Optimal Design of Digital Low Pass Finite Impulse Response Filter using Particle Swarm Optimization and Bat Algorithm

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Abstract: In this paper, the traditional metaheuristic Particle Swarm Optmization (PSO) and the Bat Algorithm (BA) are used to optimal design digital low pass (LP) Finite Impulse Response (FIR) filters. These filters have a wide range of applications because of their characteristics. They are easy to be designed, they have guaranteed bounded input-bounded output (BIBO) stability and can be designed to present linear phase at all frequencies. Traditional optimization methods based on gradient are susceptible to getting trapped on a local optima solution when they are applied to optimize multimodal problems, such as the FIR filter design. Here, to overcome this drawback, the aforementioned metaheuristics are adopted to obtain the coefficients of low pass FIR filters of order 20 and 24. The performance of BA and PSO algorithms are compared with the classical Parks and McClellan (PM) filter design algorithm, which is a deterministic procedure. For this comparison is considered the filters pass band and stop band ripples, transition width and statistical data. The simulation results demonstrate that the proposed filter design approach using BA algorithm outperforms PM and PSO.

1 INTRODUCTION

The digital filters have a relevant role in digital signal processing systems. By performing mathematical operations in a given signal, they are able to reduce or amplify certain aspects of this signal. Digital filters are used in a large number of applications, such as, video and audio processing, control and communication systems, systems for medical purposes, among others (Mandal et al., 2012a). The Finite Impulse Response (FIR) and Infinite Impulse Response (IIR) filters are the two major types of digital filters (Mandal et al., 2012b).

The impulse response of a FIR filter has finite duration. On the other hand, the same response of a IIR filter theoretically extends to infinity. The FIR filters are guaranteed to be bounded input-bounded output (BIBO) stable once they are non-recursive filters. The IIR filters are known as recursive filters, so their application requires caution regarding their stability. The IIR filters are useful for high-speed designs because they typically require a lower number of multiplies compared to FIR filters, however their implementation is more complicated (Litwin, 2000). Therefore, the FIR filters are an attractive approach in practical applications. They are easily implemented in digital systems and do not present instability issues. Furthermore, the FIR filter coefficients can be designed to be symmetrical about the center coefficient position, what guarantees a linear phase characteristic (Litwin, 2000; Mandal et al., 2012a).

The optimal design of a filter consists in choosing a set of coefficients of the filter to have a frequency response that optimally approximates the desired response (Ouadi et al., 2013). The FIR filter design is a nonlinear, non-differentiable and multimodal optimization problem that requires a suitable objective function to provide an accurate control of the various parameters of frequency spectrum. Therefore, the traditional optimization methods based on gradient do not represent a proper approach to solve this problem.

The Remez Multiple Exchange routine is used by the classical Parks and McClellan (PM) algorithm to design an optimal Chebyshev FIR filter (Parks and McClelland, 1972). This deterministic algorithm executes an iterative process based on the Chebyshev's alternation theorem in order to minimize the ripple value, considering another design specifications, such as the edge frequency and the filter size. However, this algorithm does not allow explicit selection of the maximum of the absolute ripple in the pass band and stop band (δ_p, δ_s), instead one can only specify their

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ratio (Ababneh and Bataineh, 2008).

In literature, different metaheuristics have been used to optimal design digital FIR filters, such as Particle Swarm Optimization (PSO) (Mandal et al., 2012a; Mandal et al., 2012b; Saha et al., 2013), Genetic Algorithms (GA) (Najjarzadeh and Ayatollahi, 2008; Ababneh and Bataineh, 2008), Gravitational Search Algorithm (GSA) (Saha et al., 2012), Cuckoo Search Algorithm (CSA) (Singh and Josan, 2014). Among the different metaheuristics, PSO and Bat Algorithm (BA) stand out for their simplicity of implementation and the low number of parameters that control their performance and convergence.

The PSO was proposed by Kennedy and Eberhart (Kennedy and Eberhart, 1995). It is based on simulating the social behavior of swarm of bird flocking, bees, and fish shooling (Ababneh and Bataineh, 2008). The BA was proposed by Yang (Yang, 2010), based on the echolocation behavior of bats. Due to the echolocation, the microbats can find their prey and discriminate different types of insects even in complete darkness. Both of these nature inspired metaheuristics are able to solve multi-dimensional and multi-modal optimization problems, overcoming some drawbacks of optimization gradient based methods.

In this work, the BA and PSO metaheuristics are employed to optimal design digital low pass (LP) Finite Impulse Response filters of order 20 and 24. The performance of these algorithms are compared with the Parks and McClellan filter design method. The stop band and pass band ripples, the transition width and statistical information are evaluated in this comparison. The simulation results demonstrated that in general BA presented the best performance in this specific study. It is noteworthy that according to the No Free Lunch Theorem of optimization affirms that a general purpose universal optimization strategy is impossible, and the only one strategy can outperform another is if it is specialized to the structure of the specific problem under consideration (Ho, 2001).

The remainder of this paper is organized as follows. Next section presents some basical concepts of the FIR filter and the objective function used by PSO and BA. Section 3 briefly describes the mechanisms of traditional PSO and BA metaheuristics. In Section 4, the obtained LP FIR filters are presented and the filter design approaches are compared. Finally, in Section 5 concluding remarks are given.

2 FIR FILTER DESIGN

Depending on what criteria are used, filters can be classified in several different ways. The two major types of digital filters are Finite Impulse Response (FIR) and Infinite Impulse Response (IIR) filters. The digital FIR filter, that is the focus of this study, can be mathematically described as follows:

$$H(z) = \sum_{n=0}^{N} h(n)z^{-1} \tag{1}$$

where N is the FIR filter order with $(N+1)$ coefficients to be set. Once the designed filters in this paper are positive and features even symmetry, N is an even number and only $(N/2+1)$ coefficients of $h(n)$ need to be designed. After the optimization they are concatenated to obtain all the $(N+1)$ low pass FIR filters coefficients. In this work, the PSO and BA are used to find these coefficients. Therefore, each individual (particle and bat) of these metaheuristics corresponds to a coefficient vector $\{h(0), h(1), \ldots, h(N/2)\}$.

The frequency response of a FIR filter can be defined as follows:

$$H\left(e^{j\omega_k}\right) = \sum_{n=0}^{N} h(n)e^{-j\omega_k n} \tag{2}$$

where $\omega_k = \frac{2\pi k}{N}$ and $H\left(e^{j\omega_k}\right)$ is the complex vector of the Fourier transform, which provides the FIR filter frequency response. The frequency is sampled from 0 to π with N samples. The PM algorithm for filter design uses the approximate error presented in (3).

$$E(\omega) = G(\omega)\left[H_d(e^{jw}) - H_i(e^{jw})\right] \tag{3}$$

where $H_d(e^{jw})$ is the frequency response of the designed filter and $H_i(e^{jw})$ is the frequency response of the ideal filter. $G(\omega)$ is the weighting function that provides the suitable weights for $E(\omega)$ in its different frequency bands. The $H_i(e^{jw})$ of an ideal filter can be expressed by the following relation:

$$H_i(e^{jw}) = \begin{cases} 1, & 0 \le \omega \le \omega_c \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

where ω_c is the edge frequency. The fixed ratio between the pass band (δ_p) and stop band (δ_s) ripples, presented by δ_p/δ_s is the major drawback of the PM algorithm. In order to obtain more flexibility in the optimization of the error function, allowing to specify the desired levels of δ_p and δ_s, equation (5) has been used to design digital filters (Ababneh and Bataineh, 2008; Mandal et al., 2012b; Singh and Josan, 2014). The cost function J used by the metaheuristics evaluated in this paper is given by (5).

$$J = \max_{\omega \le \omega_p}\left(|E(\omega) - \delta_p|\right) + \max_{\omega \ge \omega_s}\left(|E(\omega) - \delta_s|\right) \tag{5}$$

3 OPTIMIZATION ALGORITHMS

3.1 Particle Swarm Optimization

The Particle Swarm Optimization is a nature based metaheuristic that uses a swarm of particles based on the social behavior of bird flocking and fish schooling to search the best solution for a problem to be optimized. Each particle is a possible optimal solution that moves inside the space of feasible solutions. The actual position in the search space and the velocity of the i-th particle of the swarm are given by $\mathbf{x}_i$ and $\mathbf{v}_i$, respectively. Besides, after each iteration, the best position ($Pbest_i$) or solution reached by the i-th particle is determined. Regarding the design of FIR filters, each dimension of a particle position corresponds to one coefficient of the filter to be designed. Therefore, a particle represents a set of coefficients of a digital filter.

The swarm also have a global feature which corresponds to the best global position ($\mathbf{G}best$) already visited by all particles of the swarm. $\mathbf{x}_i$, $\mathbf{v}_i$, $\mathbf{P}best_i$ and $Gbest$ are n-dimensional vectors, where n is the dimension of the search space. In traditional PSO the velocity and position of the i-th particle can be updated using (6) and (7).

$$\mathbf{v}_i^{t+1} = \omega \mathbf{v}_i^t + r_1 \phi_1 \left(\mathbf{P}best_i^t - \mathbf{x}_i^t\right) + r_2 \phi_2 \left(\mathbf{G}best - \mathbf{x}_i^t\right) \tag{6}$$

$$\mathbf{x}_i^{t+1} = \mathbf{x}_i^t + \mathbf{v}_i^{t+1} \tag{7}$$

The updated values of velocity and position of the i-th particle is $\mathbf{v}_i^{t+1}$ and $\mathbf{x}_i^{t+1}$, respectively. ω is the inertia weight, ϕ_1 and ϕ_2 are constants that indicate the confidence of the particle in its own experience (local search) and the confidence of the particle in the swarm experience (global search). r_1 and r_2 are uniform random numbers between 0 and 1.

3.2 Bat Algorithm

The Bat Algorithm (BA) is inspired in the echolocation behavior used by bats during their flight movements. The echolocation is based on the emission of ultrasonic waves and the measurement of the time spent by these waves to return to their source after reach the prey or obstacle. The BA pseudocode is presented in Figure 1.

Initially, a set of N bats is randomly generated inside the search space of feasible solutions. The bats are described according to their position ($\mathbf{x}_i^t$), velocity ($\mathbf{v}_i^t$), emission frequency (f_i), loudness (A_i^t) and rate of pulse emission (r_i^t). Regarding the digital filter design, each dimension of the bat position represents one of the digital filter coefficients. Therefore, a solution described by one bat is a set of coefficients of the digital filter. f_{max} and f_{min} are the maximum and minimum emission frequency defined as 0 and 0.002 in this work, respectively. $\beta \in [0,1]$ is a vector of random numbers with normal distribution. ε is a random variable between -1 and 1. α and γ are constant parameters defined as 0.35 in this work. The variable $\mathbf{x}^*$ is the current global best position, which is located after comparing all the solutions among all the bats.

```
While (t < maximum number of iterations)
  For i = 1:N
     Generate a new bat (B_new) using (8), (9) and (10)
     If rand > r_new
        Select one among the best solutions and
        generate a local solution around this one, using (11)
     Else
        Select randomly a solution and generate a local
        solution around this one, using (11)
     End if
     Evaluate the bats
     If (rand < A_i) and (B_new < x_i)
        x_i = B_new
        Increase r_i and reduce A_i, using (12) and (13)
     End if
  End for
  Rank bats to find the best solutions in population
  Find the best bat
End while
```

Figure 1: Pseudocode of the Bat Algorithm.

$$f_i = f_{min} + (f_{max} - f_{min})\beta \tag{8}$$

$$\mathbf{v}_i^t = \mathbf{v}_i^{t-1} + (\mathbf{x}_i^t - \mathbf{x}^*) f_i \tag{9}$$

$$\mathbf{x}_i^t = \mathbf{x}_i^{t-1} + \mathbf{v}_i^t \tag{10}$$

$$\mathbf{x}_{new} = \mathbf{x}_{old} + \varepsilon A^t \tag{11}$$

$$A_i^{t+1} = \alpha A_i^t \tag{12}$$

$$r_i^{t+1} = r_i^0 \left[1 - exp(-\gamma t)\right] \tag{13}$$

4 RESULTS AND DISCUSSION

In this section the simulation results of the optimal design for LP FIR filters are presented. FIR filters

Table 1: Optimized coefficients of the low pass FIR filter of order 20.

Coefficient	PM	PSO	BA
$h(1) = h(21)$	0.000016462026203	-0.000695081641133	-0.000783617834824
$h(2) = h(20)$	0.048051046361716	0.036323142108915	0.034504912734454
$h(3) = h(19)$	-0.000023455414888	0.002404782366930	0.002078236516173
$h(4) = h(18)$	-0.036911143268907	-0.039388521163442	-0.035558833869864
$h(5) = h(17)$	-0.000014804257488	-0.002228795879323	-0.002848286056078
$h(6) = h(16)$	0.057262893095235	0.058189214448380	0.054878934074757
$h(7) = h(15)$	0.000000677226645	0.005997644375591	0.005087725645620
$h(8) = h(14)$	-0.102172983403192	-0.100284102843516	-0.104969693269015
$h(9) = h(13)$	0.000011850968750	-0.002421489345895	-0.002895822658702
$h(10) = h(12)$	0.316962289494363	0.317676093032592	0.318201104593910
$h(11)$	0.500018538901555	0.503677615564522	0.495333894867404

Table 2: Optimized coefficients of the low pass FIR filter of order 24.

Coefficient	PM	PSO	BA
$h(1) = h(25)$	-0.000037922722817	-0.001877155973793	-0.002252398212624
$h(2) = h(24)$	-0.032842059514712	-0.016508725107988	-0.017785448160513
$h(3) = h(23)$	0.000037994123700	-0.002070692843938	-0.002036074999033
$h(4) = h(22)$	0.025333659737802	0.024672556072237	0.024771221865898
$h(5) = h(21)$	-0.000000925584777	0.000991516825629	0.000942322200383
$h(6) = h(20)$	-0.037284360801762	-0.035767575797290	-0.034104768282211
$h(7) = h(19)$	0.000006294744005	0.000324605801132	0.000357668464521
$h(8) = h(18)$	0.057619569600668	0.056157880748866	0.055305256615506
$h(9) = h(17)$	0.000004540157503	0.000780019565850	0.000754780858245
$h(10) = h(16)$	-0.102383739808734	-0.100264251794438	-0.100759858394058
$h(11) = h(15)$	0.000000730442022	0.000062471248465	0.000056128814183
$h(12) = h(14)$	0.317060668831323	0.318357596099371	0.320874554639799
$h(13)$	0.499978577680726	0.496606219356213	0.495598720020515

of order 20 and 24 is evaluated after the adjust of their 21 and 25 coefficients, respectively. The value of the sampling frequency adopted is $f_s = 1$ Hz and the number of sampling points is taken as 256. The specification parameters of the LP FIR filters to be designed are the pass band ripple (δ_p) = 0.1, the stop band ripple (δ_s) = 0.01, pass band (normalized) edge frequency (ω_p) = 0.45; stop band (normalized) edge frequency (ω_s) = 0.55; and transition width = 0.1.

In order to establish a fair comparison between PSO and BA, different parameter values was evaluated for these algorithms. The PSO and BA was executed 100 times for each combination of parameters. In each execution, 20 particles/bats move inside the search space in 1000 epochs/generations. The empirical best parameters found for PSO and BA to design the FIR filters in this work are: $\omega = 0.002$, $\phi_1 = 2$, $\phi_2 = 1.4$, $\alpha = 0.35$, $\gamma = 0.35$, $A_i^0 = 0.35$, $r_i^0 = 0.35$, $f_{max} = 0.002$, $f_{min} = 0.0$. The following results were obtained using these parameters.

The Tables 1 e 2 list the coefficients of the best filters of order 20 and 24, respectively, designed by PSO and BA. The coefficients of the filters obtained by PM are also presented.

The Table 3 presents a summary of the simulation results obtained by PM, PSO and BA. This table presents the worst (maximum) ripple value, the average of the ripples and the best transition width found in 100 executions of PSO and BA. For the LP FIR filter of order 20, we can notice that the best values of maximum and average pass band ripple was obtained by PM. However, the average pass band ripple of the filters designed by BA is only slightly higher. Regarding the stop band, the PSO presented higher values for the maximum and average ripple than PM. The BA obtained the best average stop band ripple. The PSO and BA obtained smaller values for the transition width than PM. The BA presented the best performance considering this last parameter.

Regarding the LP FIR filters of order 24, the maximum and average values of the pass band ripples obtained by PSO are higher than the same values obtained by PM. The BA presented the best average pass band ripple value. Considering the stop band, the PM algorithm presented better results than PSO. The BA obtained an average stop band ripple better than PSO and only slightly higher than PM algorithm. The PSO and BA obtained smaller values for the transition width than PM. The BA presented the best performance considering this evaluation index for the LP filter of order 24.

The Table 4 presents the maximum, mean, vari-

Table 3: Comparative results of performance parameters for the LP FIR filters.

Algorithm	LP filter of order 20		
	Max, average pass band ripple (normalized)	Max, average stop band ripples (normalized)	Transition width (normalized)
PM	0.066452, 0.066452	0.066514, 0.066514	0.13297
PSO	0.091927, 0.071713	0.091673, 0.072021	0.12321
BA	0.098898, 0.067153	0.084058, 0.064921	0.11204
	LP filter of order 24		
PM	0.045192, 0.045192	0.045033, 0.045033	0.090225
PSO	0.068548, 0.052805	0.073533, 0.052655	0.072856
BA	0.054303, 0.038822	0.075469, 0.045328	0.071878

Table 4: Statistical parameters of stop band attenuation.

Algorithm	Order 20			
	Maximum	Mean	Variance	Standard Deviation
PM	23.5417	23.5417	–	–
PSO	20.7552	22.8866	0.62149	0.78835
BA	21.5084	23.7845	0.55516	0.74509
	Order 24			
PM	26.9293	26.9293	–	–
PSO	22.6704	25.6322	1.0551	1.0272
BA	22.4446	26.9313	0.95559	0.97754

ance and standard deviation for dB attenuation in the filters stop band. For the order 20 filters, PSO obtained a maximum attenuation of 20.7552 dB and mean attenuation of 22.8866 dB. The BA filters obtained a maximum attenuation of 21.5084 dB and mean attenuation of 23.7845 dB. Considering the order 24 FIR filters, PSO obtained a maximum attenuation of 22.6704 dB and mean attenuation of 25.6322 dB. The BA filters obtained a maximum attenuation of 22.4446 dB and mean attenuation of 26.9313 dB. The BA obtained smaller values of variance and standard deviation than PSO.

The Figures 2–11 illustrates the magnitude response for the best filters found by PSO and BA . Figures 2 and 7 shows the normalized magnitude response for LP FIR filters of order 20 and 24, respectively. Figures 3 and 8 presents the normalized magnitude response of the pass band. The normalized magnitude response of the stop band is shown in Figures 4 and 9. The magnitude response in dB can be seen in Figures 5 and 10, and the magnitude response in dB of the stop band is illustrated in Figures 6 and 11.

In Figure 3 we can notice that the pass band ripple of the best LP FIR filter of order 20 designed by PSO is slightly smaller than PM, and the BA obtained the best result regarding this performance parameter. The smallest stop band ripple was obtained by PSO, followed by BA. This is confirmed by Figure 6 that demonstrates that the highest attenuation was obtained with the LP FIR filter designed by PSO.

For the LP FIR filter of order 24, the Figure 8 illustrates that the pass band ripple of the best filters designed by PSO and BA is better than PM. The BA

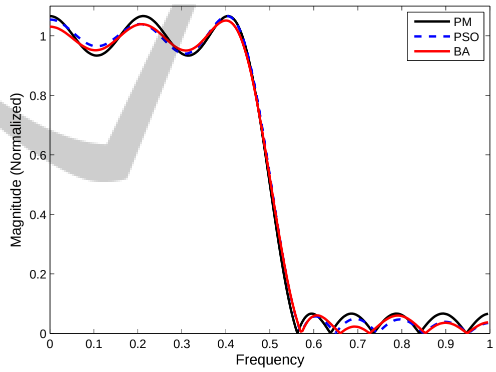

Figure 2: Normalized plot for the low pass FIR filter of order 20.

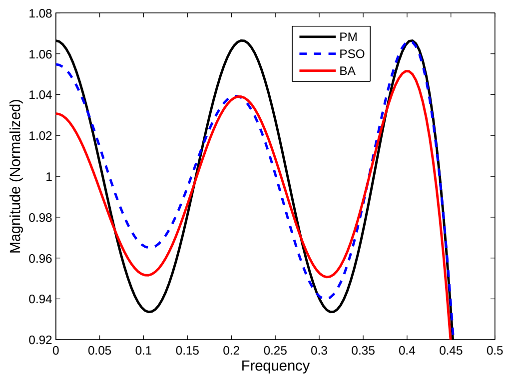

Figure 3: Normalized pass band ripple plot for the low pass FIR filter of order 20.

Table 5: Performance Parameters for the best LP FIR filters.

Algorithm	Order 20		
	Pass band ripple (normalized)	Stop band ripple (normalized)	Stop band ripple (dB)
PM	0.066452	0.066514	23.5417
PSO	0.066192	0.057021	24.8792
BA	0.051466	0.060571	24.3547
	Order 24		
PM	0.045192	0.045033	26.9293
PSO	0.032841	0.040016	27.9554
BA	0.031851	0.040027	27.9529

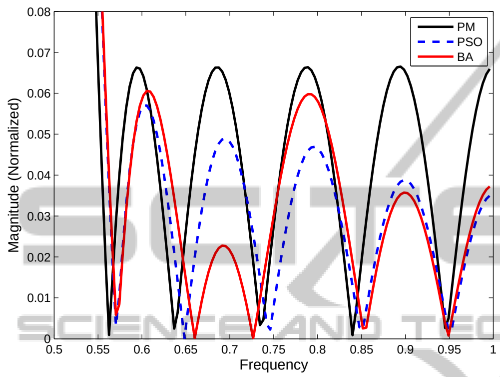

Figure 4: Normalized stop band ripple plot for the low pass FIR filter of order 20.

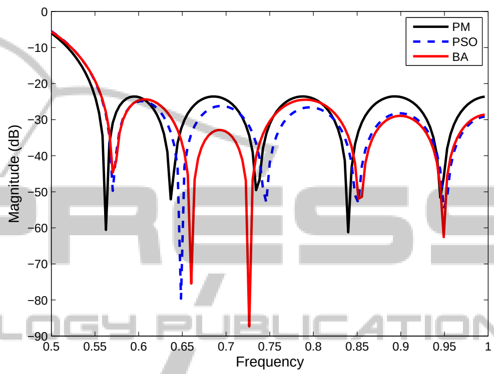

Figure 6: Magnitude (dB) plot for the stop band of low pass FIR filter of order 20.

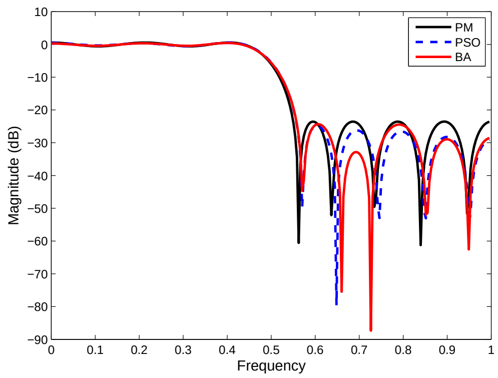

Figure 5: Magnitude (dB) plot for the low pass FIR filter of order 20.

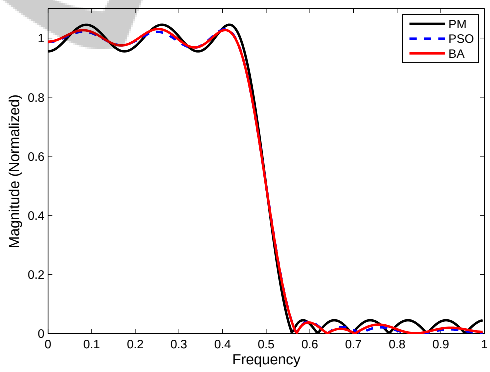

Figure 7: Normalized plot for the low pass FIR filter of order 24.

obtained the best result for this performance parameter. The smallest stop band ripple was obtained by the filter designed by BA, followed by PSO. This is confirmed in Figure 11 that demonstrates that the best BA LP FIR filter presented the highest attenuation.

Some performance parameters of the best LP FIR filters designed by PSO and BA are presented in Table 5: normalized pass band ripple, normalized stop band ripple and stop band ripple in dB.

5 CONCLUSIONS

In this work the traditional Particle Swarm Optimization and the Bat Algorithm were adopted to optimal adjust the coefficients of a low pass FIR filter. The performance of these metaheuristics were compared with the Parks and McClellan algorithm, a well known and succeeded deterministic technique to design digital filters. The simulation results demonstrated that PSO and BA are also efficient approaches to solve the problem of filter design. The average

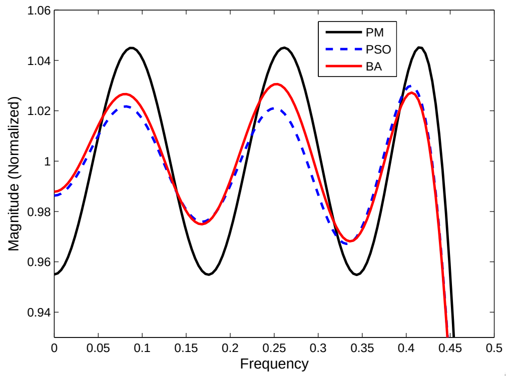

Figure 8: Normalized pass band ripple plot for the low pass FIR filter of order 24.

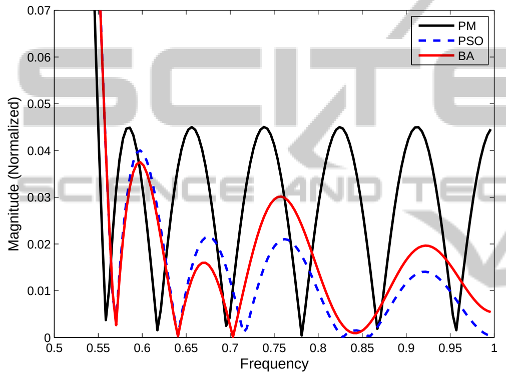

Figure 9: Normalized stop band ripple plot for the low pass FIR filter of order 24.

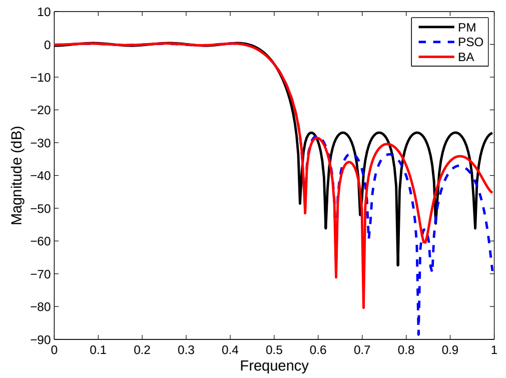

Figure 10: Magnitude (dB) plot for the low pass FIR filter of order 24.

band ripples of the filters designed by these techniques are close to the ones obtained by PM algorithm, with the closest results being presented by BA. Regarding the best LP FIR filters designed by the metaheuristics evaluated in this work, the bat algorithm presented better results for pass band ripple and transition width than PSO and PM. The best FIR filter designed by PSO obtained the smallest pass band ripple, but BA presented a close value for this same performance parameter. Therefore, the simulation results presented in this work demonstrated that the best LP FIR filters of order 20 and 24 were designed by BA. Once the PSO also have presented satisfactory results, both metaheuristic algorithms analyzed can be considered efficient optimizers to solve the problem of digital filters design. In future works, another metaheuristics and deterministic methods will be considered in order to realize a more extensive comparison between methods that can be used to project FIR and IIR digital filters. We also expect to evaluate the use of different objective functions for this problem.

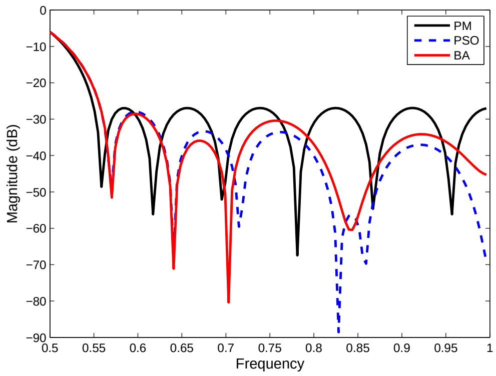

Figure 11: Magnitude (dB) plot for the stop band of low pass FIR filter of order 24.

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