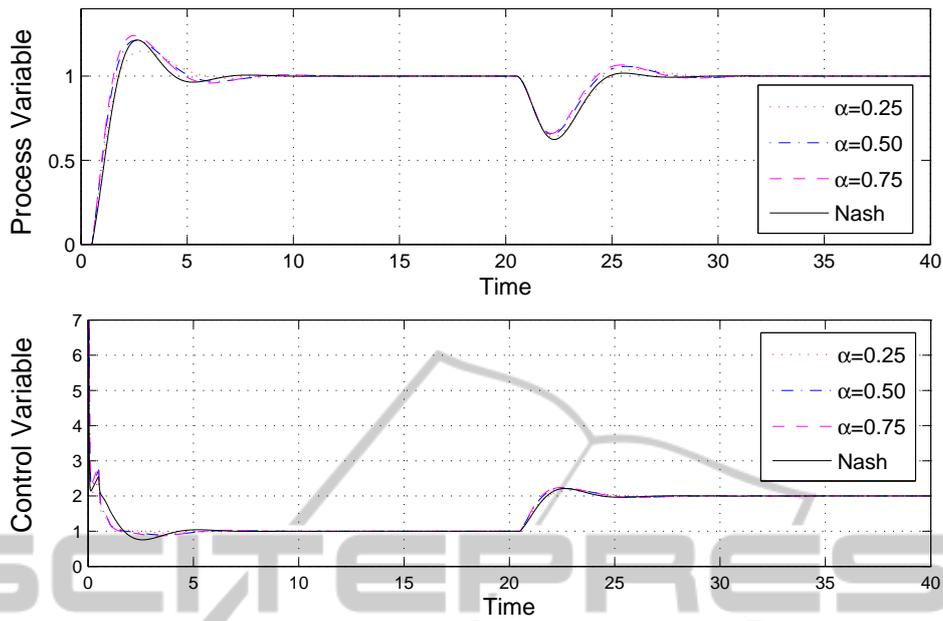


Figure 15: Step responses for the illustrative example.

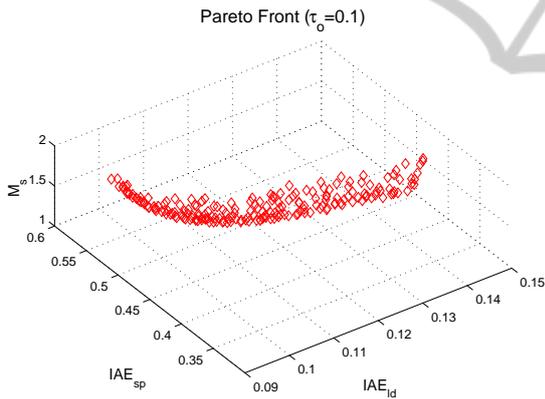


Figure 16: Pareto front for  $\tau_o = 0.1$ .

and Atherton, 1993)):

$$P(s) = \frac{e^{-0.5s}}{(s+1)^2} \approx \frac{e^{-0.99s}}{1+1.65s} \quad (12)$$

The PID controller parameters determined by using the Nash tuning rules and the intermediate tuning rules ((Arrieta et al., 2010)) are shown in Table (9) while the corresponding performance indices and maximum sensitivity values are shown in Table 10. The set-point and load disturbance step responses are plotted in Figure (15).

From the results it can be seen that the performance obtained with the Nash tuning is similar to that obtained with the intermediate tuning despite the maximum sensitivity is  $M_s = 1.73$  for the Nash tuning while in the other cases it ranges from 2.26 to 2.77.

Table 9: PID controller parameters.

| Tuning          | $K_p$  | $T_i$  | $T_d$  |
|-----------------|--------|--------|--------|
| NS              | 1.9652 | 1.6477 | 0.3464 |
| $\alpha = 0.25$ | 1.791  | 1.378  | 0.520  |
| $\alpha = 0.5$  | 1.949  | 1.234  | 0.527  |
| $\alpha = 0.75$ | 2.016  | 1.177  | 0.531  |

Table 10: Performance and robustness value for the system (12).

| Tuning          | $ISE_{ld}$ | $ISE_{sp}$ | $M_s$ |
|-----------------|------------|------------|-------|
| NS              | 0.2674     | 1.1511     | 1.73  |
| $\alpha = 0.25$ | 0.2562     | 1.1133     | 2.26  |
| $\alpha = 0.5$  | 0.2265     | 1.1793     | 2.59  |
| $\alpha = 0.75$ | 0.2172     | 1.2224     | 2.77  |

## 6.8 Future Work

The future research efforts will be conducted:

1. **FOPID Controllers:** to apply the same approach we use in (Sánchez et al., 2015) to Fractional-order proportional-integral-derivative (FOPID) controllers. A MOOD procedures will be implemented to obtain a set of tuning rules for FOPID controllers. Using the Nash solutions as the MCDM technique. The tuning rules will be devised in order to minimise the integrated absolute error with a constraint on the maximum sensitivity. The trade-off between the performance in the set-point following and in the load disturbance rejection task it will be taking into account, a preliminary result is shown in Figure (16).

**2. Multistage Procedure for PI Controller Tuning:** a multistage approach is proposed merging a deterministic and evolutionary algorithm, the Normalized Normal Constraint (NNC) and Multiobjective Differential Evolution Algorithm with Spherical Pruning (sp-MODE), respectively. This technique is formulated through design of a multi-objective optimization procedure, to ensure the construction of Pareto frontier that guarantee well distribution and exclude the non-Pareto and local Pareto points. This procedure focuses on reliability-based optimization instances. To validate the approach, we will consider the Boiler Control Benchmark and the Peltier cell; the results of the improvement of the performance is demonstrated and its usefulness for controller tuning.

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