

# Optimization of Parallel-DEVS Simulations with Partitioning Techniques

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Abstract: With the emergence of parallel computational infrastructures at low cost, reducing simulation time becomes again an issue of the research community in modeling and simulation. This paper presents an approach to improve time of discrete event simulations. For that, the Parallel Discrete Event System formalism is coupled to a partitioning method in order to parallelize the graph of models. We will present the graph partitioning method to realize this cutting and quantify the resulting time savings of parallel implementation. This article highlights the importance of considering the dynamic of the model when partitioning to improve performances. Many tests are performed from graphs with different sizes and shapes on several hardware architectures.

## 1 INTRODUCTION

Modeling complex systems are becoming increasingly costly in time and memory capacity, it is necessary to develop efficient modeling and simulation tools to address them. DEVS formalism (Zeigler et al., 2000) and Parallel-DEVS variant (Chow, 1996) *Discrete Event Specification* is a good candidate to develop a response to both formal and technical. That is a discrete events modeling and simulation theory with a hierarchical approach. The global model, called *structure of the model* in DEVS terminology, is a graph of coupled models. We propose to work from this models graph to optimize the simulation.

The use of parallel and distributed infrastructure can make a efficient response of optimization problem. Our approach is to use a partitioning algorithm on the graph models in order to parallelize their execution as efficiently as possible.

In (Herbez et al., 2015), we presented this approach as well as the relative gains obtained for two types of partitioning. One is based on the connectivity of the graph, and the other is oriented modeler. In these examples, the gain obtained by the introduction of a good partitioning is about 20% compared to an initial model hierarchy.

The goal of this paper is to show how partitioning is used to optimize the Parallel-DEVS structure including through load balancing between threads and minimization of exchanges between them. We will also show the limitations of this approach, and pro-

pose ways to address them. To achieve this, tests are carried out for two types of graphs and multiple hardware architectures.

In the first part, we describe the Parallel-DEVS formalism and partitioning graph method used for our tests. Then, various tests on several hardware architectures will be presented by illustration of the results. The results will be analyzed to show that it is possible to evaluate retrospectively the parallel capabilities of the models. And finally, a discussion will attempt to suggest ways to improve the method.

## 2 FORMALISMS AND METHODS

In this section, the Parallel-DEVS formalism is formally presented and through the main algorithms for implementation of the formalism. Moreover, we present the graph partitioning method chose to optimize the Parallel-DEVS simulations.

### 2.1 Parallel-DEVS

DEVS (Discrete Event Specification) (Zeigler et al., 2000) is a high level formalism based on the discrete events for the modeling of complex discrete and continuous systems. The model is a network of interconnections between atomic and coupled models. These models are in interaction via time-stamped events exchanges.

More specifically, we present the Parallel-DEVS (PDEVs) formalism (Chow and Zeigler, 1994; Chow, 1996). This extension of the classic DEVS introduces the concept of simultaneity of events essentially by allowing bags of inputs to the external transition function. Bags can collect inputs that are built at the same date, and process their effects in future bags.

PDEVs defines an atomic model as a set of input and output ports and a set of state transition functions:

$$M = \langle X, Y, S, \delta_{int}, \delta_{ext}, \delta_{con}, \lambda, ta \rangle$$

With:  $X, Y, S$  are respectively the set of input values, output values and sequential states

$ta : S \rightarrow \mathbb{R}_0^+$  is the time advance function

$\delta_{int} : S \rightarrow S$  is the internal transition function

$\delta_{ext} : Q \times X^b \rightarrow S$  is the external transition function

where:

$$Q = \{(s, e) | s \in S, 0 \leq e \leq ta(s)\}$$

$Q$  is the set of total states,

$e$  is the time elapsed since last transition

$X^b$  is a set of bags over elements in  $X$

$\delta_{con} : S \times X^b \rightarrow S$  is the confluent transition function, subject to  $\delta_{con}(s, \emptyset) = \delta_{int}(s)$

$\lambda : S \rightarrow Y$  is the output function

If no external event occurs, the system will stay in state  $s$  for  $ta(s)$  time. When  $e = ta(s)$ , the system changes to the state  $\delta_{int}$ . If an external event, of value  $x$ , occurs when the system is in the state  $(s, e)$ , the system changes its state by calling  $\delta_{ext}(s, e, x)$ . If it occurs when  $e = ta(s)$ , the system changes its state by calling  $\delta_{con}(s, x)$ .

Every atomic model can be coupled with one or several other atomic models to build a coupled model. This operation can be repeated to form a hierarchy of coupled models. A coupled model is defined by:

$$N = \langle X, Y, D, \{M_d\}, \{I_d\}, \{Z_{i,d}\} \rangle$$

Where  $X$  and  $Y$  are input and output ports,  $D$  the set of models and:

$\forall d \in D, M_d$  is a PDEVs model

$\forall d \in D \cup \{N\}, I_d$  is the influencer set of  $d$  :

$I_d \subseteq D \cup \{N\}, d \notin I_d, \forall d \in D \cup \{N\},$

$\forall i \in I_d, Z_{i,d}$  is a function,

the  $i$ -to- $d$  output translation:

$Z_{i,d} : X \rightarrow X_d$ , if  $i = N$

$Z_{i,d} : Y_i \rightarrow Y$ , if  $d = N$

$Z_{i,d} : Y_i \rightarrow X_d$ , if  $i \neq N$  and  $d \neq N$

The influencer set of  $d$  is the set of models that interact with  $d$  and  $Z_{i,d}$  specifies the types of relations between models  $i$  and  $d$ .

PDEVs is an operational formalism. This means that the formalism is executable and thus it provides algorithms for its execution. These algorithms define the sequence of the different functions of the PDEVs structure. Moreover, the atomic and coupled models are respectively associated with simulators and coordinators. The aim of simulators is to compute the various functions while the coordinators manage the synchronization of exchanges between simulators (or coordinators in a hierarchical view).

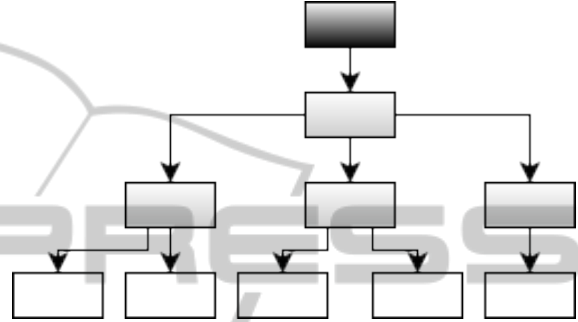


Figure 1: Hierarchy of coordinators and simulators. Black box is the root coordinator (manage simulation), Grey boxes are coordinators (simulate coupled models) and white boxes are simulators (simulate atomic models).

## 2.2 PDEVs Algorithms

In this article, we explain the PDEVs abstract simulators especially the algorithms of the coordinator which allows concurrent simulation between the components of the coordinator. Indeed, the Parallel-DEVS approach to parallelize simulation uses a risk-free and strict causality adherence. It uses a global minimum time synchronization and allows a concurrent and simultaneous output collection and distribution of events

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### Algorithm 1: Parallel-DEVS-Coordinator.

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- 1: **procedure** VARIABLES
  - 2:  $DEVS = (X, Y, D, \{M_d\}, \{i_d\}, \{Z_{i,d}\})$
  - 3:  $parent$ : parent coordinator
  - 4:  $tl, tn$
  - 5:  $event - list$ : list of elements  $(d, tn_d)$  sorted
  - 6:  $IMM$ : imminent children
  - 7:  $mail$ : output mail bag
  - 8:  $y_{parent}$
  - 9:  $\{y_d\}$
- 

In Figure 1, coordinator are represented by the grey boxes. Each coordinator manages a scheduler and route messages between children. The scheduler stores internal events (one internal event per child)

sorted by time to wake up models. These times are produced using the  $ta$  function for atomic models and the current date of the simulation. For each iteration, coordinator build a set of immediate message (all events with the same wake up time).

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**Algorithm 2:** Parallel-DEVS-Coordinator.
 

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```

1: procedure WHEN RECEIVE I-MESSAGE( $i, t$ ) AT
   TIME  $t$ 
2:   for each  $d \in D$  do
3:     send i-message to child  $d$  in parallel way
4:   sort event-list according to  $tn_d$ 
5:    $tl = \max\{tl_d | d \in D\}$ 
6:    $tn = \min\{tn_d | d \in D\}$ 
    
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**Algorithm 3:** Parallel-DEVS-Coordinator.
 

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1: procedure WHEN RECEIVE *-MESSAGE( $*, t$ )
2:   if  $t \neq tn$  then
3:     error: bad synchronization
4:    $IMM = \{d | (d, tn_d) \in (\text{event-list} \wedge tn_d = tn)\}$ 
5:   for each  $r \in IMM$  do
6:     send *-message ( $*, t$ ) to  $r$  in parallel way
    
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**Algorithm 4:** Parallel-DEVS-Coordinator.
 

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1: procedure WHEN RECEIVE X-MESSAGE( $x, t$ )
2:   if  $\neg(tl \leq t \leq tn)$  then
3:     error: bad synchronization
4:   receivers =  $\{r | r \in D, N \in I_r, Z_{N,r}(x) \neq \emptyset\}$ 
5:   for each  $r \in$  receivers do
6:     send x-message ( $Z_{N,r}(x), t$ ) to  $r$  in parallel
       way
7:   for each  $r \in IMM \wedge \neg \in$  receivers do
8:     send x-message ( $\emptyset, t$ ) to  $r$  in parallel way
9:   sort event-list according to  $tn_d$ 
10:   $tl = t$ 
11:   $tn = \min\{tn_d | d \in D\}$ 
    
```

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Messages received by a coordinator are described in the algorithms 1, 2, 3, 4 and 5. i-message is used to initialize children. \*-message is used to compute output of children. x-message and y-message are used to route messages. In PDEVS, all imminents ( $IMM$ ) are allowed to execute concurrently in contrast to DEVS where imminents were sequentially activated. The outputs of  $IMM$  are collected into a bag called the mail in previous algorithms. The mail is analyzed for the part going out because of the  $EOC$  and the parts to be distributed internally to the components due to the  $IC$  coupling. The internal transition functions of the imminents are not executed immediately

since they may also receive input at the same model time.

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**Algorithm 5:** Parallel-DEVS-Coordinator.
 

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1: procedure WHEN RECEIVE Y-MESSAGE( $y_s, t$ )
   WITH OUTPUT  $y_d$  FROM  $d$ 
2:   if this is not the last  $d$  in  $IMM$  then
3:     add  $(y_d, d)$  to mail
4:     mark  $d$  as reporting
5:   else
6:     if this is the last  $d \in IMM$  then
7:        $y_{parent} = \emptyset$ 
8:     for each  $d \in I_N \wedge d$  is reporting do
9:       if  $Z_{d,N}(y_d) \neq \emptyset$  then
10:        add  $y_d$  to  $y_{parent}$ 
11:    send y-message( $y_{parent}, t$ ) to parent
12:    for each child  $r$  with some  $d \in I_r \wedge d$  is reporting
        $\wedge Z_{d,r}(y_d) \neq \emptyset$  do
13:      for each  $d \in I_r \wedge d$  is reporting
          $\wedge Z_{d,r}(y_d) \neq \emptyset$  do
14:        add  $Z_{d,r}(y_d)$  to  $y_r$ 
15:        send x-message( $y_r, t$ ) to  $r$ 
16:      for each  $r \in IMM \wedge y_r = \emptyset$  do
17:        send x-message( $\emptyset, t$ ) to  $r$ 
18:    sort event-list according to  $tn_d$ 
19:     $tl = t$ 
20:     $tn = \min\{tn_d | d \in D\}$ 
    
```

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## 2.3 Graph Partitioning and Parallel Mode

Using graph partitioning to transform the model hierarchy in another in order to be optimized for parallel simulations, this work is possible thanks to the *closure under coupling* property of DEVS (Zeigler et al., 2000). This property formally describes the coupled model is equivalent to an atomic model. Thus an atomic model can be move into a new coupled model and all the hierarchy of coupled model can be merge into a unique coupled model.

The result of the coupled model hierarchy merge give an oriented graph. In mathematics, a graph is defined by  $G = (V, E)$  where  $V$  is the vertices set and  $E$  the edges set. For the simulation,  $V$  describe the atomic models and  $E$  the connection network between them. Moreover, a weight could be associated to each vertex and edge. For vertices, the weight quantify the execution time and for edges quantify the data proportion transmitted between models. Slower is a model, bigger is his weight.

The k-way graph partitioning allows to cut a graph  $G$  into k subgraphs  $\{G_1, G_2, \dots, G_k\}$ , while minimiz-

ing one or more criterion. They are represented by functions named “objective function”. This cutting provides  $k$  subsets of vertices  $P_k = \{V_1, V_2, \dots, V_k\} \subset V$  named partition. Each vertex of a part  $V_i$  is executed on the same simulation node or logical process (LP). For reduce the simulation time, it’s necessary equalize the execution time on each LP and minimize the events exchange between them. This can be reflected by the partition quality. To be good quality, a partition must respect some conditions: the parts weight must be similar and connections between parts must be minimal.

The objectives of our research are to reduce execution time for very large simulations (more than 20000 models). These simulations give very large model graphs. It is necessary to use partitioning graph methods efficient for this graph size. We use a multilevel scheme in order to solve the problem.

The following subsection present the objective functions used for partitioning in order to minimize the simulation time.

### 2.3.1 The Objective Function

The partition quality is given by the objective functions. Smaller is the result, better is the partition quality. They revolve around two concepts: cost cutting between partition parts and parts weight.

Given a partition  $P_k = \{V_1, V_2, \dots, V_k\}$ , the edge cut of two parts is the weight sum of edges connecting  $V_1$  and  $V_2$ :

$$\text{Cut}(V_1, V_2) = \sum_{v_1 \in V_1, v_2 \in V_2} \text{weight}(v_1, v_2) \quad (1)$$

For a partition  $P_k$ , the edge cut is the weight sum of edges connecting partition parts:

$$\text{Cut}(P_k) = \sum_{i < j} \text{Cut}(V_i, V_j) \quad (2)$$

This objective function was already used by Brian Kernighan and Shen Lin in (Kernighan and Lin, 1970).

Another function allows simultaneous management the minimization of the edge cut and weight balance between parts: the ratio cut:

$$\text{Ratio}(P_k) = \sum_{i=1}^k \frac{\text{Cut}(V_i, V - V_i)}{\text{weight}(V_i)} \quad (3)$$

It was introduced by Yen-Chuen Wei and Chung-Kuan Cheng in (Wei and Cheng, 1989). In our works, we seek to minimize this objective function.

### 2.3.2 The Multilevel Method

As introduce in (Herbez et al., 2015), we used a multilevel schemes to create quickly a graph partition of big size. It consists of three phases:

- Coarsening: Graph reduction by successive vertices matching, while keeping the nature of the original graph. Iterative process generating a graph base  $\{G_1, \dots, G_n\}$ , where  $G_1 = G$  the original graph and  $G_n$  the contracted graph. The Heavy Edge Matching introduced in (Karypis and Kumar, 1998) is implemented for this phase.
- Partitioning: Creating of a partition  $P_k$  of the coarsening graph  $G_n$  using a partitioning heuristic. We choose an expanding region method: the Greedy Graph Growing Partitioning (Karypis and Kumar, 1998).
- Uncoarsening: Projection of the partition  $P_k$  on each contraction graph levels  $G_i$  ( $i = n - 1, \dots, 1$ ). But after each projection it is necessary to realize a refinement for keep a good quality. We use a local optimization algorithm based on Kernighan-Lin algorithm (Kernighan and Lin, 1970)

For convenience, the multilevel implementation using GGGP as partitioning phase will be call GGGP.

## 3 DATA, SOFTWARE AND HARDWARE

This section presents the data on which the tests were conducted, as well as the different used hardware architectures.

### 3.1 Data Description

Tests were realized from two classical graph types inspired by the water flow model: a grid and a “tree” (abusively named). We have choose these names because they reflect the graph form, even if in the literature a “tree graph” is a hierarchical graph. It’ is not the case here. For each graph, the vertices weight is equal to 1 because the execution time of the models is the same. The edge weight is equal to 1 because the message transfer cost is the same between each model. We work on large simulations, where graphs have 20000 vertices. They are presented in Figure 2.

The left graph consists of several levels, where there are a single vertex source and outlet. The vertex source is the starting model of the simulation and the outlet is the ending model. Each vertex of level  $n$  is connected with two vertices of level  $n - 1$ . For the

penultimate level, vertices are connected only to the outlet.

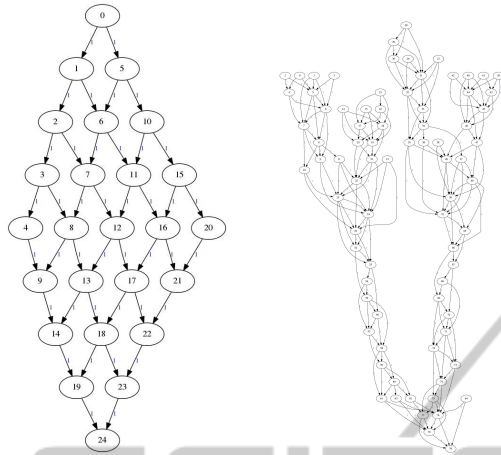


Figure 2: Little graphs size examples. On the left, a “grid graph” and on the right a “tree graph” (abusively named).

The right graph is composed of several branches, where each vertex is connected to one or more vertices following a single direction. The branches are branched until reaching the single outlet. This graph have several source vertices on each top branch ( $n$  sources by branches).

### 3.2 Software and Hardware Architectures

The tests were performed on a PDEVS simulation kernel written in C++11. This simulation kernel is part of the VLE project (*Virtual Laboratory Environment*) a modeling and simulation software suite (Quesnel et al., 2009). The VLE software suite is used in many projects in the French National Institute for Agronomical Research and several French Universities.

The simulations were done on an three different hardware architectures:

- Intel Core i5-2520M processor - 2.5 GHz: 2 cores with 4 threads (hyperthreading mode)
- Intel Core i7-3840QM processor - 2.8 GHz: 4 cores with 8 threads (hyperthreading mode)
- Samsung Exynos 5422 with a Cortex A15 2.1 Ghz quad core and a Cortex A7 1.5 GHz quad core processor: 4 big cores and 4 little cores

These four used architectures to test different algorithms on architectures with different possibilities in terms of cores. The smallest configuration allows to have two cores with a small speedup factor of four threads (2C + 2H). The second doubles possibilities

(4C + 4H). The third offers a hybrid solution (4C + 4LC) slightly greater than the second.

## 4 RESULTS AND DISCUSSION

In this section, we present simulation results on classical models which we employ in our scope (nitrogen and water management in catchment area). For that, PDEVS formalism and the partitioning algorithm, introduced in section 2, are used on several hardware architectures.

The results are then discussed to evaluated the performance and limitations of our approach. These tests are performed from the graphs presented in Section 3.1.

### 4.1 Results Analysis

The simulation results presented in this subsection are obtained for a computation time of about 1 ms by models. Our goal is to compare the performance obtained with those expected in theory.

In absolute terms, the expected performance for parallelization tend to have a speedup equal to the available thread number. It is named absolute theoretical speedup. However, given the dynamic of the models, it is very difficult to achieve this performance in pessimistic approach. In order to have an effective comparison basis, we propose to compute a theoretical speedup including the dynamics of the models.

The following subsection present this theoretical speedup and an illustration to explain its operation.

#### 4.1.1 Theoretical Speedup Definition

For a given transition, the theoretical speedup is defined by the ratio of the sum of active atomic models and the maximum number of active models in one of the coordinators. This is expressed mathematically by:

$$\text{Speedup} = \frac{\sum_{i=1}^k n_i}{\max_{i \in \{1, \dots, k\}} n_i} \quad (4)$$

where  $k$  is the coordinator number and  $n_i$  the active models number in the coordinator  $i$ .

The active models are the models included in the *IMM* set when the transition function is executed. The active models in a same coordinator form a bag. For a given date, the bags size varies according to the event propagation in the global model. If the bag size is the same then the theoretical speedup is equal to the number of active coordinators (since there is

one thread per coordinator and that, depending on the hardware architecture, all threads can be executed in parallel modulo the memory access). This concept is illustrated in Figure 3.

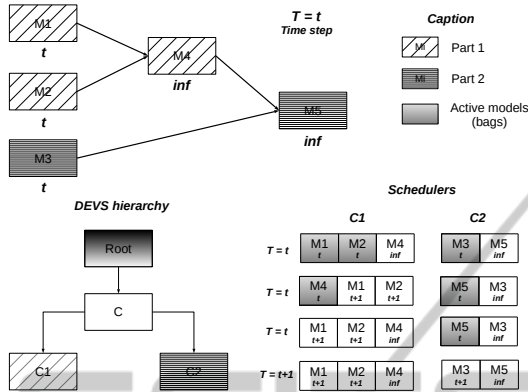


Figure 3: Illustration of the schedulers evolution for a time step on a small example.

This diagram shows different information from a simulation at time  $t$ . The sub-diagram at top show a graph models partitioned in two parts. His hierarchical representation is given at bottom on the left. Each coordinator has a scheduler. Their evolution, for each transition, is shown at the bottom right. Active models are represented by the shaded boxes. All active models of a same coordinator form a bag. For the first transition ( $M_1, M_2$ ) and ( $M_3$ ) form two bags, so the speedup of this transition is 1.5. The speedup is computed for each transition. Here, there are 3 transitions with a respective speedup 1.5, 2 and 1.

For a give date  $t$ ,  $n_t$ -speedup are computed ( $n_t$  is the transition number). Figures 6 et 7 show this variation at the date  $t = 0$ . The theoretical speedup of a date  $t$  is the mean of speedups at each transition:

$$\text{Speedup}(t) = \text{mean}(\text{Speedup}) \quad (5)$$

And the speedup of the simulation is:

$$\text{Theoretical\_Speedup} = \text{mean}(\text{Speedup}(t)) \quad (6)$$

This theoretical speedup is closely linked to the hierarchy of coordinators / simulators. It is important to create balanced sub-models. But this is not enough, it is necessary to have a balance between bags at each transition to ensure a perfect balance. In our case, all models have the same charge in terms of calculation, that's why we talk about model number and not model weight.

#### 4.1.2 Influence of the Hardware Architecture on Speedup

Figures 4 and 5 compare the evolution of the speedup for different hardware architectures to the theoretical

speedup. For that, we vary the number of threads (partition number) and we observe the impact on the evolution of the speedup.

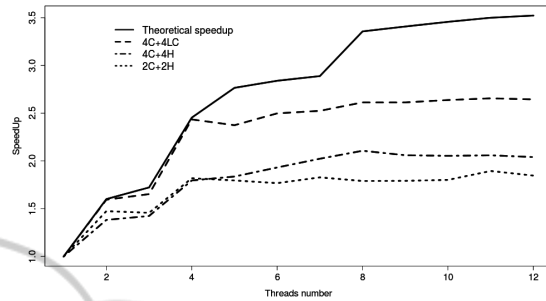


Figure 4: Speedup for tree graphs with 3 hardware architectures and theoretical speedup.

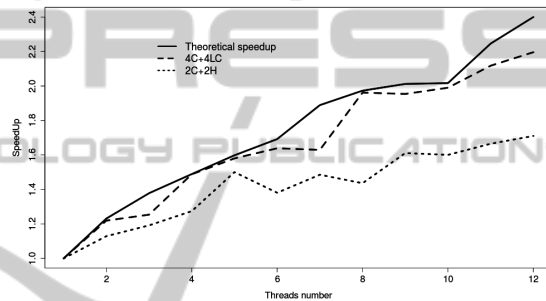


Figure 5: Speedup for grid graphs with 2 hardware architectures and theoretical speedup.

These curves show the influence of the hardware architecture on the speedup. Indeed, until the number of threads is less than or equal to the number of cores, the speedup is very close to the theoretical value. For architecture 4C + 4LC, a slight inflection is observed with 8 threads because the 4 additional cores are less efficient than the first 4 cores.

#### 4.1.3 Link between Theoretical Speedup and Partitioning Quality

The results presented in this subsection are obtained from graph of size 1000 and a hierarchical structure with four sub-models. The theoretical speedup can be a partition quality indicator, as shows this subsection.

In the Figure 6, the transition function is computed 63 times where 1/3 of them the number of active sub-models is equal to 1. So the parallelization is not used during this phase. By against, the remaining two thirds show a efficiency close to the optimum (4). Given the graphs structure, it is hard to beat.

For the grid (Figure 7), the conclusion is not the same. We are far to the absolute theoretical speedup with 4 threads (max = 2.2). This is explained by the dynamics of the grid model: the events propagate by

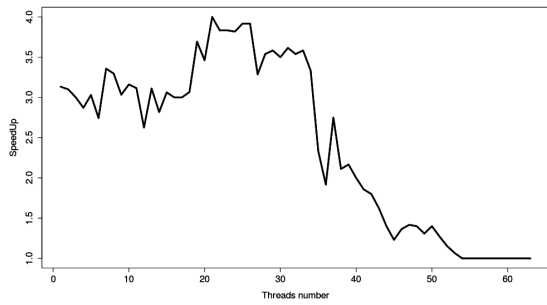


Figure 6: Variation of theoretical speedup at  $t = 0$  for "tree".

wave from the top left corner to the bottom right corner. For the partitioning, the grid is divided in 4 almost regular sub-grids. The number of models computed in parallel can be at most 2 (or 3 in some limit cases).

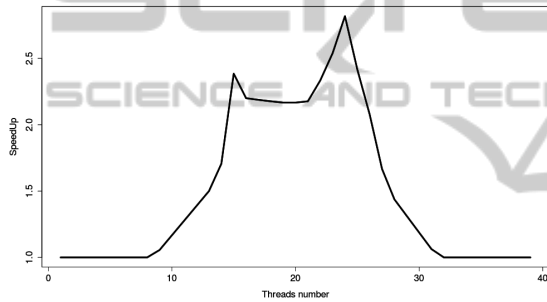


Figure 7: Variation of theoretical speedup at  $t = 0$  for "grid".

Figure 7 suggests that partitioning is not optimal in this case. It does not take sufficient account of the dynamics of the model. However, the particular structure of this graph does that the theoretical speedup can not always be equal to the absolute because the number of parallelizable models is lower than the number of threads available at times. This is particularly the case in the beginning and end of the simulation.

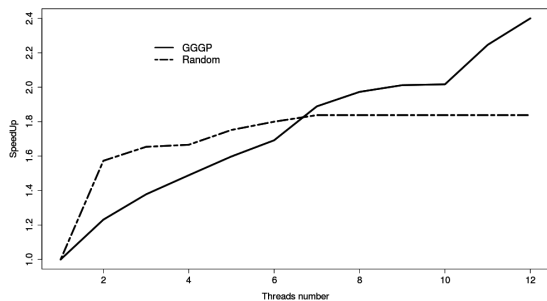


Figure 8: Speedup of gggp and random method on "grid".

To be convinced of the phenomenon, we generate a random cutting and compare the theoretical speedup

that obtained with our partitioning method. Figure 8 shows that the random cutting has a greater theoretical speedup for a parts number less than 8.

## 4.2 Results Discussion

The results show that in one case (tree), our partitioning method leads to the construction of a hierarchy similar to the optimum of the theoretical speedup perspective. The second case (grid), the results are not up to par. In fact, there are more suitable cuts which follow the dynamics of the model. The optimal cutting is computable and depends on the graph structure and of the dynamics of the model. For each bag, dividing the cardinality of the *IMM* set by the number of coordinators, we take the integer part. If the modulus of the two terms is not zero then added one (see 7). We then obtain the optimal size of bags that are processed by each coordinator. The average is then carried out on all the transitions ( $2N - 1$  transitions for a grid size  $N$ ) for a time step. We obtain then the theoretical speedup in optimum for the grid. The equation of optimal speedup in this case is:

$$S\_bag(k) = \lfloor \frac{k}{P} \rfloor + \langle k \bmod P \rangle \quad (7)$$

$$Speedup = \text{mean}(2 \sum_{i=1}^{N-1} S\_bag(i) + S\_bag(N)) \quad (8)$$

We can then compare the theoretical speedup of our partitioning compared to the theoretical speedup of the best partitioning (see Figure 9).

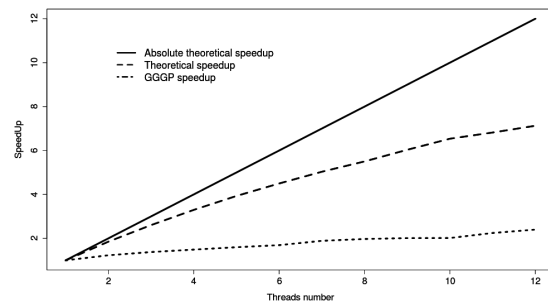


Figure 9: Speedup comparison to the absolute for a "grid".

The objective of partitioning is to obtain a speedup equal to the number of cores provided on the machine during almost all of the simulation. To be a good quality, the partition shall enable at the results to be close to this speedup. Which is not actually our case here. The partitioning method must not only balance the loads among threads, it must also take into account the dynamics of the graph. The knowledge of the dynamics minimizes the load difference between

the bags at each transition. This allows better management of threads throughout the simulation.

If the models were fully synchronous as in the case of a cellular automaton, the issue of balancing would be easy to solve if all models have the same computational load. In this case, we observe no change in the number of transitions to be made between two time steps. The partitioning becomes useless. In contrast, if the models are completely asynchronous, the *IMM* sets have a single model. Parallelization is completely useless in a pessimistic context. In this case, it was essential to work on algorithms optimistic parallel simulation.

## 5 CONCLUSION AND PROSPECTS

In this paper, we have shown that in some cases, we improve the simulation time by using a partitioning based solely on the model structure. These simulations are performed using an implementation of PDEVs algorithms in a risk-free mode. However, we have shown that it is also necessary to consider the dynamic of models for have a better models balance.

We have shown that the measure of the theoretical speedup (see equation 4) based on the *IMM* set gives us accurate information. We can generalize this measure so that it becomes an parallelization ability indicator of a model. This indicator can vary from 1 to  $P$  where  $P$  is the parts number of the graph. The minimum value is obtained for fully asynchronous model and the maximum value for fully synchronous model.

In our case, the indicator takes values close to the maximum value. This means that a coupling between a partitioning method and risk-free simulation is an excellent approach. However, it is necessary to go further if the indicator is close to 1. May be introduce a conservative or optimistic simulation engine coupled with partitioning methods. The model structure must be consider, but also his dynamics and the conservative algorithms with look-head properties (Chandy and Misra, 1979; Chandy and Misra, 1981) or optimistic (Time-Wrap (Jefferson, 1985), for example). Look-head is the ability of a model to predict that it will not have output for a certain period in future. The complexity of the optimization algorithm will be increased. It will be necessary to understand the interactions between look-head, for example, dynamics and the models graph.

Furthermore, our strategy of optimized hierarchy building has the overall objective to integrate distributed hardware architecture where the communication time between processes are not negligible.

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