A Path-based Equivalence Checking Method for Petri Net based Models of Programs

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Abstract: Programs are often subjected to significant optimizing and parallelizing transformations. It is therefore important to model parallel behaviours and formally verify the equivalence of their functionalities. In this work, the untimed PRES+ model (Petri net based Representation of Embedded Systems) encompassing data processing is used to model parallel behaviours. Being value based with inherent scope of capturing parallelism, PRES+ models depict such data dependencies more directly; accordingly, they are likely to be more convenient as the intermediate representations (IRs) of both the source and the transformed codes for translation validation than strictly sequential variable-based IRs like Finite State Machines with Datapath (FSMDs) (which are essentially sequential control data-flow graphs (CDFGs)). In this work, a path based equivalence checking method for PRES+ models is presented.

1 INTRODUCTION

Recent advancement of multi core and multi processor systems has enabled incorporation of concurrent applications in embedded software systems through extensive optimizing transformations for better time performance and resource utilization (Gupta et al., 2003). If such optimizations are carried out by untrusted compilers, they can result in *software bugs*. Hence, it is important to verify whether the optimized code faithfully represents the intended functionality.

A comprehensive list of models proposed to represent programming systems and their validation can be found in (Edwards et al., 1999; Akl, 1997; Milner, 1989). Petri nets have long been popular modeling paradigm for concurrent behaviours. The untimed one safe PRES+ model (Petri net based Representation for Embedded Systems) (Cortés et al., 2003) *enhances the classical Petri net model to capture natural concurrency and well defined semantics of computations over integers, reals and general data structures*. Analyses of dependencies among the operations in a program lie at the core of many optimizing and parallelizing transformations. Being value based with inherent scope of capturing parallelism, PRES+ models depict such data dependencies more directly; accordingly, they are likely to be more convenient as the intermediate representations (IRs) of both the source and the transformed codes for translation validation than strictly sequential variable-based IRs like all types of control data-flow graphs, communicating sequential processes, etc.

Behavioural verification involves demonstrating the input-output equivalence of all computations represented by the original behavioural description with those of the transformed behavioural description. From the success of path based equivalence checking of CDFG models (essentially FSMDs) recorded in (Banerjee et al., 2014), it is perceived that a similar approach is worth pursuing for PRES+ models. Path structures in PRES+ models, however, are far more complex than those in CDFG models due to the presence of parallel threads of computations in the former. The major *contributions* of the present paper are as follows:

1. A formal definition of computations of untimed PRES+ models is provided.
2. An algorithm for path construction of PRES+ models is provided and formally treated.
3. An algorithm for path based validation of PRES+ models is provided and formally treated.

Organization: The rest of the paper is organized as follows. Section 3 describes an overview through a motivating example. Section 4 formally introduces the untimed PRES+ model, its computational semantics and also the notion of computational equivalence between two PRES+ models. The concept of cut-

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points and paths in a PRES+ model is introduced and a path construction algorithm is also given in section 5; an equivalence checking algorithm is given in section 6. Experiments on some benchmarks can be found in section 7. Section 8 states the related work on translation validation. The paper is concluded in section 9.

2 FRAMEWORK

Figure 1 depicts the actual framework of the present work. A software program is compiled using some compiler transformation techniques and an optimized intermediate code is produced. Validation of such transformations is a *undecidable problem*. On the other hand, if we are able to establish the computational equivalence between the original and the transformed programs, we can claim that the transformations applied for the specific run is correct. Therefore, our main target is to develop an equivalence checker for translation validation.

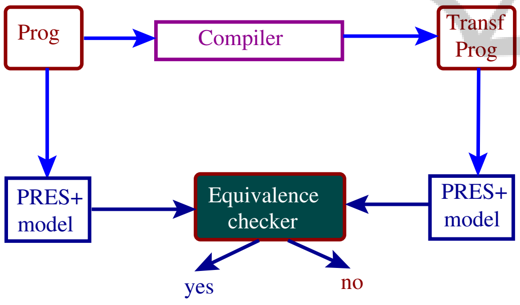


Figure 1: Basic framework.

For program analysis, it is necessary to translate any program to its equivalent formal model representation. As the main target of this work is to validate code optimizing and several parallelizing transformations, a parallel model of computation (MoC) is necessary. In this work, the PRES+ model, whose underlying structure is a one safe Petri net model with token holding values, is selected as the parallel MoC. Therefore, PRES+ models are constructed from both the original and the transformed program. The automated PRES+ construction method from high level language is reported in (Cortes et al., 2000). Here, our main task is to devise a PRES+ equivalence checker which takes two PRES+ models as inputs and returns either "yes" or "no" as its output. If the equivalence checker gives a "yes" response, then the two programs are equivalent, i.e., particular transformations which are carried out by the compiler are correct; if it gives a "no" response, then the two programs may not be equivalent. Hence, our method is sound but not complete and it may give a *false negative result*. The basic steps of the equivalence checking procedure are as follows: (1) In the first step, a PRES+ model is partitioned into several fragments which are called *paths*; the paths are obtained by cutting a loop in at least one cut-point which is adapted from (Floyd, 1967); any computation of the model can now be represented as a concatenation of these paths. (2) Next, checking whether for all paths in the PRES+ model N_0, say, which corresponds to the source program, there exists a path in the PRES+ model N_1, say, which corresponds to the transformed program such that the two paths are equivalent, i.e., their data computations and conditions of execution are identical and their input and output places have correspondence. (3) Repeat steps 1 and 2 with N_0 and N_1 interchanged. The major challenges of this work are as follows:

1. Path construction procedure for a PRES+ model
2. Equivalence checking method for PRES+ models.

3 METHODOLOGY

Before describing the formal notion of path based equivalence checking mechanism between two PRES+ models, in this section, we underline through an example some of the relevant issues which arise during development of an equivalence checker for PRES+ models.

```
int i = 1, j = 1;
int k;
while (i*7 <=100)
    i++ ;
while ((j+1)*11 <=100)
    j++;
k = i+j;
```

(a)

```
int i = 1, j = 1;
int k;
while ((j+1)*11 <=100)
    j++;
while ( i*7 <=100)
    i++ ;
k = i+j;
```

(b)

```
int i = 1, j ;
int k;
j = i;
#parbegin
  while (i*7 <=100)
      i++ ;
|| while ((j+1)*11 <=100)
      j++;
#parend
k = i+j;
```

(c)

Figure 2: Initial and Transformed Behaviour.

Example 1. *Figure 2 (a) represents an initial program which computes* $\lceil \frac{100}{7} \rceil + \lfloor \frac{100}{11} \rfloor$. *Figure 2(b) and 2(c) pertain to programs transformed using loop swapping transformation and thread level parallelizing transformations, respectively. Figures 3 (a), 3(b) and 3(c) depict the PRES+ models corresponding to the programs given in Figures 2(a), 2(b) and 2(c), respectively. Automated model construction from high level program to its equivalent PRES+ model is reported in (Cortes et al., 2000).*

Let us now address the issues in finding equivalence between the PRES+ models of Figures 3(a) and 3(c). In Figure 3(a), the fragments $p_1.(p_3)^n.p_5$ *computes the first term* $\lceil \frac{100}{7} \rceil$ *and the fragment*

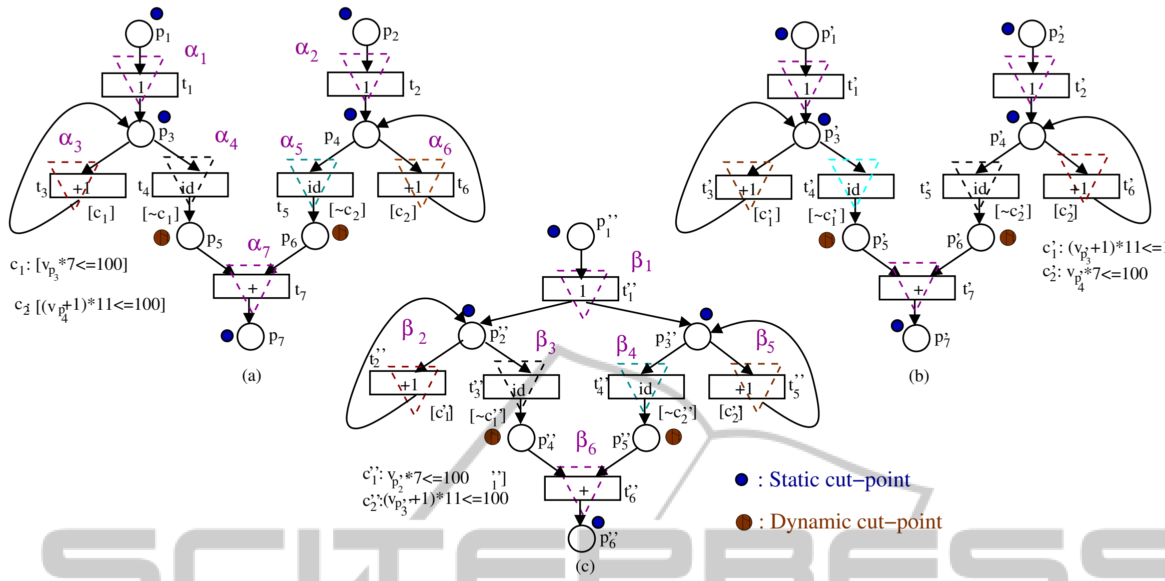


Figure 3: Initial and transformed PRES+ models.

$p_2.(p_4)^m.p_6$ computes the second term $\lfloor \frac{100}{11} \rfloor$*; correspondingly, in Figure 3(c), $p''_1.(p''_2)^n.p''_4$ computes the first term and $p''_1.(p''_3)^m.p''_5$ computes the second term ($n \neq m$).* In general, *in a computation, the number of traversals of a loop in a PRES+ model depends upon the input token values. Since equivalence has to be established for all computations, the notion of finite computation paths is used so that any computation can be captured in terms of these paths. To do so, we cut the loops by introducing cut-points so that every loop is* cut in at least one cut-point; *each path will originate from* a set of cut-points and extend up to a cut-point without having any intermediary cut-points. *In Figure 3(a), suppose the cut-points are p_1, p_2 (as in-ports), p_3, p_4 (for cutting the loops), p_7 (for the out-port) and p_5, p_6 (extra cut-points). A path is represented as* a sequence of maximally parallelizable transitions*; hence the corresponding paths are also depicted in Figure 3(a) using inverted dotted triangular boxes, such as $\alpha_7 = \langle\{t_7\}\rangle$ and the computation μ_{p_7} can be represented as $(\alpha_1 \parallel \alpha_2).(\alpha_3 \parallel \alpha_4)^m.(\alpha_3 \parallel \alpha_6).(\alpha_3)^n.\alpha_5.\alpha_7$. Thus, introducing extra cut-points judiciously is an important step which will be discussed in detail in the subsequent sections.*

Similarly, in Figure 3(c), the paths are as shown by inverted dotted triangular boxes with cut-points $\{p''_1, p''_2, p''_3, p''_4, p''_5, p''_6\}$. Two entities are used to characterize a path α: (1) the condition of execution R_α, and (2) the data transformation r_α; these are computed by the forward substitution method. For the paths α_1 and α_3 in Figure 3(a), $R_{\alpha_1}(v_{p_1})$: "true" and $R_{\alpha_3}(v_{p_3}) : v_{p_3} * 7 \leq 100$; and the data transformations are $r_{\alpha_1}(v_{p_1}) = 1$ and $r_{\alpha_3}(v_{p_3}) = v_{p_3} + 1$.

Let the in-port association $f_{in} \subseteq inP_0 \times inP_1$ be $\{\langle p_1, p''_1\rangle, \langle p_2, p''_1\rangle\}$; let out-port association f_{out} : $outP_0 \leftrightarrow outP_1$ be $p_7 \mapsto p''_6$. For each path of Figure 3(a), the equivalent path of Figure 3(c) is obtained. Specifically, we obtain $\alpha_1 \simeq \beta_1$, $\alpha_2 \simeq \beta_1$, $\alpha_3 \simeq \beta_2$, $\alpha_4 \simeq \beta_3$, $\alpha_5 \simeq \beta_4$, $\alpha_6 \simeq \beta_5$ and $\alpha_7 \simeq \beta_6$; also, their input and output places have correspondence. Since all the paths of the original behaviour have some equivalent paths in the transformed behaviour, and vice versa, the models are asserted to be equivalent. We can also establish the equivalence between the models of Figures 3(a) and 3(b) identically.

Path Extension: *Code motion transformations move code segments beyond the basic block boundaries; consequently, some paths of one model may be found to have no equivalent paths in the other model. Such paths will have to be extended through its subsequent path(s) till paths equivalent to the resulting concatenated path(s) are obtained. The idea of path extension is similar to that of path based FSMD equivalence checking mechanism (Banerjee et al., 2014). Intricacies, however, arise due to the presence of paths parallel to the path being extended. This situation is presented in section 6 through Example 5.* ■

4 THE PRES+ MODEL AND ITS COMPUTATION

A PRES+ model is a 6−tuple $N = \langle P, T, I, O, inP, outP\rangle$, where the members are defined as follows. P: a finite non-empty set of places. A place p is capable of holding a token having a value v_p from a domain D_p. A token value may be of type Boolean, integer, etc., or a user-defined type of any complexity (for instance, a structure or a set). T is a finite non-empty set of transitions; the relation $I \subseteq P \times T$ is a flow relation from places to transitions; a place p is said to be an input place of a transition t if $(p,t) \in I$; $^\circ t$ denotes the input places of t. The relation $O \subseteq T \times P$ is a flow relation from transitions to places; a place p is said to be an output place of a transition t if $(t,p) \in O$; t° denotes the output places of t. A place $p \in P$ is said to be an in-port iff $(t,p) \notin O$, for all $t \in T$. Likewise, a place $p \in P$ is said to be an out-port iff $(p,t) \notin I$, for all $t \in T$. The set $inP \subseteq P$ is a non-empty set of *in-ports* and the set $outP \subseteq P$ is a non-empty set of *out-ports*. The pre-set $^\circ p$ (post-set p°) of a place p comprises all the transitions of which p is an output (input) place. A function f_t and a guard condition g_t are associated with a transition t. The function f_t captures the functional transformation that takes place on the token values in $^\circ t$ to produce the same token value at all the post-places of t°. The model is deterministic and completely specified; that is, for any set P_g of places, (i) for any two transitions $t_i, t_j \in P_g{}^\circ$, if $^\circ t_i \cap {}^\circ t_j \neq \emptyset$, then $g(t_i) \wedge g(t_j) = false$, and (ii) $\bigvee_{t \in P_g^\circ} g(t) = true$.

A marking M is an ordered 2-tuple of the form $\langle P_M, val_M \rangle$ where, P_M is a subset of the places of M where tokens are present and $val_M : P \rightarrow D_P$ is a mapping from places to token values. The token value $val_M(p)$ in p for the marking M is also denoted as v_p^M, where $p \in P_M$; otherwise it is undefined, denoted as ω. A marking M_0 is an initial marking with $P_{M_0} = inP$. In a PRES+ model, a transition $t \in T$ is *bound* for a given marking M:$\langle P_M, val_M \rangle$ iff all its input places are marked, i.e., $^\circ t \subseteq P_M$. A bound transition $t \in T$ for a given marking M is *enabled* iff $g_t(v_{p_1}^M, v_{p_2}^M, \cdots, v_{p_n}^M)$ holds, where $^\circ t = \{p_1, \cdots, p_n\}$. The set of enabled transitions for a marking M is denoted as T_M. All the enabled transitions are assumed to fire simultaneously. A marking M^+ is said to be a successor of the marking M if M^+ contains all the post-places of the enabled transitions of M and also all the places of M whose post-transitions are not enabled; symbolically, $P_{M^+} = \{p \mid p \in t^\circ$ and $t \in T_M\} \cup \{p \mid p \in P_M$ and $p \notin {}^\circ T_M\}$; for any marking M and its successor marking M^+, for any place $p \in t^\circ$, where $t \in T_M$ having $^\circ t = \{p_1, \cdots, p_n\}$ and associated with function f_t, $v_p^{M^+} = f_t(v_{p_1}^M, v_{p_2}^M, \cdots, v_{p_n}^M)$; for any place $p \in t^\circ$, where $t \notin T_M$, $v_p^{M^+} = v_p^M$. Specifically, for an assignment statement of a high level language of the form $x := y + c/d * 4$, the transition t will have $^\circ t = \{p_1, p_2, p_3\}$, $t^\circ = \{p\}$ and f_t will be maintained as $v_{p_1} + v_{p_2}/v_{p_3} * 4$.

Let $p_1 \prec p_2 \prec \ldots \prec p_n$ be an ordering over the places $P = \{p_1, \ldots, p_n\}$. Let $\overline{val_M}$ represent the vector $\langle v_{p_1}^M, v_{p_2}^M, \ldots, v_{p_n}^M \rangle |_{P_M \subseteq P}$ of values associated with the places restricted to the subset P_M of places. We refer to $\overline{val_M}$ as the value vector for the marking M and $\overline{val_M}(P_M)$ to represent its restriction $\overline{val_M}|_{P_M \subseteq P}$. It is to be noted that the transitions may also have delay and deadline time parameters; models having these features are called timed PRES+ models. We deal with only untimed PRES+ models. Also, we consider one-safe PRES+ models whose structures ensure that at any point a place may hold at most one token. Henceforth, by a PRES+ model we only mean a one-safe untimed PRES+ model.

Definition 1 (Successor Relation Between Two Transitions)**.** *A transition t_i succeeds a transition t_j, denoted as $t_i \succ t_j$, if (i) $^\circ t_i \cap\, t_j^\circ \neq \emptyset$ or (ii) $\exists t_{k_1}, t_{k_2}, \ldots, t_{k_n}, n > 1$ such that $t_i \succ t_{k_1} \succ t_{k_2} \succ \ldots \succ t_{k_n} \succ t_j$. $t_i \not\succ t_j$ is used as a shorthand for $\neg t_i \succ t_j$.*

Definition 2 (Set of Maximally Parallelizable Transitions)**.** *Two transitions t_i and t_j are said to be parallelizable, denoted as $t_i \asymp t_j$, if (i) $t_i \not\succ t_j$ and $t_j \not\succ t_i$ and (ii) $\forall t_k, t_l \in T_M, (t_k \neq t_l \wedge t_i \succeq t_k \wedge t_j \succeq t_l) \rightarrow {}^\circ t_k \cap {}^\circ t_l = \emptyset$, where $t_i \succeq t_k$ holds iff t_i succeeds t_k or t_i is the same as t_k. A set $T = \{t_1, t_2, \ldots, t_k\}$ of transitions is said to be parallelizable if $\forall t_i, t_j \in T, t_i \neq t_j \rightarrow t_i \asymp t_j$ holds. The set T is said to be maximally parallelizable if there is no set T' of parallelizable transitions which contains T.*

Definition 3 (Computation in a PRES+ Model)**.** *In a PRES+ model N a computation $\mu_{N,p}$ of an out-port p is a sequence $\langle T_1, T_2, \ldots, T_i, \ldots, T_l \rangle$ of sets of maximally parallelizable transitions where, $^\circ T_1 \subseteq inP$, $p \in T_l^\circ$ and if $T_i^\circ \subseteq P_{M_i}$ in the marking M_i and $T_{i+1}^\circ \subseteq P_{M_{i+1}}$ in the marking M_{i+1}, then $M_{i+1} = M_i^+$, for all i, $1 \leq i < l$. When we need to refer explicitly to the the initial value vector $\overline{val_{M_0}}(^\circ T_1)$, we represent the computation $\mu_{N,p}$ as an ordered pair $\langle \langle ^\circ T_1, T_1^\circ, {}^\circ T_2, \ldots, T_l^\circ \rangle, \overline{val_{M_0}}(^\circ T_1) \rangle$.*

If there are k out-ports, then for each initial marking M_0, there are at most k computations, one for each out-port. (We drop the suffix(es) of μ when they are clear from the context. Thus, more specifically, when there is no other PRES+ model we use the symbol μ_p.) There are two entities associated with a computation μ_p of an out-port p:

1. The *condition* $R_{\mu_p}(\overline{val_{M_0}}(^\circ T_1))$ *of* μ_p on the initial token values at $^\circ T_1$ under which μ_p takes place.
2. The *data transformation* $r_{\mu_p}(\overline{val_{M_0}}(^\circ T_1))$ *of* μ_p which provides the token value in the out-port p after μ_p is completed.

Two PRES+ models N_0 and N_1 will not be functionally equivalent unless they are cardinality-equivalent (Cortés et al., 2003), or more specifically, input-output cardinality-equivalent, that is, their respective in-ports and out-ports are bijective; the corresponding bijections are denoted as $f_{in} : inP_0 \leftrightarrow inP_1$ and $f_{out} : outP_0 \leftrightarrow outP_1$.

Let $N_0 : \langle P_0, T_0, I_0, O_0, inP_0, outP_0\rangle$ and $N_1 : \langle P_1, T_1, I_1, O_1, inP_1, outP_1\rangle$ be two cardinality-equivalent PRES+ models with in-port bijection f_{in} and out-port bijection f_{out}. We now define the notions of computations of an out-port, containment of PRES+ models and the computational equivalence of PRES+ models.

Definition 4 (Equivalence of PRES+ Computations). *Let $\mu_{0,p}$ be a computation of an out-port p of N_0 of the form $\langle T_{0,1}, T_{0,2}, \ldots, T_{0,n_p}\rangle$ starting with an initial marking $M_{0,0} \supseteq {}^\circ T_{0,1}$; let $\mu_{1,f_{out}(p)}$ be a computation of the out-port $f_{out}(p)$ of N_1 of the form $\langle T_{1,1}, T_{1,2}, \ldots, T_{1,n_{f_{out}(p)}}\rangle$ starting with the initial marking $M_{1,0} \supseteq {}^\circ T_{1,1}$, where $val_{M_{0,0}}(p) = val_{M_{1,0}}(f_{in}(p))$, $\forall p \in inP_0$. The computations $\mu_{0,p}$ and $\mu_{1,f_{out}(p)}$ are said to be equivalent (represented as $\mu_{0,p} \simeq \mu_{1,f_{out}(p)}$), if $R_{\mu_{0,p}}(\overline{val_{M_{0,0}}}({}^\circ T_{0,1})) \equiv R_{\mu_{1,f_{out}(p)}}(\overline{val_{M_{1,0}}}({}^\circ T_{1,1}))$ and $r_{\mu_{0,p}}(\overline{val_{M_{0,0}}}({}^\circ T_{0,1})) = r_{\mu_{1,f_{out}(p)}}(\overline{val_{M_{1,0}}}({}^\circ T_{1,1}))$.*

Definition 5 (Containment of PRES+ Models). *A PRES+ model N_0 is said to be contained in a PRES+ model N_1, represented as $N_0 \sqsubseteq N_1$, if, $\forall p \in outP_0$, for every computation $\mu_{0,p}$ of the out-port of N_0 starting with an initial marking $M_{0,0} \supseteq {}^\circ T_{0,1}$, there exists a computation $\mu_{0,f_{out}(p)}$ of the out-port of N_1 starting with an initial marking starting with the initial marking $M_{1,0} \supseteq {}^\circ T_{1,1}$, where $val_{M_{0,0}}(p) = val_{M_{1,0}}(f_{in}(p))$, $\forall p \in inP_0$, such that $\mu_{0,p} \simeq \mu_{1,f_{out}(p)}$.*

Definition 6 (Computational Equivalence of PRES+ Models). *The PRES+ models N_0 and N_1 are said to be computationally equivalent if $N_0 \sqsubseteq N_1$ and $N_1 \sqsubseteq N_0$.*

5 PATHS OF A PRES+ MODEL

It is to be noted that the notion of paths and concatenated paths has already been covered in Example 1. In this section, we describe the formal notion of a path of a PRES+ model.

Definition 7 (Back Edge). *An edge $\langle t, p\rangle$ from a transition t to a place $p \in t^\circ$ is said to be a back edge with respect to an arbitrary DFS traversal, if p is an ancestor of t in that traversal of the PRES+ model.*

Definition 8 (Static Cut-point). *A place p is designated as a static cut-point with respect to an arbitrary DFS traversal if (i) p is an in-port, or (ii) p is an out-port or (iii) there is an edge $\langle t, p\rangle$ which is a back edge with respect to that DFS traversal.*

Definition 9 (Path in a PRES+ Model). *A finite path α in a PRES+ model from a set T_1 of transitions to a transition t_j is a finite sequence of distinct sets of parallelizable transitions of the form $\langle T_1 = \{t_1, t_2, \ldots, t_k\}, T_2 = \{t_{k+1}, t_{k+2}, \ldots, t_{k+l}\}, \ldots, T_n = \{t_j\}\rangle$ satisfying the following properties:*

1. All the members of ${}^\circ T_1$ are cut-points.
2. All the members of T_n° are cut-points.
3. There is no cut-point in T_m°, $1 \le m < n$.
4. $\forall i, 1 < i \le n, \forall p \in {}^\circ T_i$, if p is not a cut-point, then $\exists k, 1 \le k \le i-1, p \in T_{i-k}^\circ$; thus, any pre-place of a transition which is not a cut-point must be a post-place of some preceding transition in the path.
5. There do not exist two transitions t_i and t_l in α such that ${}^\circ t_i \cap {}^\circ t_l \neq \emptyset$.
6. $\forall i, 1 \le i \le n$, T_i is maximally parallelizable within the path, i.e., $\forall t \in T_l$ in the path such that $l \neq i$, $T_i \cup \{t\}$ is not parallelizable.

The set ${}^\circ T_1$ of places is called the pre-set (pre-places) of the path α, denoted as ${}^\circ\alpha$; similarly, the post-set (post-places) α° of the path α is T_n°. We can synonymously denote a path $\alpha = \langle T_1, T_2, \ldots, T_n\rangle$ as the sequence $\langle {}^\circ T_1, {}^\circ T_2, \ldots, {}^\circ T_n, T_n^\circ\rangle$ of the sets of places from the place(s) ${}^\circ T_1$ to the place(s) T_n°.

In the following example, we show the need of extra cut-points such that any computation can be represented in terms of paths.

Example 2. *In Figure 3(a), suppose the cut-points are p_1, p_2 (as in-ports), p_3, p_4 (for cutting the loops), p_7 (for the out-port); hence, the corresponding paths are as follows: $\alpha_1 = \langle\{t_1\}\rangle, \alpha_2 = \langle\{t_2\}\rangle, \alpha_3 = \langle\{t_3\}\rangle, \alpha_4 = \langle\{t_6\}\rangle$ and $\alpha_5 = \langle\{t_4, t_5\}, \{t_7\}\rangle$. Let a computation μ_{p_7} of the out-port p_7, depicted as a sequence of maximally parallelizable transitions, be $\langle\{t_1, t_2\}, (\{t_3, t_6\})^m, \{t_3, t_5\}, \{t_3\}^n, \{t_4\}, \{t_7\}\rangle$;* the computation, however, cannot be obtained as a sequence of concatenation of paths from the set $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5\}$. *Instead, suppose the cut-points are $p_1, p_2, p_3, p_4, p_5, p_6$ and p_7; the corresponding paths are also depicted in Figure 3(a) using inverted dotted triangular boxes, such as $\alpha_7 = \langle\{t_7\}\rangle$. Now, μ_{p_7} can be represented as* $(\alpha_1 \parallel \alpha_2).(\alpha_3 \parallel \alpha_4)^m.(\alpha_3 \parallel \alpha_6).(\alpha_3)^n.\alpha_5.\alpha_7$. ■

If a path starts from the set of in-port cut-points, then the computation of the path starts with an initial marking M_0. Example 1 underlines the need for introducing further cut-points and the notion of parallel paths and their concatenation for capturing computations. For the former, a notion of *token tracking execution* is necessary which essentially captures all computations of the model with the token values abstracted out and every loop traversed exactly once. Thus, a token tracking execution starts with an initial marking comprising tokens at the in-ports and tracks the progress of the tokens through the successor markings avoiding repetitions of subsequences of markings. If a given marking involves a token holding place with more than one outgoing transition, then firing of such transitions will be mutually exclusive of each other; hence there may be more than one alternative set of successor markings all of which are covered in a DFS manner by the token tracking execution mechanism. Note that the number of times a loop is executed varies from one execution to another depending upon the input. Hence if a given marking involves more than one place with at least one place having a back edge leading to itself, then the execution falls under *a degenerate case* whereupon dynamic cut-points have to be introduced exhaustively in all the places of the markings as captured by the following definition.

Definition 10 (Dynamic Cut-point)**.** *A place is designated as a dynamic cut-point if during a token tracking execution of the model (with static cut-points already incorporated), it occurs in a marking containing at least one cut-point. If the token tracking execution falls under the degenerate case, all the places occurring in the subsequent markings are also marked as dynamic cut-points until a marking is reached with a single post-transition.*

The designation of dynamic cut-points is described through Example 3. The definition of path (Definition 9) is modified with "cut-points" read as both static and dynamic cut-points.

5.1 Path Construction Algorithm

We now describe the dynamic cut-point designation and the path construction algorithm through Example 3 where all the intricacies are covered. The pseudo code of the path construction algorithm is given below. The functional modules along with complexity and correctness analysis are given in (Bandyopadhyay,) where, it is also shown that the complexity of the path construction algorithm is $O((\frac{|T|}{|P|})^{|P|}.(|T|^2))$ which is further shown to reduce to $O(|T|^2)$, where $|T|$ is the cardinality of the set T of transitions. Towards correctness of the algorithm, it is shown that all functional modules terminate, paths constructed by this algorithm satisfy the properties of paths (Definition 9) and the paths constructed by this algorithm give a path cover. *It is to be noted that designation of dynamic cut-points and the path construction procedure goes in hand in hand.*

```
1. Input: A PRES+ model $N$; Output: Set $Q$ of all paths.
2. $M_h \Leftarrow inP$, $Q \Leftarrow \emptyset$, $T_{sh} \Leftarrow \langle\rangle$; /* $M_h$ : marking at hand − initialized to in-ports; set $Q$ of all paths − initially empty; $T_{sh}$ : transition sequence at hand − initially empty.*/
3. $\mathcal{T}$ = ComputeAllSetsOfConcurrentTransitions ($M_h, N$); /* it takes $M_h$ and forms all possible sets of concurrent transitions that are bound to $M_h$ */
4. $\forall T \in \mathcal{T}$
    • $Q \Leftarrow Q \bigcup$ obtainAllthePaths ($T_{sh}, M_h, T, N$); /* obtainAllthePaths appends the set $T$ of enabled transitions to the transition sequence $T_{sh}$ at hand. Token tracking execution and designation of dynamic cut-points is carried out by this module. For each cut-point, it calls the recursive function constructOnePath which constructs the path from a single cut-point to the set of cut-points by backward cone of foci method (using $T_{sh}$) − proceed beyond $T$ (recursively) */
5. Return $Q$;
```

Example 3. *Consider the model given in Figure 3(a). The token tracking execution starts with the initial marking $\{p_1, p_2\}$. After firing of t_1 and t_2, marking becomes $\{p_3, p_4\}$ with a back edge leading to p_3 and another leading to p_4. As p_3 and p_4 are cut-points, the paths are $\langle\{t_1\}\rangle$ and $\langle\{t_2\}\rangle$ which are constructed using backward cone of foci method. Since, the cardinality of the current marking $\{p_3, p_4\}$ is greater than one, the situation falls under degenerate case. Both p_3 and p_4 have two out-transitions each, i.e., $\{t_3, t_4\}$ and $\{t_5, t_6\}$. Therefore, four alternative sets of enabled transitions are obtained from the given marking, namely, $\{t_3, t_5\}, \{t_3, t_6\}, \{t_4, t_5\}$ and $\{t_4, t_6\}$. These four alternatives are explored in a DFS manner. For the set $\{t_3, t_5\}$, the successor marking becomes $\{p_3, p_6\}$ indicating a loop since the marking $\{p_3, p_6\}$ is repeated. Hence this DFS branch is not pursued. As the marking $\{p_3, p_6\}$ do not have a single post-transition, p_6 is designated as a dynamic cut-point. Therefore, the paths are $\langle\{t_3\}\rangle$ and $\langle\{t_5\}\rangle$. Similarly, when the set $\{t_4, t_6\}$ is processed, the place p_5 is designated as a dynamic cut-point. The paths are $\langle\{t_4\}\rangle$ and $\langle\{t_6\}\rangle$. For the set $\{t_4, t_5\}$, the successor marking becomes $\{p_5, p_6\}$. As $p_5^\circ = p_6^\circ = t_7$, at this point the token tracking execution ceases to exist in the degenerate case. Finally, $\{p_7\}$ is reached after firing of the* *transition t_7. The path is $\langle\{t_7\}\rangle$. Therefore, the set of dynamic cut-points is computed as $\{p_5, p_5\}$ and the set of paths is $\langle\{t_1\}\rangle$, $\langle\{t_2\}\rangle$, $\langle\{t_3\}\rangle$, $\langle\{t_4\}\rangle$, $\langle\{t_5\}\rangle$, $\langle\{t_6\}\rangle$ and $\langle\{t_7\}\rangle$.* ■

5.2 Characterization of a Path

We associate with a path α two entities namely, R_α, the condition of execution of the path α, and r_α, the data transformation along the path α. For any computation μ_α of the form $\langle T_1, T_2, \dots\rangle$ of the path α with $^\circ T_1 \subseteq P_{M_1}$, say, the condition R_α depicts the condition that must be satisfied by $\overline{val_{M_1}}(^\circ\alpha)$ so that α is executed. The data transformation r_α depicts the token value obtained in α° after computation μ_α. Thus, the places in α° contain the value $r_\alpha(\overline{val_{M_1}}(^\circ\alpha))$ after execution of the path α.

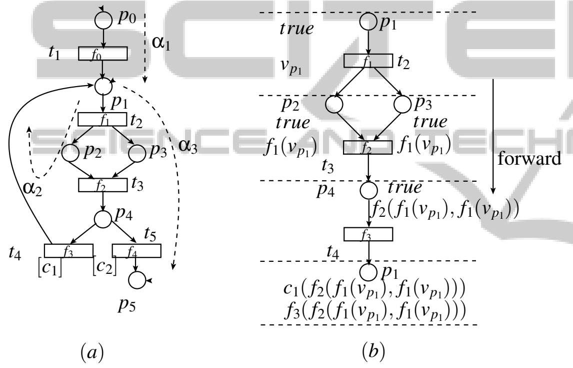


Figure 4: Characterization of a path

Example 4. *Figure 4(a) depicts a PRES+ model having p_0, p_1 and p_5 as cut-points following the cut-point introduction rules given in the previous section. So the corresponding paths are $\alpha_1 = \langle\{t_1\}\rangle$, $\alpha_2 = \langle\{t_2\},\{t_3\},\{t_4\}\rangle$ and $\alpha_3 = \langle\{t_1\},\{t_3\},\{t_5\}\rangle$, respectively. Figure 4(b) depicts how the data transformation (r_{α_2}) and the condition of execution (R_{α_2}) for the path α_2 are computed. A forward traversal along the forward direction of the edges of the path α_2 from p_1 to p_1 is used for this purpose. We may use backward traversal (along the edges in the reverse direction) also as an alternative. In Figure 4(b), let the token value at p_1 be v_{p_1} and the condition be true. The token value at both p_2 and p_3 after t_1 fires is $v_{p_2} = v_{p_3} = f_1(v_{p_1})$ and the condition is true since t_1 fires unconditionally. After firing of t_3, the token value at p_4 becomes $v_{p_4} = f_2(v_{p_2}, v_{p_2}) = f_2(f_1(v_{p_1}), (f_1(v_{p_1}))$ and condition still remains true. When the condition c_1 associated with the transition t_4 is satisfied by v_{p_4}, t_4 fires. After firing of t_4, the token value at p_1 becomes $v_{p_1} = f_3(v_{p_4}) = f_3(f_2(f_1(v_{p_1})), f_2(f_1(v_{p_1})))$ which is identical to the data transformation r_{α_2} and the condition of execution R_{α_2} is $c_1(f_2(f_1(v_{p_1})), f_2(f_1(v_{p_1})))$.* ■

The following theorem captures the uniqueness of the set of paths obtained from the set of (static and dynamic) cut-points.

Theorem 1. *For any PRES+ model N, for the set of cut-points obtained by Definition 8 and the token tracking execution of N, the set of paths covering all the transitions is unique.*

The proof sketch of the theorem is given bellow and the detail proof is given in (Bandyopadhyay,). Let there be two distinct sets Q_1 and Q_2 of paths where each of the sets covers all the transitions of the given PRES+ model N. Let $\alpha = \langle T_1, T_2, \dots, T_n\rangle$ be a path such that $\alpha \in Q_1 - Q_2$. We argue that any member T_i of α, $1 \le i \le n$, represents the only way to group the transitions of T_i into a maximally parallelizable set and hence conclude that α must be in Q_2 as well. We prove it by induction on i.

Similar to the parallelizable transitions, we can also define parallelizable paths in the same manner. The notion of the concatenated path is captured by the following definition.

Definition 11 (Concatenated Path)**.** *A path α is said to be a concatenated path obtained by concatenation of a path α' to a set $Q_P = \{\alpha_1, \cdots, \alpha_k\}$ of parallelizable paths if $\alpha_i^\circ \cap {}^\circ\alpha' \neq \emptyset$, $1 \le i \le k$. The path α is denoted as $(\alpha_1 \parallel \cdots \parallel \alpha_k).\alpha'$, where . stands for concatenation operation. The intermediary cut-points $(\bigcup_{1 \le i \le k} \alpha_i^\circ) \cap {}^\circ\alpha'$ lose their cut-point designation so that the concatenated path α does not have any intermediary cut-points.*

Note that a concatenated path follows the properties of a path. The characterization of a concatenated path is given in (Bandyopadhyay,).

Definition 12 (Path Cover)**.** *A finite set of paths $\Pi = \{\alpha_0, \alpha_1, \cdots, \alpha_k\}$ is said to be a path cover of a PRES+ model N if any computation μ of any out-port of N can be represented as a sequence of concatenations of parallelizable paths from Π.*

In Figure 3 of Example 1, it is noted that the set $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5\}$ of paths which are obtained only from the static cut-points is not a path cover. Whereas, the set $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7\}$ of paths which are obtained from both static and dynamic cut-points is a path cover.

Theorem 2. *Let C be a set of cut-points obtained by Definition 8 and a token tracking execution of a PRES+ model N. The set of paths corresponding to the set C is a path cover of N.*

The proof sketch of the theorem is given bellow and detail is in (Bandyopadhyay,). Let μ_p be a computation of an out-port p of the form $\langle T_1, T_2, \dots, T_l\rangle$ where, $^\circ T_1 \subseteq inP$, $p \in T_l^\circ$, $T_i^\circ \subseteq P_{M_i}$, $1 \le i < l$, where M_i is a marking and $M_{i+1} = M_i^+$, the successor marking of M_i, for all i, $1 \leq i < l$. The sequence μ_p can be represented as the sequence $\langle T_1, \ldots, T_{i_1}, T_{i_1+1}, \ldots, T_{i_2}, \ldots, T_{i_m}, \ldots, T_l \rangle$, where $T_{i_j}^\circ$, $1 \leq j \leq m$ and T_l° are all members of C (cut-points) and there are no other transitions in the above sequence whose output places are members of C.

6 VERIFICATION METHOD

In this section, we describe the formalism of equivalence checking procedure between two out-port cardinality equivalent PRES+ models.

Definition 13 (Path Equivalence)**.** *Let N_0 and N_1 be two out-port cardinality-equivalent PRES+ models. A path α of N_0 is said to be computationally equivalent to a path β of N_1, denoted as $\alpha \simeq \beta$, if their data transformation functions are same and their conditions of execution are equivalent, i.e., $r_\alpha = r_\beta$ and $R_\alpha \equiv R_\beta$.*

Definition 14 (Corresponding Transitions)**.** *Let $N_0 = \langle P_0, T_0, I_0, O_0, inP_0, outP_0 \rangle$ and $N_1 = \langle P_1, T_1, I_1, O_1, inP_1, outP_1 \rangle$ be two out-port cardinality-equivalent PRES+ models having the in-port relation $f_{in} \subseteq inP_0 \times inP_1$ and the out-port bijection $f_{out} : outP_0 \leftrightarrow outP_1$. A transition t of N_0 corresponds to a transition $t^{'}$ of N_1, if*

1. *$t^\circ \in outP_0 \Rightarrow (t')^\circ \in outP_1$ and $f_{out}(t^\circ) = (t')^\circ$ and*
2. *$\exists \alpha \in \Pi_0, \beta \in \Pi_1$ such that $\alpha \simeq \beta$, $t = last(\alpha)$ and $t' = last(\beta)$ where $last(\alpha)(last(\beta))$ is the last transition of $\alpha(\beta)$. The set of corresponding transitions is denoted as η_t;*

Definition 15 (Corresponding Places)**.** *Two places p of N_0 and p' of N_1 are said to be corresponding if $\langle p, p' \rangle \in f_{in}(p)$ or $p \in t^\circ$ and $p' \in (t')^\circ$, where $\langle t, t' \rangle \in \eta_t$. The set of corresponding places is denoted as η_p.*

The basic step for path-based equivalence checking is to find a path β of N_1 for any path α in a path cover Π_0 of N_0 such that $\alpha \simeq \beta$. For the path α, the mechanism of selecting the subset of *candidate paths* of N_1 for checking equivalence with α is as follows. If there is a place $p \in {}^\circ\alpha$ such that $\langle p, p' \rangle \subseteq f_{in}$ where, $p \in inP_0$ and $p' \in inP_1$, then the paths from p' in N_1 are candidate paths of α. Otherwise, the set η_t of corresponding transitions is used to choose paths of N_1 for examining the equivalence. More specifically, for the latter case, the paths leading to ${}^\circ\alpha$ are identified; let $\alpha^{'}$ be such a path, i.e., $(\alpha^{'})^\circ \in {}^\circ\alpha$; a transition t' of N_1 is found such that $\langle \text{last}(\alpha^{'}), t' \rangle \in \eta_t$. Let T be the set of all such transitions (t') of N_1. Any path β satisfying the set equality $T^\circ = {}^\circ\beta$ will be a candidate for checking equivalence with α.

Theorem 3. *A PRES+ model N_0 is contained in another PRES+ model N_1, denoted as $N_0 \sqsubseteq N_1$, if there exists a finite path cover $\Pi_0 = \{\alpha_{0,0}, \alpha_{0,1}, \cdots, \alpha_{0,l}\}$ of N_0 for which there exists a set $\Pi_1 = \{\alpha_{1,0}, \alpha_{1,1}, \cdots, \alpha_{1,l}\}$ of paths of N_1 such that $\alpha_{0,i} \simeq \alpha_{1,i}$ and the places in ${}^\circ\alpha_{0,i}$ have correspondence with these ${}^\circ\alpha_{1,i}$, $0 \leq i \leq l$.*

The proof of the theorem is soundness of equivalence checking algorithm.

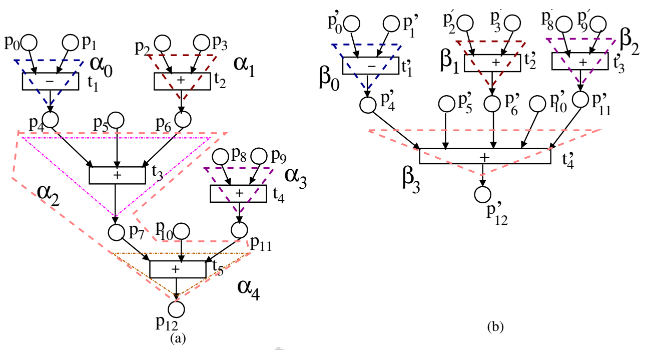
6.1 Broad Outline of the Equivalence Checking Algorithm

The basic method for checking equivalence of two PRES+ models consists of the following steps:

1. Introduce static and dynamic cutpoints and hence construct the paths of N_0 and N_1.
2. Construct the initial path covers Π_0 of N_0 and Π_1 of N_1, comprising paths from a set of cutpoints to another cutpoint without having any intermediate cutpoint. Let $\Pi_0 = \{\alpha_{0,0}, \alpha_{0,1}, \cdots, \alpha_{0,k}\}$ and $\Pi_1 = \{\beta_{0,0}, \beta_{0,1}, \cdots, \beta_{0,l}\}$.
3. Show that $\forall \alpha_{0,i} \in \Pi_0$, there exists a path $\beta_{1,j}$ of N_1 such that $\alpha_{0,i} \simeq \beta_{1,j}$.
4. Let $\Pi_1^E \subseteq \Pi_1$ be the paths of N_1 which have already been found to be equivalent to some paths in N_0. $\forall \beta_{1,k} \in \Pi_1 - \Pi_1^E$, find its equivalent path of N_0.

Step 3 may fail because of code motion transformations where the code segments move beyond the basic block boundaries. In this situation, some paths $\alpha_{0,i} \in \Pi_0$ have no equivalent paths in N_1. In such a case either $\alpha_{0,i}$ or one of its candidate paths is to be extended till equivalence of the resulting concatenated path(s) are obtained. The idea of path extension is similar to that of path based FSMD equivalence checking mechanism (Banerjee et al., 2014). Intricacies, however, arise due to presence of paths parallel to the path being extended. The mechanism is illustrated through Example 5 (rather than presenting a formal algorithm). All the functional modules as well as complexity and correctness of the equivalence checking algorithm are presented in (Bandyopadhyay,).

Example 5. *Fig 5(a) depicts the PRES+ model N_0 for some program. Let the code corresponding to the transition t_4 of N_0 be moved ahead of t_3 and the transitions t_3 and t_5 be combined into a single transition $t_4^{'}$ resulting in a program whose model N_1 is depicted in Fig 5(b). In Fig 5(a), the initial path cover*



p_0 p_1 α_0 t_1
p_2 p_3 α_1 t_2
p_4 p_5 p_6 t_3
p_8 p_9 α_3 t_4
α_2
p_7 p_{10} p_{11} t_5 α_4
p_{12}
(a)
p'_0 p'_1 β_0 t'_1
p'_2 p'_3 β_1 t'_2
p'_8 p'_9 β_2 t'_3
p'_4 p'_5 p'_6 p'_{10} p'_{11}
β_3 t'_4
p'_{12}
(b)

Figure 5: Illustrative example on Verification Algorithm.

Π'_0 of N_0 is $\{\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4\}$ and in Fig 5(b), the initial path cover Π'_1 of N_1 is $\{\beta_0, \beta_1, \beta_2, \beta_3\}$. (Note that the computation $\mu_{0,p_{12}}$ can be represented as $(((\alpha_0 || \alpha_1).\alpha_2) || \alpha_3).\alpha_4$. Similarly, $\mu_{1,p'_{11}}$ can be represented as $(\beta_0 || \beta_1 || \beta_2).\beta_3$)). Establishing $\alpha_0 \simeq \beta_0$ and $\alpha_1 \simeq \beta_1$ is straight forward. For α_2, the path β_3 is the candidate path because β_3 is the only path some of whose pre-places have correspondence with (all) the pre-places of α_2. However, as $|{}^{\circ}\alpha_2| = 3 < |{}^{\circ}\beta_3| = 5$, an extension of α_2 is required through all its post-paths. The path α_4 is the only post path of α_2. The pre-paths of α_4 contain α_3, in addition to α_2 (which is being extended). The path α_3 is equivalent to β_2, which is also a pre-path of α_2's candidate path β_3. So, the path α_2 is extended through the path α_4 to obtain the path $\alpha_2.\alpha_4$ which is found to be equivalent with β_3. Since all the paths of N_0 or their extensions have some equivalent paths in N_1, and vice versa, the models N_0 and N_1 are asserted to be equivalent. ■

7 RESULTS

The path construction and the equivalence checking procedures have been implemented in *C* on a 3.0-GHz Intel(R) Core(TM)2 Duo CPU machine with 2-GB RAM and satisfactorily tested on both sequential (Gupta et al., 2003) and parallel benchmarks. The translation is carried out by one HLS (high level synthesis) compiler, i.e., SPARK (Gupta et al., 2003) and two thread level parallel compilers PLuTo and Par4All. For checking equivalence between two paths the SMT solver has been used (Z3,). For each of these benchmarks, the original behaviour (in *C*) is fed to all of the above mentioned compilers to obtain the transformed behaviours (again, in *C*). In all the cases, the PRES+ models are obtained from the corresponding *C* code manually. To nullify human errors, each of these models is checked for validity using the CPN tool (Jensen et al., 2007).

Table 1 depicts the equivalence checking times taken by our tool and that of another method (Banerjee et al., 2014). Equivalence checking time always includes path construction time. Note that our method is somewhat faster than the other method as our PRES+ model of computation being a value based one, the costly path extension is needed only in MODN. In (Bandyopadhyay et al., 2012), the untimed PRES+ models for parallel behaviours are converted into FSMD models and then the FSMD equivalence checker (Banerjee et al., 2014) is used. In this method extra space and time are needed for model translation. Although the asymptotic complexity for our method is found to be exponential, for the set of benchmarks it did not hit this bound.

For PLuTo and Par4All, we have applied the following thread level parallelizing transformation techniques: (1) loop-nesting operation, (2) parallelization on affine loop nests, (3) coarse-grained parallelism and (4) data locality.

Table 1: Results for several sequential benchmarks.

Benchmark	Orig		Trans		FSMD EC Time (sec)	Our EC Time (sec)
	Pl	Tr	Pl	Tr	(sec)(Banerjee et al., 2014)	(sec)[our]
MODN	28	21	27	20	0.22931	0.20633
SUMOFDIGITS	11	9	10	9	0.12345	0.07834
PERFECT	19	13	14	10	0.73414	0.61910
GCD	31	27	19	17	0.12767	0.05980
TLC	39	36	28	26	0.75410	0.59801
DCT	25	18	20	10	0.10762	0.04980
LCM	32	28	20	18	0.12867	0.07180
LRU	42	41	40	39	0.52767	0.25980
PRIMEFAC	12	11	11	10	0.92767	0.70980
MINMAX	35	23	34	21	NA	0.07180

Table 2: Results for several parallel benchmarks.

Benchmark	Original PRES+		Transformed PRES+				Eqv Chk Time (sec)	
			PLuTo		Par4All		PLuTO	Par4All
	place	trans	place	trans	place	trans		
BCM	10	6	10	5	10	5	0.0112	0.0110
MINMAX	35	23	30	22	30	21	0.1672	0.1523
LUP	55	53	50	47	55	50	0.3212	0.3111
DEKKER	34	32	30	25	28	27	0.2713	0.2931
PATTERSON	30	28	28	26	29	28	0.1161	0.1159

Table 2 shows the equivalence checking time for the parallelizing compilers PLuTo and Par4All. The FSMD equivalence checking method fails to validate these transformations because thread level parallelism is not supported by FSMD models. In course of this experiment our equivalence checker has identified a bug of the PLuTo compiler (Bandyopadhyay,) (possibly due to faulty usage of a variable name in the source program).

8 RELATED WORKS

Translation validation, whereby each individual translation is followed by a validation phase to establish the behavioural equivalence of the source code and the target code, was introduced by Pnueli et al. in (Pnueli et al., 1998) and were demonstrated by Necula in (Necula, 2000) and Rinard et al. (Rinard and Diniz, 1999). This method is further enhanced by Kundu et al. (Kundu et al., 2008) to verify the high-level synthesis tool SPARK capturing parallel execution of statements and Vafeiadis et al. (V. Vafeiadis, 2015) to verify $C11$ compiler. A bisimulation method for concurrent programs is reported in Milner et al. (Milner, 1989). A major limitation of these methods (Necula, 2000; Kundu et al., 2008; V. Vafeiadis, 2015; Milner, 1989) is that they can verify only structure preserving transformations and fail for schedulers that alter the control structure of a program. To alleviate this shortcoming, a path based equivalence checker for the FSMD model is proposed for sophisticated uniform and non-uniform code motions and code motions across loops (Karfa et al., 2012; Banerjee et al., 2014). They, however, are presently unable to handle loop swapping transformations and also thread-level parallelizing transformations mainly because FSMDs, being a sequential model of computation, cannot capture parallel behaviours straightway; modeling concurrent behaviours via CDFGs is significantly more complex due to all possible interleavings of the parallel operations. In (S Bandyopadhyay, 2015; Bandyopadhyay et al., 2015), some issues for translation validation of concurrent programs are addressed.

9 CONCLUSION

A formal notion of computation of an untimed PRES+ model is presented. The concept of finite paths capturing computations on the PRES+ model has been incorporated based on a notion of dynamic cut-points. The path construction and path based equivalence checking methods are described through two examples. The implementation is satisfactorily tested on a set of fifteen benchmark problems encompassing the various speculative and non speculative code optimization techniques (Karfa et al., 2012) as well as thread level loop parallelizing transformations for scalar programs.

The *limitations* of the present work is that it cannot handle loop-shifting, software pipelining based transformations as well as several loop transformations for array handling programs. Being a value based model, it captures data-path more vividly. In course of code transformation, only data movement takes place. If we consider only those portions of the model where the exact code transformations have taken place, the present method can be fast and scalable. Hence, scalability analysis for this method is one of our future goals. Enhancing the equivalence checking procedure to encompass the limitations mentioned above seems to be a promising future endeavor; investigating alternate proof based verification techniques, such as (Lengauer, 2011), can be useful in this regard.

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