Estimation of Uniform Static Regression Model with Abruptly Varying Parameters

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Abstract: A modular framework for monitoring complex systems contains blocks that evaluate condition of single signals, typically of sensors. The signals are modelled and their values must be found within the prescribed bounds. However, an abrupt change of the signal increases the estimated parameters' variance, which raises uncertainty of the sensor condition although it operates correctly. This increase affects the whole system in evaluation of condition uncertainty. The solution must be fast and simple, because of runtime application requirements. The signal is modelled by a static model with uniform noise, variance increase is tested and if detected, the model memory is cleared. The fast and efficient algorithm is demonstrated on industrial rolling data. The method prevents the parameters' variance from the artificial increase.

1 INTRODUCTION

Fault detection and condition monitoring is a permanently developing area (Isermann, 2011; Toliyat et al., 2012; Marwala, 2012). Currently, a hierarchical condition monitoring framework (ProDisMon) is developed (Dedecius and Ettler, 2014) where the system in question is decomposed into a set of mutually logically interconnected basic components. To each component, a binomial opinion on its particular health is assigned. This opinion includes also uncertainty of users' judgement. It can be interpreted as a characteristics of a condition of the investigated system or unit. The particular opinions on basic components are subsequently combined using rules of subjective logic (Jøsang, 2008) to obtain information on overall system health.

Sensors comprise an important part of the above mentioned basic system components. For the purpose of ProDisMon project, several methods were proposed that evaluated a health of the sensor signal (Ettler and Dedecius, 2014; Pavelková and Jirsa, 2014). These methods take into consideration the inaccuracy of a measured signal with respect to user given bounds and build this inaccuracy in the binomial opinion as an uncertainty. This uncertainty is the higher, the closer the signal values are to user given bounds. Nevertheless, a situation may occur during the evaluation that the signal value changes abruptly within the permitted area. Then, a variance of the signal estimate rapidly increases. Consequently, the uncertainty in the opinion unnecessarily increases. To prevent these unwanted uncertainty increases, a method using the change point detection might help.

Change-point problems (Basseville, 1988) arise when different subsequences of a data series have different probability distributions. In (Chib, 1998), the change-point model is described by a latent state variable that indicates the mode from which a particular observation has been drawn. This state variable is specified to evolve according to a discrete-time discrete-state Markov process with the transition probabilities constrained so that the state variable can either stay at the current value or jump to the next higher value. The paper (Hawkins, 2001) develops an exact approach for finding maximum likelihood estimates of the change points and within-segment parameters when the functional form is within the general exponential family. The paper (Lebarbier, 2005) deals with the problem of detecting change-points in the mean of a signal corrupted by an additive Gaussian noise. The number of changes and their position are unknown. From a non-asymptotic point of view, their estimation is proposed using a method based on a penalized least-squares criterion. In (Zhang and Basseville, 2014), a statistical approach to fault de-

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tection and isolation for linear time-varying systems subject to additive faults with time-varying profiles is described. The proposed approach combines a generalized likelihood ratio test with a recursive filter that cancels out the dynamics of the monitored fault effects.

In this paper, we propose a method that considers an abrupt change in a sensor signal values. The signal is estimated by a static regression model with bounded noise on a sliding window. An unwanted increasing of estimate variance, that indicates a change point, is prevented by the window resetting.

The implementation in practice requires fast algorithms that can run in real time with a relatively high sampling frequency (200 Hz or higher) for a system composed of tens of units to be observed. Therefore, another criterion is a computational simplicity.

The choice of the method is given by demands of the application being developed. The method must be compatible with the mechanisms already implemented, particularly probabilistic (subjective) logic (Jøsang, 2008) as a tool to build a hierarchical structure of the basic components (blocks).

Because a sensor deterioration can manifest itself, among others, by increase of the signal noise, the signal variance is used as an input quantity to evaluate uncertainty of the sensor condition (Ettler and Dedecius, 2015). This is one of several sensor tests.

The purpose of this work is to propose an estimator of a scalar signal's variance, resistant to abrupt changes, i. e. to jumps in the data.

2 BASICS OF THE SUBJECTIVE LOGIC

Subjective logic is a kind of probabilistic logic, introduced by (Jøsang, 2008). Except of terms "true" and "false", used by a traditional binary logic, it operates with a term "not known". We present basic terms of this field, details can be found e. g. in (Jøsang, 2008).

According to analysis or observation, a *binomial opinion* ω on truth value of a statement x is formulated. Formally, $\omega = (b, d, u, a)$. The items of the vector ω are

- b — probability that x is true (**b**elief),
- d — probability that x is false (**d**isbelief),
- u — probability that state of x is unknown (**u**ncertainty),
- a — prior belief in x being true (base rate).

It holds $b + d + u = 1$.

The subjective logic defines logical operations on binomial opinions like addition, multiplication, comultiplication, averaging fusion etc. If a state of each component of a complex system is described by its binomial opinion, these opinions can be composed by the logical operations mentioned above according to the logical composition of the system. In this way, the structure of the system can be described hierarchically and binomial opinion on the whole system state can be derived.

The binomial opinion ω can be mapped e. g. to parameters of beta distribution.

A relation between the signal variance and uncertainty of the respective sensor condition has been proposed in (Ettler and Dedecius, 2015). Increased uncertainty of modules' condition would negatively affect uncertainty of the whole plant ("false alarm"), although values of measured quantities are located in usual intervals and the signal variance without the jump is proper.

3 SENSOR MODEL AND ITS ESTIMATION

To avoid construction and identification of a complex generic model for particular type of sensor data (e. g. dynamic probabilistic mixture), the model is chosen simple with a mechanism to resits abrupt changes. The purpose of the model is estimation of the signal variance as an input quantity for evaluation of uncertainty of the sensor condition.

User given bounds on values of the data given by the sensor motivated us to choose a model with bounded noise, particularly uniformly distributed, which is the simplest case of bounded distributions.

3.1 Uniform Model of Sensor Signal

A sensor signal y_t is described by the following model ($t = 1, 2, \ldots, T$ is discrete time)

$$y_t = K + e_t \qquad (1)$$

where K is an unknown parameter and e_t is an uniformly distributed white noise e_t, i.e. $e_t \sim \mathcal{U}(-r, r)$; $r > 0$ is unknown. The equivalent description of y_t by probability density function (pdf) is

$$f(y_t) = \mathcal{U}(K - r, K + r) = \mathcal{U}(L, U), \qquad (2)$$

where $L = K - r$, $U = K + r$.

3.2 Bayesian Estimation

To estimate parameters K and r in (2), we use a Bayesian maximum a posteriori (MAP) estimation.

Parameters $\Theta = [K, r]'$ are estimated on a sliding window of the maximal length Δ. According to (Pavelková and Kárný, 2014), the MAP estimation converts to a problem of linear programming which has very simple form in the case of static model (1).

The statistics used for estimation are counter ν_t and data vector $w_t \equiv [y_t, y_{t-1}, \ldots, y_{t-\Delta_{Mt}}]$, $\Delta_{Mt} = \min(\Delta, \nu_{t-1})$ The statistics are updated

$$\nu_t = \nu_{t-1} + 1 \tag{3}$$

$$w'_t = [y_t, w'_{t-1}(1:\Delta_{Mt})] \tag{4}$$

$w'_{t-1}(1:\Delta_{Mt})$ denotes the vector created from the first Δ_{Mt} entries of w'_{t-1}. The estimation starts with $\nu_1 = 1$, $w_1 = y_1$.

Then, for $\tau^* = \{\tau; \tau = t - \Delta_{Mt}, \ldots, t-1, t\}$, MAP estimates are as follows

$$\hat{L}_t = \min_{\tau\in\tau^*}(w_\tau), \qquad \hat{U}_t = \max_{\tau\in\tau^*}(w_\tau),$$

$$\hat{K}_t = \frac{\hat{U}_t + \hat{L}_t}{2} \tag{5}$$

$$\hat{r}_t = \frac{\hat{U}_t - \hat{L}_t}{2} \tag{6}$$

$$\mathrm{var}(K)_t = \frac{(\hat{U}_t - \hat{L}_t)^2}{12} = \frac{\hat{r}_t^2}{3} \tag{7}$$

where $\max(w_\tau)$ and $\min(w_\tau)$ denotes the maximal and minimal entry of w_τ, respectively. $\hat{X}_t$ denotes the estimate of X in time t.

3.3 Considering Abrupt Signal Changes

When the estimation (5) – (7) is performed with the updates of statistics (3) and (4), then the sliding window length Δ_{Mt} continuously grows from 1 up to the maxima Δ after Δ steps. When the signal value abruptly changes, the variance of estimate rapidly increases. To prevent these rapid jumps, the estimation procedure is adapted as described below.

We describe variance increase between time instants $t-1$ and t by the ratio

$$R_{\mathrm{var}} = \frac{\mathrm{var}(K)_t}{\mathrm{var}(K)_{t-1}} \tag{8}$$

where $\mathrm{var}(K)_t$ means the current value of variance, $\mathrm{var}(K)_{t-1}$ is the value of variance in previous step. We define B as a limit for variance increase between time instants $t-1$ and t. The value $R_{\mathrm{var}} > B$ indicates undesirable variance change. If this case arises, then the statistics ν and w are reset, i.e. $\nu_t = 1$, $w_t = y_t$. The current estimation step is repeated with the revised statistics. Then, the estimation continues in a usual way.

4 EXPERIMENTS

Here, an example is given to illustrate the proposed method. The following real data from rolling mill are used.

- Hydraulic pressure upper front P_{u0} [Mpa]. This is a partial pressure composing the total pressure exerted on a metal strip. Technological (hard) bounds of the signal are $\underline{y}_H = -10$ MPa and $\overline{y}_H = 31$ MPa, expected (soft) range is between $\underline{y}_S = -2$ MPa and $\overline{y}_S = 28$ MPa.
- Slide valve actual position front $u_{ist\,0}$ [%]. The valve position determines the pressure change. Technological bounds of the signal are $\underline{y}_H = -101\,\%$ and $\overline{y}_H = 110\,\%$, expected range is between $\underline{y}_S = -100\,\%$ and $\overline{y}_S = 90\,\%$.

The data sets have the origin in the same functional unit of the rolling mill, although they describe different quantities. The abrupt changes in the data, therefore, are observed in the same time instants (this is not visible in the figures, where different data blocks are shown).

The memory, represented by a sliding window, was set to the length $\Delta = 25$ data vectors, which corresponds to the forgetting factor 0.96 in analogy of the exponential forgetting (Kárný et al., 2005). This value was chosen as a reasonable compromise between information in the past data and ability to track slow parameter changes (adaptivity) (Pavelková and Jirsa, 2014).

The limit for variance increase B was experimentally set a preliminary constant 2. Because of situations, when the model excitation is low (data are almost constant), the memory reset option is considered if $\mathrm{var}(K) > 0.2$ (7), which is another constant found experimentally.

The data were modelled by a static model (1). According to (5) and (7), the mean value $\hat{K}$ and variance $\mathrm{var}(K)$ of the absolute term were computed in each time step. If $R_{\mathrm{var}} = \mathrm{var}(K)_t / \mathrm{var}(K)_{t-1} > B$ (see (8)), i.e. the data values changed abruptly, the model memory was reset. For illustration, estimation results of the model without resetting are plotted, too. The figures show representative selections of the data to illustrate the effect of the method in typical situations.

The influence of memory resetting to estimation quality with data P_{u0} is shown in Figures 1 and 2. Figure 1 shows estimates of parameter mean value $\hat{K}$ with and without memory resetting, compared with the data.

Figure 2 shows estimates of parameter variance $\mathrm{var}(K)$ with and without memory resetting.

Influence of memory can be observed in case of abrupt change of a high magnitude. The data in the

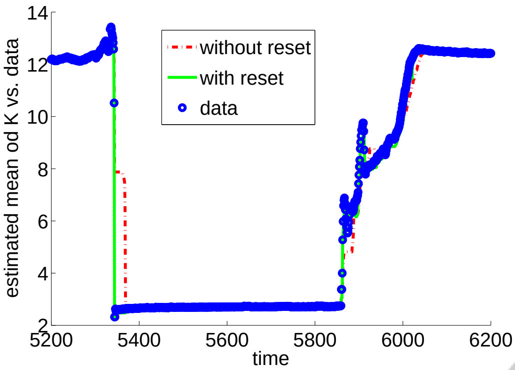

Figure 1: Hydraulic pressure upper front P_{u0}: comparison of estimated mean $\hat{K}$ for algorithms with memory resettting (solid line) and without memory resetting (dash-dot line). Data are shown as circles

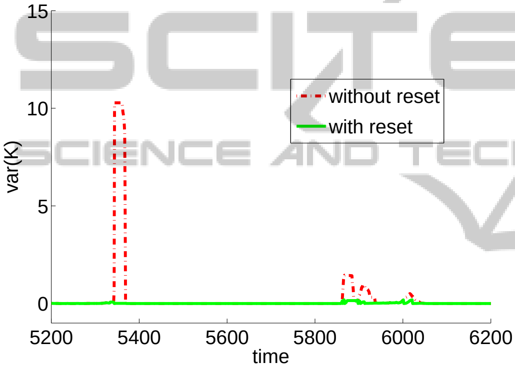

Figure 2: Hydraulic pressure upper front P_{u0}: comparison of estimated variance $\mathrm{var}(K)$ for algorithms with memory resettting (solid line) and without memory resetting (dash-dot line).

memory distort estimates of the first and second moments of the parameter K. The length of the memory, Δ, can be seen in the figures as a feature of the unreset estimates. Resetting the memory, after the abrupt change is detected, feeds the model with the data without influence of the history when the system was in a different "mode" with respect to the static model. Variance of the parameter is then not increased artificially although the system behaves properly and the values stay within the soft limits. A slight disagreement of the reset estimate and data about time 5 900, may be caused by very fast, almost chaotic changes in the signal.

The results for experiments with data $u_{ist\,0}$ are presented in Figures 3 and 4. Figure 3 shows estimates of parameter mean value $\hat{K}$ with and without memory resetting, compared with the data.

Figure 4 shows estimates of parameter variance $\mathrm{var}(K)$ with and without memory resetting.

The effect of memory resetting is even more significant than in Figures 1 and 2 and it gives better re-

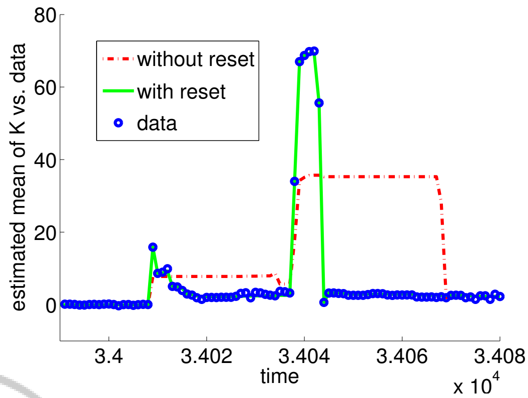

Figure 3: Slide valve actual position front $u_{ist\,0}$: comparison of estimated mean $\hat{K}$ for algorithms with memory resettting (solid line) and without memory resetting (dash-dot line). Data are shown as circles

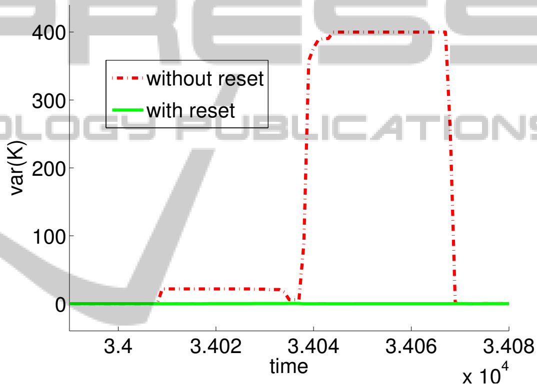

Figure 4: Slide valve actual position front $u_{ist\,0}$: comparison of estimated variance $\mathrm{var}(K)$ for algorithms with memory resettting (solid line) and without memory resetting (dash-dot line).

sults in estimation of both $\hat{K}$ and $\mathrm{var}(K)$, probably because of the subsequent abrupt changes within the interval Δ. Again, the estimates without memory resetting are influenced by the previous values up to Δ, which is also visible in Figures 3 and 4.

5 CONCLUSION

The paper proposes a simple, fast and efficient method for estimation of a signal variance, if the signal contains abrupt changes (jumps) in data. The method prevents the estimator from variance increase caused by the change. The variance increase would affect uncertainty of the binomial opinion ω.

A simple static model with a bounded uniform noise is identified. The problem of abrupt changes is dealt by detection of the parameter variance step by step and resetting the model memory, if the variance increase is higher than the requested bound.

The effectivity of the method is illustrated. Figures 3 and 4 demonstrate a case when estimated variance was unaffected by the abrupt change. If the change is faster, as in Figures 1 and 2 after time 5 900), variance increase is substantially reduced.

The future work can be focused on adaptive setting of B according to nature of the data (noise, oscillations, outliers, change of variance) and exploring other methodology of abrupt change detection, e. g., testing of hypotheses etc.

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