LQG/LTR Versus Smith Predictor Control for Discrete-time Systems with Delay

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Abstract: A simple LQG control with no control cost is considered for discrete-time systems with input delay. In such case the loop transfer recovery (LTR) effect can be obtained especially for minimum-phase systems. The robustness of this control is analyzed and compared with state prediction control whose robustness stability is formulated via LMI. The robustness with respect to uncertain time-delay is considered including the control systems with Smith predictor-based controllers. Computer simulations of a second-order stable, unstable and nonminimum-phase systems with time-delay are given to illustrate the robustness and performance of the considered controllers.

1 INTRODUCTION

The LQG/LTR control for discrete-time systems is a well known problem investigated for example in (Tadjine et al., 1994) where the general design aspects of loop transfer recovery (LTR) both at the input and at the output of the system are presented. In (Maciejowski, 1985) the asymptotic case of LQG control, i.e. when the control weighting factor tends to zero is considered for both prediction and filtering type of controller. The case of nonminimum-phase (nmph) system is also discussed. Robust LQG/LTR control of discrete-time systems with time-delay at the input (or computation delay) is a specific problem within a general LQG/LTR framework. In this context some results are given in the literature like: (Kinnaert, 1990), (Kinnaert and Peng, 1990), (Zhang and Freudenberg, 1993). In (Kinnaert, 1990) the LQG/LTR problem with respect to the system input is solved for the square minimum-phase (mph) system with d-sample delays. The generalization of results in (Kinnaert, 1990) are given in (Kinnaert and Peng, 1990) where the recovery at both system input and system output is investigated and the corresponding recovered loop transfer matrices are derived. Further extension of these results can be found in (Zhang and Freudenberg, 1993) where LQG/LTR problem was solved for nmph systems with time-delays and explicit expressions of sensitivity and loop matrices are derived for the asymptotic behaviour of control system.

In this paper,the discrete-time Kalman filter based LQG control with no control cost for input-delayed systems with application of LTR technique is considered. The resulting robustness with respect to uncertain delay for mph and nmph systems is analyzed and compared to prediction based control (Gonzales et al., 2012). Additionally, the Smith predictor-like controls and their robustness properties to time-delay uncertainty are analyzed by simulations of second-order systems.

2 LQG/LTR FOR DISCRETE-TIME SYSTEMS WITH DELAY

The state-space discrete-time SISO system is given by

$$\underline{x}_{t+1} = F\underline{x}_t + \underline{g}u_{t-d} + \underline{w}_t \quad (1)$$

$$y_t = \underline{h}^T\underline{x}_t + v_t \quad (2)$$

where $\{\underline{w}_t\}$ and $\{v_t\}$ are sequences of independent random vector and scalar variables with zero mean and covariances $E\underline{w}_t\underline{w}_s^T = \Sigma_w\delta_{t,s}$, $Ev_tv_s = \sigma_v^2\delta_{t,s}$, and d is a delay given as multiplicity of sampling period. The system (1), (2) can be transformed to

$$\underline{x}_{t+1}^p = F\underline{x}_t^p + \underline{g}u_t + \underline{w}_t^p \quad (3)$$

$$y_t = \underline{h}^T\underline{x}_{t-d}^p + v_t \quad (4)$$

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where $\underline{x}_t^p = \underline{x}_{t+d}$ and the Kalman filter estimate of $\underline{x}_t^p$ is given by

$$\hat{\underline{x}}_{t/t}^p = F^p[\hat{\underline{x}}_{t/t}^T, u_{t-d}, \cdots, u_{t-1}]^T \tag{5}$$

where $F^p = [F^d, F^{d-1}\underline{g}, F^{d-2}\underline{g}, \cdots, F\underline{g}, \underline{g}]$ and the filtered estimate $\hat{\underline{x}}_{t/t}$ in terms of prediction $\hat{\underline{x}}_{t/t-1}$ follows from

$$\hat{\underline{x}}_{t/t} = \hat{\underline{x}}_{t/t-1} + \underline{k}_f \tilde{y}_t^p \tag{6}$$

where $\tilde{y}_t^p = y_t - \underline{h}\hat{\underline{x}}_{t/t-1}$ is an innovation of output. The Kalman predictor for $\underline{x}_{t+1}$ in steady-state is given by

$$\hat{\underline{x}}_{t+1/t} = F\hat{\underline{x}}_{t/t-1} + \underline{g}\underline{u}_{t-d} + \underline{k}_p \tilde{y}_t^p \tag{7}$$

and its gain is

$$\underline{k}_p = FP_f\underline{h}[\underline{h}^T P_f \underline{h} + \sigma_v^2]^{-1} \tag{8}$$

where P_f is the solution of Riccati equation

$$P_f = FP_fF^T + \Sigma_w - FP_f\underline{h}[\underline{h}^T P_f \underline{h} + \sigma_v^2]^{-1}\underline{h}^T P_f F^T \tag{9}$$

The filter gain is

$$\underline{k}_f = P_f\underline{h}[\underline{h}^T P_f \underline{h} + \sigma_v^2]^{-1}, \tag{10}$$

so $\underline{k}_p = F\underline{k}_f$ in view of (8) and (10). Finally, combining (6) and (7) one gets

$$\hat{\underline{x}}_{t/t-1} = F\hat{\underline{x}}_{t-1/t-1} + \underline{g}u_{t-d-1} \tag{11}$$

The LQG control law

$$u_t = \underline{k}_c^T \hat{\underline{x}}_{t/t}^p \tag{12}$$

aims to minimize the cost function

$$J = E\sum_{t=0}^{\infty} y_t^2, \tag{13}$$

so the gain $\underline{k}_c$ is

$$\underline{k}_c^T = -[\underline{g}^T P_c \underline{g}]^{-1}\underline{g}^T P_c F \tag{14}$$

and P_c is the solution of Riccati equation

$$P_c = F^T P_c F - F^T P_c \underline{g}[\underline{g}^T P_c \underline{g}]^{-1}\underline{g}^T P_c F + Q \tag{15}$$

When the weighting matrix Q is $Q = \underline{h}\underline{h}^T$ and assuming that the system (1), (2) is stabilizable, detectable, mph and $d = 0$ in (1) then it can be shown (Tadjine et al., 1994), (Maciejowski, 1985) that $\underline{k}_c$ takes very simple form

$$\underline{k}_c^T = -(\underline{h}^T\underline{g})^{-1}\underline{h}^T F. \tag{16}$$

under the condition that $\underline{h}^T\underline{g} \neq 0$ which implies that system has a natural one-step delay in control channel.
If $G(z) = \underline{h}^T(zI - F)^{-1}\underline{g}$ is mph and $\underline{k}_c$ takes a form (16) then the transfer function $G_f(z)$ of compensator defined by (6) and (12) can be manipulated into the form

$$\begin{aligned} G_f(z) &= -z\underline{k}_c^T[zI - (I - \underline{k}_f\underline{h}^T)(F - \underline{g}\underline{k}_c^T)]^{-1}\underline{k}_f = \\ &= -z\underline{k}_c^T[zI - F + \underline{g}\underline{k}_c^T]^{-1}\underline{k}_f, \end{aligned} \tag{17}$$

and the perfect recovery takes place, that is

$$\Delta(z) = \Phi(z) - G(z)G_f(z) = 0, \tag{18}$$

where the filter's open-loop return ratio $\Phi(z)$ is

$$\Phi(z) = \underline{h}^T(zI - F)^{-1}\underline{k}_p. \tag{19}$$

When $G(z)$ is nmph then the perfect recovery is in general not possible (this will be commented later on). Similarly, it is interesting to see what happens when the LTR procedure is applied for system (1), (2) with time-delay.
Time-delay in control channel of the system (1), (2) can alternatively be characterized by taking $d = 0$ in (1) and assuming that delay is incorporated in the system $(F, \underline{g}, \underline{h})$ with the Markov parameters fulfilling the following properties

$$\underline{h}^T\underline{g} = \underline{h}^T F\underline{g} = \cdots = \underline{h}^T F^{r-2}\underline{g} = 0, \quad \underline{h}^T F^{r-1}\underline{g} \neq 0 \tag{20}$$

for $r \geq 1$. It is known that the smallest integer r satisfying the above properties is the *relative degree* of the system. It is worthy noting that for relative degree r and time-delay d in (1) it holds $r = d + 1$.
In (Zhang and Freudenberg, 1993), (Kinnaert and Peng, 1990) it was shown that for mph systems the error function $\Delta(z)$ for

$$\underline{k}_c^T = -(\underline{h}^T F^{r-1}\underline{g})^{-1}\underline{h}^T F^r. \tag{21}$$

has a form

$$\Delta(z) = \underline{h}^T(I - z^{-(r-1)}F^{r-1})(zI - F)^{-1}\underline{k}_p \tag{22}$$

for $r \geq 1$. In general $\Delta(z) \neq 0$, so the perfect recovery cannot be obtained except the case $r = 1$ that corresponds to (18).

2.1 Comments on nmph Systems

As already mentioned LTR for nmph systems is recommended because the partial recovery could be achieved (Zhang and Freudenberg, 1993). The result for mph systems can be modified for the nmph systems after the proper factorization of $\Phi(z)$ (Zhang and Freudenberg, 1993). For every nmph system the all-pass factorization is possible

$$\begin{aligned} G(z) &= \underline{h}^T(zI - F)^{-1}\underline{g} = G_a(z)G_m(z) = \\ &= G_a(z)\underline{h}_m^T(zI - F)^{-1}\underline{g} \end{aligned} \tag{23}$$

where $G_a(z)$ is all-pass and $G_m(z)$ is mph stable transfer function. Partial recovery ($\Delta(z) \neq 0$) for time-delayed system is then possible with LQG control gain

$$\underline{k}_c^T = -(\underline{h}_m^T F^{r-1} \underline{g})^{-1} \underline{h}_m^T F^r. \tag{24}$$

where $\underline{h}_m$ can be easily obtained as a function of system parameters.
The recovery error is now

$$\Delta(z) = (\underline{h}^T - z^{-(r-1)} G_a(z) \underline{h}_m^T F^{r-1})(zI - F)^{-1} \underline{k}_p. \tag{25}$$

It is worth noting, as shown in (Zhang and Freudenberg, 1993), that full recovery is possible in the sense of loop transfer function $\Phi(z)$ if the following conditions are fulfilled

- $\Phi(z) = G_a(z) \underline{h}_m^T (zI - F)^{-1} F \underline{k}_f$,
- $\underline{h}^T F \underline{k}_f = \underline{h}^T F^2 \underline{k}_f = \cdots = \underline{h}^T F^r \underline{k}_f = 0$.

This means that the observer loop has the same nmph structure and at least as many delay steps as the system.

2.2 LMI Approach

In (Gonzales et al., 2012) an LMI condition for robust stability of noise-free system (1) with unknown time-delay belonging to known interval, i.e. $d_l \leq d \leq d_u$ is given. The system is under the state feedback prediction based controller $u_t = \underline{k}_c^T \hat{\underline{x}}_{t+h/t}$ with a given gain $\underline{k}_c$ and a given prediction horizon h. This approach is adopted for our comparison study where $h = d$ and $\hat{\underline{x}}_{t+d/t}$ can be obtained e.g. from (5), (6), (7) neglecting the noise terms. Then the following corollary follows: the global closed-loop stability result given in (Gonzales et al., 2012) reads: for any $\underline{k}_c^T$ such that $F + \underline{g}\underline{k}_c^T$ is Hurwitz and for $d_l = d_u = h = d$ there exists a feasible solution, i.e. there exist matrices $P, L, Q, Q_m, Q_M, Q_d, Z, Z_1, Z_2, M > 0$ that satisfy

$$\begin{bmatrix} \Gamma & 0 \\ 0 & -Z_M \end{bmatrix} < 0 \tag{26}$$

where

$$\Gamma = \begin{bmatrix} \Gamma_1 & 0 & 0 & 0 & \Gamma_3 & \Gamma_4 & \Gamma_5 \\ * & \Gamma_6 & 0 & 0 & 0 & \Gamma_9 & \Gamma_{10} \\ * & * & \Gamma_{11} & 0 & Z_1 & 0 & 0 \\ * & * & * & \Gamma_{12} & 0 & 0 & 0 \\ * & * & * & * & \Gamma_{13} & -\Gamma_9 & -\Gamma_{10} \\ * & * & * & * & * & -L & 0 \\ * & * & * & * & * & * & -M \end{bmatrix}.$$

and $\Gamma_1 = -P + Q + Q_m + Q_M + Q_d - Z_2, \Gamma_3 = Z_2, \Gamma_4 = A_1^T, \Gamma_5 = (A_1 - I)^T, \Gamma_6 = -Q, \Gamma_9 = B_1^T, \Gamma_{10} = B_1^T, \Gamma_{11} = -Q_m, \Gamma_{12} = -Q_M, \Gamma_{13} = -Q_d - Z_1 - Z_2, Z_M = Z - d^2 Z_1, A_1 = F + \underline{g}\underline{k}_c^T, B_1 = F^d \underline{g}\underline{k}_c^T, PL = I, ZM = I$, with $Q = \varepsilon_1 I, Q_m = \varepsilon_1 I, Q_M = \varepsilon_1 I, Q_d = \varepsilon_1 I, Z = \varepsilon_1 I, M = \varepsilon_1^{-1} I, Z_1 = \varepsilon_2 I, Z_2 = \varepsilon_2 I$ for some positive small enough scalars $\varepsilon_1, \varepsilon_2$.
It is interesting to note that the stabilizable and detectable system with arbitrarily large delay in the control input can be asymptotically stabilized by either linear state or output feedback as long as the open loop system is not asymptotically unstable (Zongli, 2007). The additive uncertain system with input time-delay and possible unstable poles was considered in (A. Kodjina and Ishijima, 1994), where it was shown that achievable robustness margin decreases to zero as the time-delay value increases. Problem of time-delay compensation for nonlinear systems was tackled in (Kravaris and Wright, 1989) using Smith Predictor-based controllers.

3 SMITH-PREDICTOR APPROACH

Among the variety of Smith Predictor controllers, a PID Smith Predictor (PIDSP) controller (Bobal et al., 2011) was derived so that the desired closed-loop transfer function is $\frac{1-e^{-\alpha}}{1-z^{-1}}$ where $\alpha = \frac{T_s}{T_m}$ and T_m is desired time constant of the first-order closed-loop response. For a second-order system the controller has a form

$$u_t = q_0 \varepsilon_t + q_1 \varepsilon_{t-1} + q_2 \varepsilon_{t-2} + u_{t-1} \tag{27}$$

where $q_0 = \gamma$, $q_1 = a_1 \gamma$, $q_2 = a_2 \gamma$, $\gamma = (1 - e^{-\alpha})/(b_1 + b_2)$. The error is $\varepsilon_t = r_t - \hat{y}_{p,t}$ where r_t is the reference signal and the signal $\hat{y}_{p,t}$ is calculated as $\hat{y}_{p,t} = \hat{e}_{p,t} + \hat{y}_{m,t}$ with $\hat{e}_{p,t} = y_t - \hat{y}_t$, $\hat{y}_t = G_d \hat{y}_{m,t}$, $y_t = G_p u_t$, $\hat{y}_{m,t} = G_m u_t$, and finally the PID controller (27) is described by

$$u_t = G_c(z^{-1}) \varepsilon_t.$$

This gives the output-reference closed-loop transfer function

$$G_{cl}(z^{-1}) = \frac{G_p(z^{-1}) G_c(z^{-1})}{1 + G_p(z^{-1}) G_c(z^{-1}) + G_m(z^{-1}) G_c(z^{-1})(1 - G_d(z^{-1}))} \tag{28}$$

that in case of perfect matching, i.e. $G_m(z^{-1}) G_d(z^{-1}) = G_p(z^{-1})$ ($d = d_m$) yields

$$G_{cl}(z^{-1}) = \frac{G_p(z^{-1}) G_c(z^{-1})}{1 + G_m(z^{-1}) G_c(z^{-1})} \tag{29}$$

For the second-order model

$$G_p(z^{-1}) = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} z^{-d}, \tag{30}$$

considered below in the simulations, the specific transfer functions are

$$G_m(z^{-1}) = \frac{z^{-1}(b_1+b_2)}{A(z^{-1})}$$

and

$$G_d(z^{-1}) = \frac{z^{-d_m}B(z^{-1})}{z^{-1}(b_1+b_2)}.$$

It is easy to check from (29) that for a second-order model (30) in steady state one obtains $G_{cl}(1)=1$, i.e. perfect tracking for perfect matching.

Consider now the Smith predictor idea presented in (Kravaris and Wright, 1989) for continuous-time system and apply it to discrete-time state-space framework. Then for the noise-free system (1), (2) the control law is

$$u_t = \frac{det(zI-F)}{det(zI-F)+\underline{k}_c^T Adj(zI-F)\underline{g}(1-z^{-d_m})}(v_t+\underline{k}_c^T\underline{x}_t), \tag{31}$$

where v_t is a command signal, d_m is the time-delay in the model and the state $\underline{x}_t=(zI-F)^{-1}\underline{g}z^{-d}u_t$. The closed-loop discrete-time transfer function from y_t to v_t takes a form

$$G_{cl}(z^{-1}) = \frac{\underline{h}^T Adj(zI-F)\underline{g}}{det(zI-F)-\underline{k}_c Adj(zI-F)\underline{g}(1+z^{-d}-z^{-d_m})}z^{-d} \tag{32}$$

With this form it is possible to select closed-loop poles for the delayed system according to the pole-placement method. The feedback gain $\underline{k}_c$ calculated using (14) can also be applied. To obtain the asymptotic tracking accuracy defined by the error $\varepsilon_t = r_t - y_t$ the feedforward gain k_r is introduced, i.e. $v_t = k_r r_t$ where $k_r = G_{cl}(1)^{-1}$.

Finally, the error feedback controller described for example in (Soroush and Kravaris, 1992) is considered. When the condition (20) is fulfilled then it holds

$$\begin{aligned} G(z^{-1}) &= z^{-r}[\underline{h}^T F^r(zI-F)^{-1}\underline{g}+\underline{h}^T F^{r-1}\underline{g}] = \\ &= z^{-r}G_r(z^{-1}). \end{aligned} \tag{33}$$

Suppose the required closed-loop response is of the simple first-order with time-delay

$$G_{cl}(z^{-1}) = \frac{1-\alpha}{1-\alpha z^{-1}}z^{-r}. \tag{34}$$

then the controller has the following transfer function from ε to u_t

$$G_c(z^{-1}) = \frac{1-\alpha}{1-\alpha z^{-1}-(1-\alpha)z^{-r}}\frac{1}{G_r(z^{-1})} \tag{35}$$

where $0<\alpha<1$ and the error is $\varepsilon_t = r_t - y_t$. When the time-delay mismatch occurs the relative degree r_m in the model should be used in (35), noting that $r_m = d_m + 1$. The corresponding closed-loop transfer function is then

$$G_{cl}(z^{-1}) = \frac{(1-\alpha)G_r(z^{-1})}{z^{r-1}(z-a)G_{r_m}(z^{-1})+(1-\alpha)z^{r-r_m}(G_{r_m}(z^{-1})-G_r(z^{-1}))} \tag{36}$$

Obviously, for perfect matching we get (34).

4 LTR FOR ARMAX MODEL

The ARMAX model is given by

$$y_t = G(z^{-1})u_{t-d} + G_e(z^{-1})e_t \tag{37}$$

where $G(z^{-1}) = \frac{B(z^{-1})}{A(z^{-1})}$, $G_e(z^{-1}) = \frac{C(z^{-1})}{A(z^{-1})}$, and at the same time $G(z) = \underline{h}^T(zI-F)^{-1}\underline{g}$, $G_e(z) = \underline{h}^T(zI-F)^{-1}\underline{k}_e + 1$ with $A(z^{-1}), B(z^{-1})$ and $C(z^{-1})$ polynomials in the operator z^{-1}, i.e. $A(z^{-1}) = 1 + a_1z^{-1} + \ldots + a_nz^{-n}, B(z^{-1}) = b_1z^{-1} + \ldots + b_nz^{-n}, C(z^{-1}) = 1 + c_1z^{-1} + \ldots + c_nz^{-n}$ and $\{e_t\}$ assumed to be a sequence of independent variables with zero mean and variance σ_e^2.

ARMAX model (37) has an equivalent innovation state-space representation

$$\underline{x}_{t+1} = F\underline{x}_t + \underline{g}u_{t-d} + \underline{k}_e e_t \tag{38}$$

$$y_t = \underline{h}^T\underline{x}_t + e_t \tag{39}$$

where $\underline{g} = (b_1,\ldots,b_n)^T$, $\underline{k}_e = (c_1-a_1,\ldots,c_n-a_n)^T$, $\underline{h}^T = (1,0,\ldots,0)$

$$F = \begin{bmatrix} -a_1 & 1 & \ldots & 0 \\ . & . & \ldots & 0 \\ -a_{n-1} & . & \ldots & 1 \\ -a_n & . & \ldots & 0 \end{bmatrix}.$$

Equations (38), (39) can take the following representation

$$\underline{x}_{t+1} = F^*\underline{x}_t + \underline{g}u_{t-d} + \underline{k}_e y_t \tag{40}$$

$$y_t = \underline{h}^T\underline{x}_t + e_t, \tag{41}$$

where $F^* = F - \underline{k}_e\underline{h}^T$. Kalman predictor associated with eq.(40) is

$$\hat{\underline{x}}_{t+1/t} = F^*\hat{\underline{x}}_{t/t} + \underline{g}u_{t-d} + \underline{k}_e y_t \tag{42}$$

and Kalman filter is given by

$$\hat{\underline{x}}_{t/t} = \hat{\underline{x}}_{t/t-1} + \underline{k}_f(y_t - \underline{h}^T\hat{\underline{x}}_{t/t-1}), \tag{43}$$

with filter gain

$$\underline{k}_f = P_f\underline{h}[\underline{h}^T P_f\underline{h} + \sigma_e^2]^{-1} \tag{44}$$

where P_f is the solution of Riccati equation

$$P_f = F^* P_f F^{*T} - F^* P_f \underline{h}\underline{h}^T P_f F^{*T} (\underline{h}^T P_f \underline{h} + \sigma_e^2)^{-1}. \quad (45)$$

The predictor equation derived from (42) is

$$\hat{\underline{x}}_{t+1/t} = F\hat{\underline{x}}_{t/t-1} + \underline{g}u_{t-d} + \underline{k}_p(y_t - \hat{\underline{h}}^T \hat{\underline{x}}_{t/t-1}), \quad (46)$$

where the predictor gain is now $\underline{k}_p = \underline{k}_p^* + \underline{k}_e$ and $\underline{k}_p^* = F^* \underline{k}_f$.

However, in the considered steady state case, the solution of (45) is $P_f = 0$ and consequently $\underline{k}_f = \underline{k}_p^* = 0$ and $\hat{\underline{x}}_{t/t} = \hat{\underline{x}}_{t/t-1} = \hat{\underline{x}}_t$. From (46) or directly from (42) the Kalman filter equation takes then a simple form

$$\hat{\underline{x}}_{t+1} = F^* \hat{\underline{x}}_t + \underline{g}u_{t-d} + \underline{k}_e y_t \quad (47)$$

Taking this filter form into account together with (21) or (24) one can see that in order to implement LQG/LTR control no Riccati equation has to be solved neither for $\underline{k}_c$ nor for $\underline{k}_f$.

5 SIMULATION STUDY

First, consider the stable system

$$G(s) = \frac{s+2}{(s+1)(s+3)} e^{-s}$$

discretized with ZOH and sampling period $T_s = 0.5s$ which yields the following transfer function in z^{-1}

$$G(z^{-1}) = \frac{-0.3262z^{-1} - 0.1224z^{-2}}{1 - 0.8297z^{-1} + 0.1535z^{-2}} z^{-2}, \quad (48)$$

so $d = 2$.

Next, an example of second-order unstable time-delay system is

$$G_p(s) = \frac{s+2}{(s+1)(s-3)} e^{-s}$$

and its discrete-time form with ZOH and $T_s = 0.5s$ is

$$G_p(z^{-1}) = \frac{1.352z^{-1} - 0.439z^{-2}}{1 - 5.088z^{-1} + 2.718z^{-2}} z^{-2}. \quad (49)$$

Finally, nmph time-delay system is considered

$$G_p(s) = \frac{-s+1}{(s+1)(s+2)} e^{-s}$$

which after discretization yields the following transfer function in z^{-1}

$$G_p(z^{-1}) = \frac{-0.1612z^{-1} + 0.2856z^{-2}}{1 - 0.9744z^{-1} + 0.223z^{-2}} z^{-2}. \quad (50)$$

The nominal model in z operator is

$$G(z) = \frac{-0.1612z + 0.2856}{z^2 - 0.9744z + 0.223}. \quad (51)$$

with one nmph zero at 1.772. Then one can calculate

$$G_a(z) = \frac{z - 1.772}{1 - 1.772z}$$

and according to (23) and (24)

$$\underline{h}_m^T = (0.5452, 1.3077), \quad \underline{k}_c^T = (-0.8391, -1.9091).$$

In computer tests different configurations of delay d in the system and its model d_m in the controller were tested. In other words the undermodeling $d_m < d$ and overmodeling $d_m > d$ cases are analyzed.

Simulations of closed-loop step responses with Smith predictor based controllers have been tested for stable and nmph systems as they are not suitable for unstable systems.

An example run of step responses for controller (27) is shown in Fig.1, for stable system with $d = 2$ and $d_m = 2, 6, 10$ and for $d_m = 2$ and $d = 2, 6, 10$. Responses for the same configuration of time-delays for controller (31) is shown in Fig.2, and corresponding situation for controller (35) in Fig.3.

An analogous run of step responses for nmph system and controllers (27), (31), (35) with the same time-delays configurations is shown in Figs.4, 5, 6, correspondingly.

One can observe some performance difference between all these controllers. Looking at the responses one may say that controller (35) slightly outperforms others and in case of nmph system there is no typical undershoot because of pole-zero cancelation in opem-loop.

LQG/LTR method with control (21) as well as LMI approach (26) applied to stable and nmph systems give stability for all under- and overmodeling configurations of time-delay. For unstable systems the global closed-loop stability with respect to time-delay can not be assured even in case of perfect matching $d = d_m$.

The obtained values of destabilizing time-delay for LQG/LTR method are $d_{dest} = 5$, $d_{dest} = 4$ for controllers (14) and (21), respectively and for the noise variance $\sigma_e^2 = 0.01$. For variance value $\sigma_e^2 = 0.001$ one obtains $d_{dest} = 10$, $d_{dest} = 11$ for controllers (14) and (21), respectively. One may observe that the smaller the variance the larger value of d_{dest}, so the value of d_{dest} depends on stochastic properties of noise. In considered case the performance of both controllers (14), (21) is comparable, however controller (21) is computationally simpler. Additionally, unstable nmph system was simulated with controllers (14), (21), (24) yielding the same result $d_{dest} = 5$ for $\sigma_e^2 = 0.01$. For variance $\sigma_e^2 = 0.001$, the values of $d_{dest} = 5, 6, 7$ are obtained correspondingly for controllers (14), (21), (24).

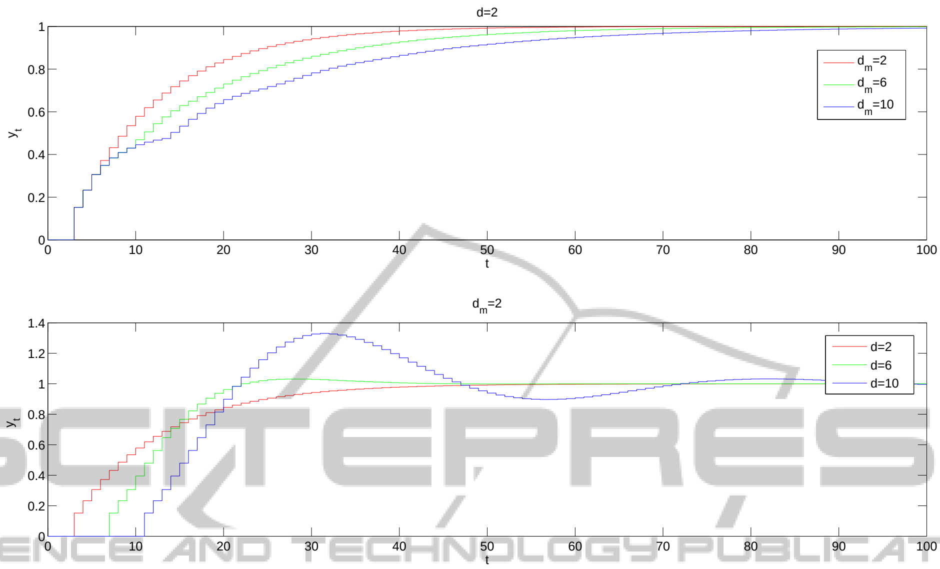

Figure 1: Step responses for stable system with controller (27).

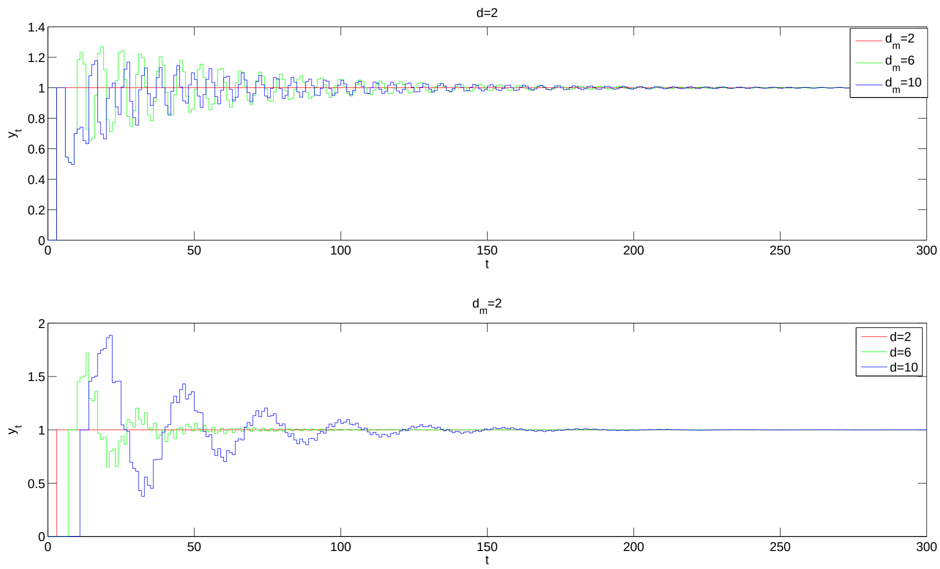

Figure 2: Step responses for stable system with controller (31).

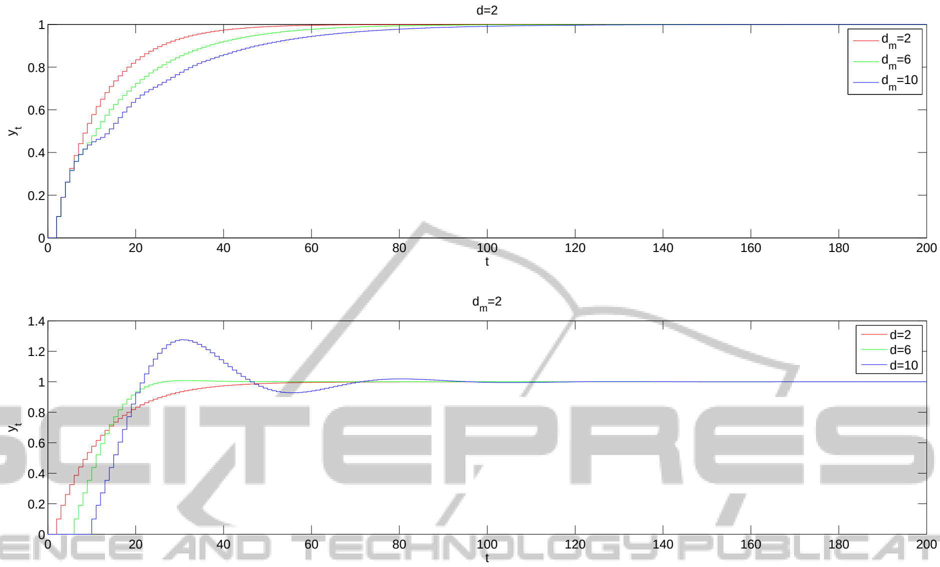

Figure 3: Step responses for stable system with controller (35).

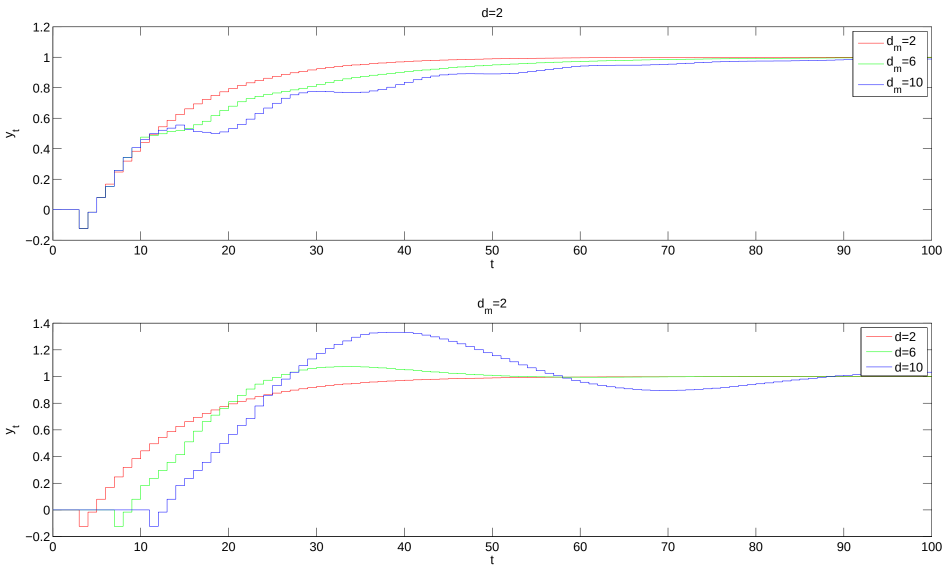

Figure 4: Step responses for nmph system with controller (27).

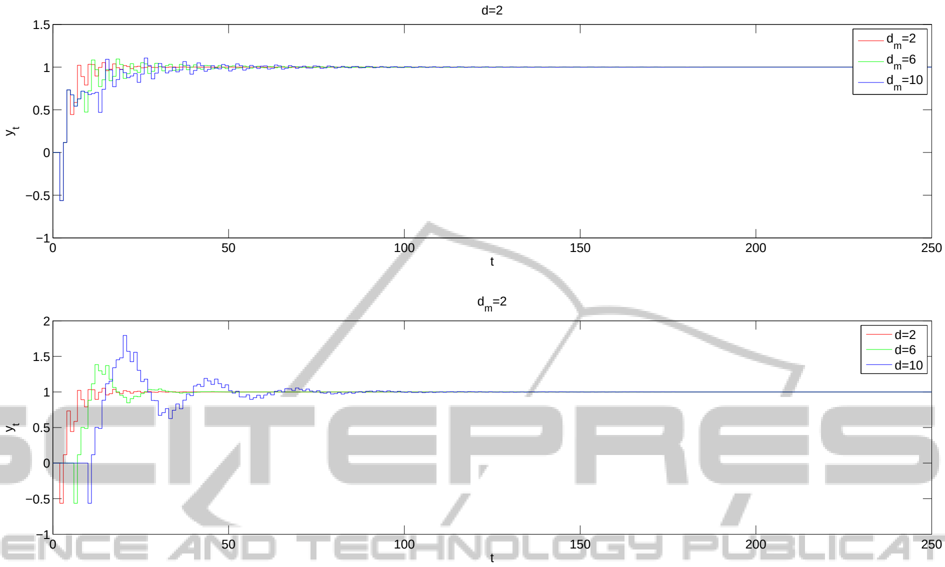

Figure 5: Step responses for nmph system with controller (31).

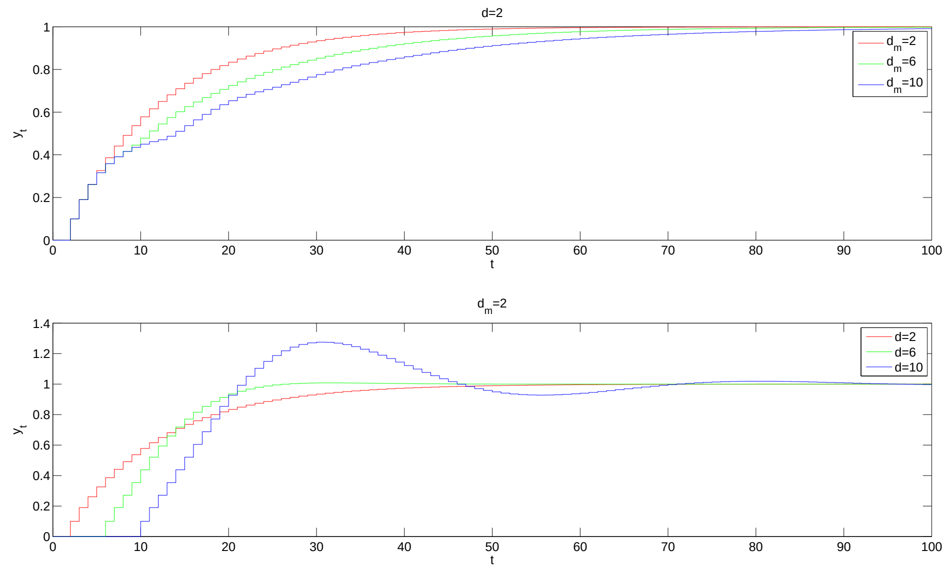

Figure 6: Step responses for nmph system with controller (35).

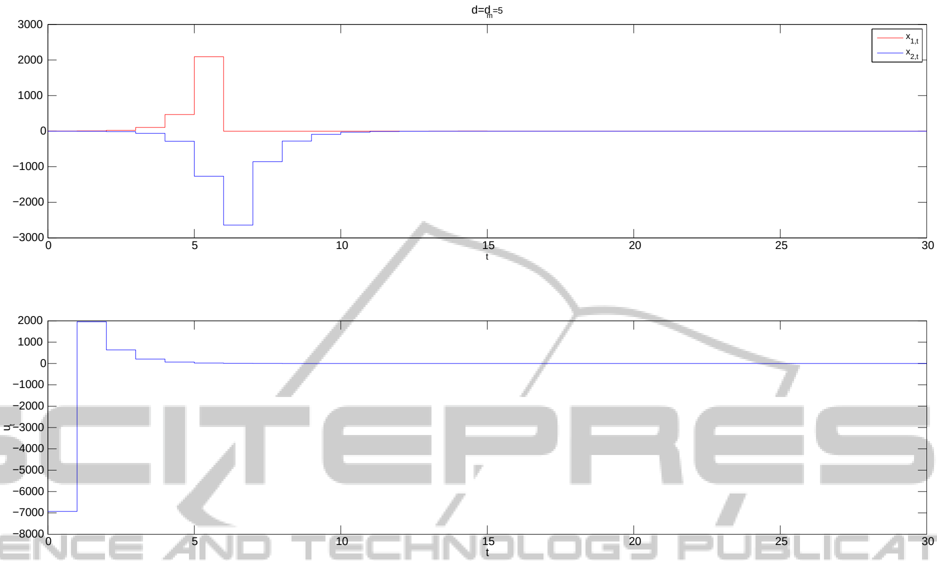

Figure 7: Control and state variables for unstable system with controller (14).

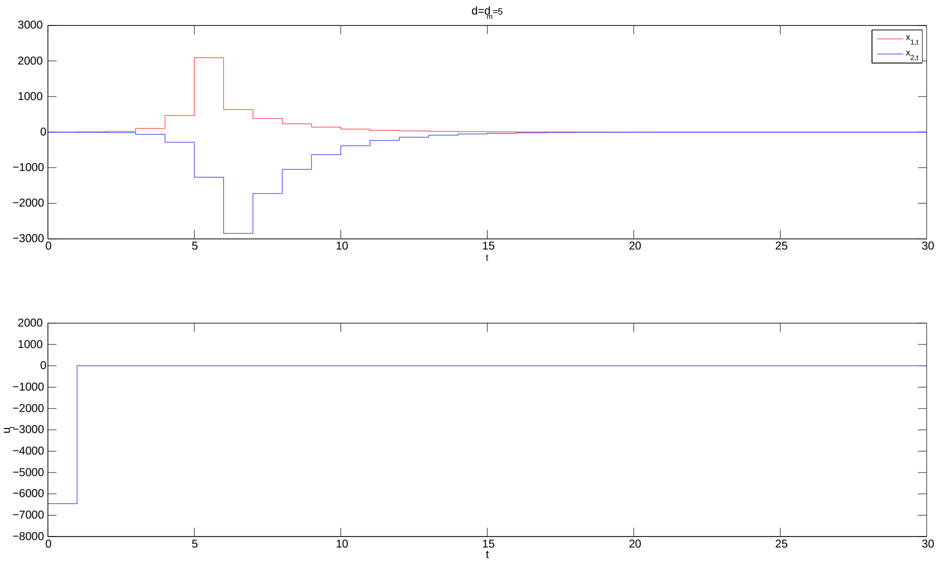

Figure 8: Control and state variables for unstable system with controller (21).

Simulation of state feedback prediction control, whose stability condition is given by LMI (26), for an unstable system is performed for the feedback gain $\underline{k}_c$ from (14). For scalars $\varepsilon_1 = \varepsilon_2 = 10^{-6}$ the obtained value of destabilizing time-delay of the system is $d_{dest} = 14$, however, it should be remembered that this is for deterministic system. This value may be considered as a limit value of d_{dest} for LQG/LTR as a noise variance decrease, i.e. as the system becomes more deterministic.
Plots of state variables and control for unstable noise-free system with non-zero initial conditions and $d = d_m = 5$ are given in Figs.7, 8 for controllers (14) and (21), respectively.

6 CONCLUSIONS

LQG control of discrete-time SISO system with time-delay in the context of LTR effect is presented and compared with LMI robust stability condition given in (Gonzales et al., 2012). Moreover, the Smith predictor approach for PID controller, state space controller and error feedback controller are included into analysis of robust stability with respect to the modeling error of time-delay. This is done on the basis of simulations of second-order system with given nominal time-delay value. Results show some potential of the LQG method with LTR effect as a way for robustifying the stability of closed-loop control for stochastic systems with time-delay and possible unstable open-loop system.

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