New Approach to the Artificial Force Concept for Skid-steering Mobile Platform*

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Abstract: In the paper control algorithm for skid-steering mobile platform is presented. For mathematical model of such an object, expressed in auxiliary coordinates, control law based on the idea of artificial force is introduced. A skid-steering mobile platform is an underactuated control system with a rectangular input matrix. In the approach explored in the paper it was assumed that there exists an additional control input, giving an additional column in input matrix and causing this matrix invertible. Because such an actuator does not exist in reality, this input was kept equal to zero equivalently. Simulations have proved proper work of this method.

1 INTRODUCTION

Wheeled mobile platforms can be treated as independent robots or as a transportation part of complex robotic systems. Depending on wheels' type and a way in which they are fixed to the chassis, motion of wheeled mobile platforms can be realized with or without slipping effect. If slippage effect between wheels and surface does not occur, then exists an equation describing forbidden directions for realized velocities of the system. Such an equation is called nonholonomic constraint in platform's motion.

A special kind of wheeled mobile platforms are platforms with tracks. They can be modeled by a chassis with more than one axis equipped with fixed wheels. These platforms are called skid-steering mobile platforms (SSMP), due to skidding effect observed in their behavior.

Designing of a control algorithm for the skid-steering platform is based on a proper mathematical model of a considered object. A problem occurs in this point, namely there is not an adequate model of objects moving with sleeping effects. There were some attempts to get strict model of wheeled mobile platforms using slow manifold idea (Campion and Motte, 2000) or singular perturbation approach (D'Andrea-Novel et al., 2007). Unfortunately, mathematical models presented in literature are inadequate because they don't cover both cases, i.e. motion with or without slippage effect. For this reason authors describing slippage phenomena in skid-steering mobile platforms use only approximate models with artificial constraints see e.g. (Caracciolo et al., 1999) or with artificial forces (Mazur et al., 2013), (Mazur and Cholewiński, 2013).

In papers treating of skid-steering mobile platforms, another approach to the problem of object modeling has appeared. In (Pazderski and Kozłowski, 2008) authors observed that an artificial constraint introduced by Caracciolo et al. is inadequate to real behavior of SSMP platform and proposed modification of such a constraint to modified form. It implied nonholonomic constraint of second order i.e. dynamically constrained model of SSMP platform. The same idea can be found in (Mohammadpour et al., 2010) and in (Maalouf et al., 2006).

In this paper a new method of modeling SSMP platform is used as a base for artificial force concept. In previous works, e.g. (Mazur et al., 2013) and (Mazur and Cholewiński, 2013), SSMP platform with four wheels was modeled as a chassis with two axes of fixed wheels coupled on both sides of platform. In this case it was impossible to obtain independent motion of each wheel separately because two wheels on one side were connected by a transmission belt. In this paper the chassis with four uncoupled wheels is considered. Taking into account different manners of modeling it was possible to verify if the way of mod-

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eling influences the resulting behavior of the platform.

The paper is organized in the following way. In Section 2 mathematical model of SSMP platform with nonholonomic constraints is developed. In Section 3, the concept of an artificial force approach to control such platforms in presented. In Section 4 the control problem is formulated. Section 5 presents a new control algorithm based on artificial force idea. Section 6 contains the simulation results. Section 7 presents some conclusions.

2 MATHEMATICAL MODEL OF SKID-STEERING PLATFORM

SSMP platforms are modeled as a cart with more than one axis of fixed wheels. In the model of such a platform we will treat it as a cart with four independently driven fixed wheels, see Fig. 1 for details.

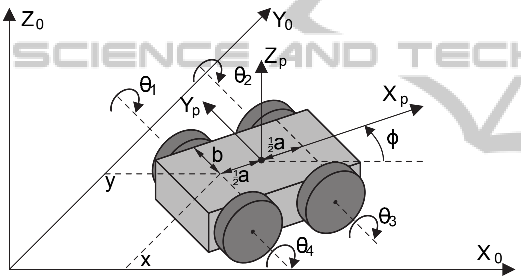

Figure 1: Scheme of skid-steering platform.

The state of such an object can be described by a vector of platform's generalized variables

$$q^T = \begin{pmatrix} x & y & \varphi & \theta_1 & \theta_2 & \theta_3 & \theta_4 \end{pmatrix},$$

where (x, y) are position coordinates of mass center expressed in relation to global frame $X_0Y_0Z_0$, φ is an orientation of skid-steering platform and θ_i is an angle of rotation of ith wheel, $i = 1, \ldots, 4$.

In further considerations we will assume that SSMP platform moves on horizontal surface and no longitudinal slippage phenomena occur. All wheels of mobile platform are identical, therefore constraints related to the absence of longitudinal slippage can be expressed in so-called Pfaffian form

$$A(q)\dot{q} = 0, \tag{1}$$

where Pfaff's matrix equals to

$$A(q) = \begin{bmatrix} \cos\varphi & \sin\varphi & -b & -r & 0 & 0 & 0 \\ \cos\varphi & \sin\varphi & -b & 0 & -r & 0 & 0 \\ \cos\varphi & \sin\varphi & b & 0 & 0 & -r & 0 \\ \cos\varphi & \sin\varphi & b & 0 & 0 & 0 & -r \end{bmatrix}. \tag{2}$$

Symbol r denotes radius of a wheel whereas b is a half of platform's width.

It is obvious that equation (1) determines nonholonomic constraints for SSMP platform.

2.1 Model in Generalized Coordinates

Dynamics of SSMP platform can be derived from Lagrange formula

$$L(q, \dot{q}) = K(q, \dot{q}) - V(q).$$

The platform moves on horizontal surface, therefore its potential energy is equal zero ($V(q) = 0$), hence, the Lagrange formula is composed only of kinetic energy of the platform and its wheels

$$L = K(q, \dot{q}) = \frac{1}{2}\dot{q}^T M(q)\dot{q}, \tag{3}$$

where $M(q)$ is positive definite inertia matrix of the platform

$$M = \begin{bmatrix} m_t & 0 & -\frac{1}{4}m_t\alpha & 0 & 0 & 0 & 0 \\ 0 & m_t & \frac{1}{4}m_t\beta & 0 & 0 & 0 & 0 \\ -\frac{1}{4}m_t\alpha & \frac{1}{4}m_t\beta & I_p & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{xx} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{xx} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I_{xx} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I_{xx} \end{bmatrix}$$

with elements defined below

- $I_p = I_z + 4I_{zz} + m_k \sum_{i=1}^{4} d_i^2$ – total inertia moment of the platform with wheels relative Z_p axis,
- $m_t = m_p + 4m_k$ – total mass of SSMP platform,
- $I_{xx} = \frac{1}{2}m_k r^2$ – inertia moment of wheel relative to rotation axis,
- $\alpha = a\sin\varphi - b\cos\varphi$,
- $\beta = a\cos\varphi + b\sin\varphi$.

Symbols m_p, m_k denote mass of a platform and a wheel respectively. In turn, I_z, I_{zz}, I_{xx} are inertia moments of a platform and wheel – for wheel expressed with the respect to horizontal (I_{xx}) and vertical (I_{zz}) axis. The distance between the platform‘s center of mass and the point of contact of ith wheel with the ground, is marked as d_i and a is the distance between the mass center and the axis of front or back wheels.

Due to nonholonomic nature of constraints, for obtaining dynamic model of SSMP platform d'Alembert Principle must be used

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + F(q, \dot{q}) = B(q)u + A^T(q)\lambda, \tag{4}$$

where:
$M(q)$ – inertia matrix of SSMP platform,
$C(q, \dot{q})$ – matrix of Coriolis and centrifugal forces,
$F(q, \dot{q})$ – ground reaction forces e.g. friction etc.,
$A(q)$ – Pfaff matrix defined by (2),
λ – vector of Lagrange multipliers,
$B(q)$ – input matrix,
u – vector of controls.

2.2 Model in Auxiliary Velocities

According to (1), since the platform velocities $\dot{q}$ are always in the null space of A, it is always possible to find a vector of auxiliary velocities $\eta \in R^3$, such that

$$\dot{q} = G(q)\eta, \tag{5}$$

where $G(q)$ is an 7×3 full rank matrix satisfying relationship $A(q)G(q) = 0$.

Now we want to express the model of dynamics using auxiliary velocities (5) instead of generalized coordinates of the mobile platform. We compute

$$\ddot{q} = G(q)\dot{\eta} + \dot{G}(q)\eta,$$

and eliminate in the model of dynamics the Lagrange multiplier using the condition $G^T A^T \equiv 0$. Substituting $\dot{q}$ and $\ddot{q}$ in (4) we get

$$M^*\dot{\eta} + C^*\eta + F^* = B^*u \tag{6}$$

with elements of matrix equation defined in the following way

$$\begin{aligned} M^* &= G^T M G, \ \ C^* = G^T M \dot{G} + G^T C G, \\ F^* &= G^T F, \ \ B^* = G^T B. \end{aligned}$$

3 ARTIFICIAL FORCE IDEA

Mobile SSMP platform REX should be considered as an underactuated system on dynamic level because it has got a rectangular input matrix (which is non-invertible). There are possible two different approaches to solve the problem of inverting the input matrix B^*:

- introducing additional artificial nonholonomic constraints. Then a model in auxiliary velocities has the same size of state variables as reduced to the number of control inputs.
- introducing an additional artificial input to the dynamics. Then a model in auxiliary velocities has the same number of state variables and control inputs. However, an artificial input (so-called "artificial force" in further considerations) has to be equal 0, i.e.

$$u_{3\ add} \equiv 0, \tag{7}$$

because it does not exist in reality.

4 FORMULATION OF CONTROL PROBLEM

In the paper, our goal is to find control law guaranteeing the trajectory tracking for SSMP platform. Our goal is to address the following control problem to such platforms:

> Determine control law u such that SSMP platform with fully known dynamics follows the desired trajectory.

To design trajectory tracking controller for the considered mobile platform, it is necessary to observe that a complete mathematical model of the nonholonomic system expressed in auxiliary variables is a cascade consisting of the two groups of equations: kinematics (5) and dynamics (6), see Fig. 2: For this reason

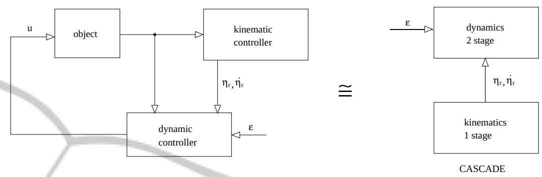

Figure 2: Structure of the proposed control algorithm: cascade with two stages.

the structure of the controller consists of the two parts working simultaneously:

- kinematic controller η_r – a vector of embedded control inputs, which ensure realization of the task for the kinematics (nonholonomic constraints) if the dynamics were not present. Such the controller generates 'velocity profile' which can be executed in practice to realize the trajectory tracking for nonholonomic SSMP platform.
- dynamic controller – as a consequence of cascaded structure of the system model, the system's velocities cannot be commanded directly, as it is assumed in the designing of kinematic control signals, and instead they must be realized as the output of the dynamics driven by u.

It can be observed that backstepping-like algorithm (Krstić et al., 1995) has been evoked to solve the presented control problem for SSMP platform. Backstepping is well-known and often used approach to control cascaded systems, e.g. the system (6) with nonholonomic constraints (5).

5 CONTROL ALGORITHM

As it has been previously mentioned, a control algorithm has to consist of the two parts, i.e. kinematic controller and dynamic controller.

5.1 Kinematic Controller

Considering nonholonomic constraints (5), for a real case of the two active controls, they are equivalent to the unicycle model. On that basis, the kinematic controller is suggested in the form given by (Samson and

Ait-Abderrahim, 1991). This algorithm allows trajectory tracking for a simple unicycle vehicle. Unicycle velocities appropriate for tracking of desired trajectory q_d are described by the following equation (first and second column of matrix $G(q)$ in equation (5))

$$\dot{q}_d = G(q_d)\eta_d = \begin{bmatrix} r\cos\varphi_d & 0 \\ r\sin\varphi_d & 0 \\ 0 & r \end{bmatrix} \begin{pmatrix} \eta_{1d} \\ \eta_{2d} \end{pmatrix}, \quad (8)$$

where $\eta_{1d} = \frac{v_d}{r}$ and $\eta_{2d} = \frac{\omega_d}{r}$ are desired linear and angular velocities of the platform.

The kinematic algorithm of the model described by (5) and desired velocities (8) requires

$$\begin{pmatrix} v_r \\ \omega_r \end{pmatrix} = \begin{pmatrix} v_d e_\varphi - e_x \\ \omega_d - k_1 e_\varphi - k_2 \frac{\sin e_\varphi}{e_\varphi} v_d e_y \end{pmatrix}, \quad (9)$$

where

- v_r, ω_r are reference linear and angular velocities for a robot vehicle (signals coming from kinematic controller),
- v_d, ω_d are desired linear and angular velocities,
- $k_1 > 0$ and $k_2 > 0$ are control parameters,
- $e_\xi = (e_x, e_y, e_\varphi)^T$ – reference trajectory errors.

The reference trajectory tracking errors are defined as below

$$e_\xi = \begin{pmatrix} e_x \\ e_y \\ e_\varphi \end{pmatrix} = Rot(z, -\varphi) \begin{pmatrix} x - x_d \\ y - y_d \\ \varphi - \varphi_d \end{pmatrix}.$$

The asymptotic convergence of tracking errors e_ξ to zero implies asymptotic trajectory tracking. Reference velocities η_{1r} and η_{2r} could be obtained from relationship

$$\eta_{1r} = \frac{v_r}{r}, \qquad \eta_{2r} = \frac{\omega_r}{r}.$$

The third component η_{3r} is responsible for maintaining the apparent force u_{3add} at 0. It can be obtained by solving the equation $u_{3add} = 0$.

5.2 Dynamic Controller

Let's consider dynamics of SSMP platform (6) expressed in auxiliary velocities. For such a system, following passivity-based control law has been proposed

$$u = (B^*)^{-1}(M^*\dot{\eta}_r + C^*\eta_r + F^* - K_d e_\eta), \quad (10)$$

where

- $K_d > 0$ – matrix of regulation parameters,
- $e_\eta = \eta - \eta_r$ – velocity tracking error.

Dynamics of the closed-loop system (6)-(10) can be described as follows

$$M^*\dot{e}_\eta + C^* e_\eta + K_d e_\eta = 0. \quad (11)$$

5.3 Proof of the Convergence

For the system (11) we propose the following Lyapunov-like function

$$V(e_\eta) = \frac{1}{2} e_\eta^T M^*(q) e_\eta \geq 0, \quad (12)$$

which is non-negative definite.

Time derivative of V along solutions of the closed-loop system (11) is equal to

$$\begin{aligned} \dot{V} &= \frac{1}{2} e_\eta^T \dot{M}^*(q) e_\eta + e_\eta^T M^*(q) \dot{e}_\eta \\ &= -e_\eta^T K_d e_\eta \leq 0. \end{aligned} \quad (13)$$

From La Salle theorem, see (Krstić et al., 1995) for details, it could be concluded that the errors e_η converge asymptotically to zero.

From the other side, the convergence of e_η to zero means that the the velocity profile generated by kinematic controller is successfully followed, and therefore one can conclude that the nonholonomic system, i.e. skid-steering mobile platform, tracks desired trajectory q_d. It ends the proof.

5.4 Artificial Force – Implicit Function

From definition of artificial force i.e.

$$u_{3\ add} \equiv 0,$$

where control inputs are given by the control algorithm (10), the third reference signal η_{3ref} can be calculated as an implicit function in the following way

$$\begin{aligned} \dot{\eta}_{3ref} &= \frac{1}{4m_t r^2}(4K_d\eta_3 - 4K_d\eta_{3ref} + \\ &+ \beta m_t r^2 \cos\varphi \dot{\eta}_{2ref} + \alpha m_t r^2 \sin\varphi \dot{\eta}_{2ref} + \\ &+ 4m_t r^2 \eta_{1ref} \dot{\varphi} + b m_t r^2 \eta_{2ref} \dot{\varphi}). \end{aligned} \quad (14)$$

6 SIMULATIONS

The simulations were run with the MATLAB package and the SIMULINK toolbox.

Parameters of the platform were equal to: $m_t = 30$ [kg], $m_k = 3$ [kg], $I_p = 11.3335$ [kg·m^2], $I_{xx} = 0.0968$ [kg·m^2], $a = 0.8$ [m], $b = 0.3$ [m], $r = 0.127$ [m]. Parameters of terrain are set on value $\mu = f_r = 1$.

The model of SSMP platform expressed in auxiliary velocities is defined by equations (5) and (6).

The elements of the mathematical model of SSMP platform expressed in auxiliary velocities have detailed form

- kinematics (5)

$$G = \begin{bmatrix} r\cos\varphi & r\sin\varphi & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & r & -b & -b & b & b \\ r\sin\varphi & -r\cos\varphi & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

- dynamics (6)

$$\begin{aligned} M^* &= \begin{bmatrix} M_{11} & M_{12} & 0 \\ M_{12} & M_{22} & M_{23} \\ 0 & M_{23} & M_{33} \end{bmatrix}, \\ M_{11} &= 4I_{xx} + m_t r^2 \\ M_{12} &= -\frac{1}{4} m_t r^2 (\alpha\cos\varphi - \beta\sin\varphi) \\ M_{22} &= 4b^2 I_{xx} + I_p r^2 \\ M_{23} &= -\frac{1}{4} m_t r^2 (\beta\cos\varphi + \alpha\sin\varphi) \\ M_{33} &= m_t r^2 \end{aligned}$$

$$C^* = \begin{bmatrix} 0 & -\frac{1}{4} a m_t r^2 \dot\varphi & m_t r^2 \dot\varphi \\ \frac{1}{4} a m_t r^2 \dot\varphi & 0 & \frac{1}{4} b m_t r^2 \dot\varphi \\ -m_t r^2 \dot\varphi & -\frac{1}{4} b m_t r^2 \dot\varphi & 0 \end{bmatrix},$$

$$F^* = \begin{bmatrix} g m_t \mu \cdot \operatorname{sgn} \dot y \\ \frac{1}{2} f_r b g m_t r [\operatorname{sgn}(\dot x + b\dot y) - \operatorname{sgn}(\dot x - b\dot\varphi)] \\ 0 \end{bmatrix},$$

$$B^* = \begin{bmatrix} 1 & 1 & 0 \\ -b & b & 0 \\ 0 & 0 & r \end{bmatrix}.$$

6.1 Desired Trajectories

The goal of the simulations was to investigate a behavior of a mobile platform with the controller (10) proposed in the paper. In simulations the two desired trajectory of the platform has been tested: a circle and a square trajectory. The first one, was a circle with a radius of $R = 10$ [m] and frequency $\omega = 0.1$ [$\frac{rad}{s}$]. The second desired trajectory was a square with side equal to 10 meters. The linear velocity v_d for this desired trajectory was set to $0.5\frac{m}{s}$.

The parameters of a kinematic controller given by (9) were equal to $k_1 = 1$, $k_2 = 1$ while a dynamic controller (10) has as regulation parameter $K_d = 50$ for both desired trajectories.

Circle. In Figures from 3 to 6 were presented the plots from a simulation with the desired trajectory set to a circle. From the plots presented in Figures 4-6 it can be observed that position tracking errors and orientation error tend to 0.

Square Trajectory. In Figures 7-10 were presented the plots for desired trajectory set to a square.

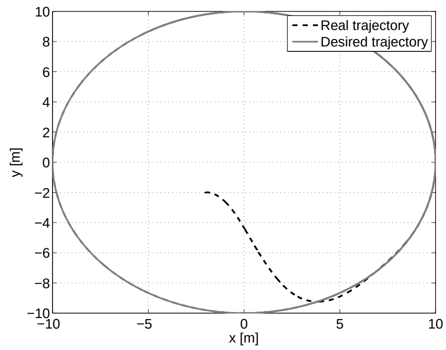

Figure 3: Real vs desired trajectory.

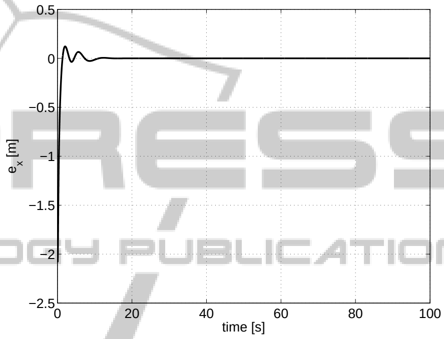

Figure 4: Error in cartesian space for x coordinate.

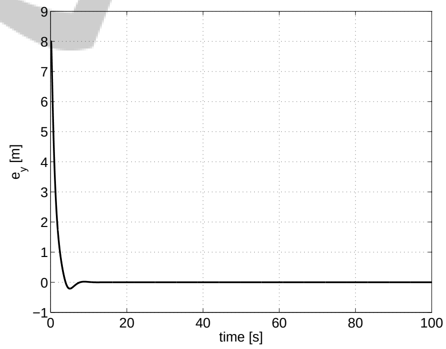

Figure 5: Error in cartesian space for y coordinate.

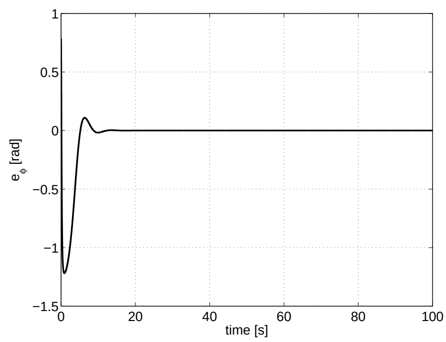

Figure 6: Error in cartesian space for φ coordinate.

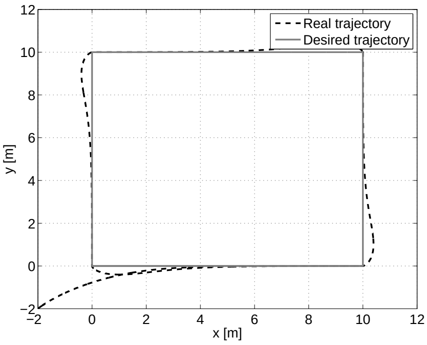

Figure 7: Real vs desired trajectory.

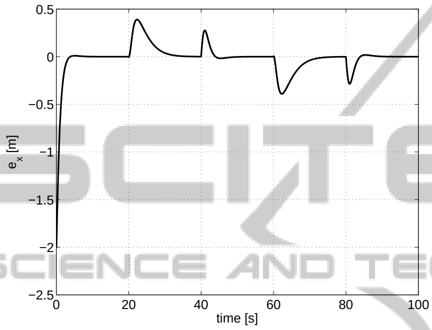

Figure 8: Error in cartesian space for *x* coordinate.

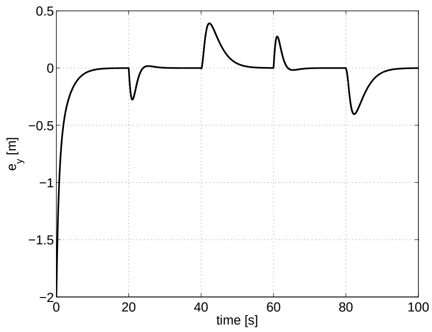

Figure 9: Error in cartesian space for *y* coordinate.

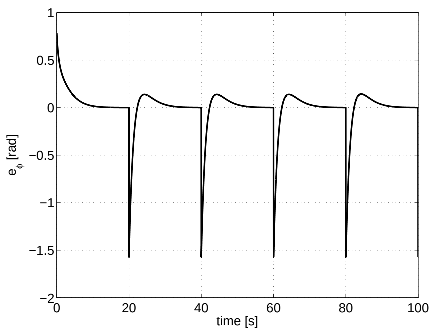

Figure 10: Error in cartesian space for φ coordinate.

7 CONCLUDING REMARKS

In the paper the method of an artificial force was used for the skid-steering mobile platform. In this method an additional input to the model of dynamics has been introduced making input matrix invertible. Differently than in previous publications, in considered approach, the platform has been modeled as a cart with four independent, not coupled wheels.

Used method has shown that it gives good results for different ways of modeling a skid-steering platform. It implies that in future research, the use of artificial force concept will lead to proper work of control algorithm and good enough trajectory tracking.

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