

# Multi-granularity Modeling of Variable Structured Rocket based on Declarative Language

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**Keywords:** Modelica, Object-orientation, Declarative Modeling, Variable Structured Rocket, Rocket Simulation Library, Multi-granularity.

**Abstract:** The development of the rocket design process depends heavily on the effective and accurate modeling method. This paper elaborates the procedure to establish the variable structured rocket simulation package with the declarative modeling language Modelica. The package includes the rocket library with basic rocket components and rocket products with subsystems and segments. Some of the components like body and measurement take full advantages of the Modelica object-oriented characters, while others like control system and aerodynamics are assembled on the basis of traditional flight dynamics. The multi-granularity library for variable-structured rocket is verified with a case study. The results indicate that the library is successfully applied to the rocket modeling for flight control system design and is capable of rocket design in different phases.

## 1 INTRODUCTION

The object-oriented modeling language, Modelica, is widely used in large scaled multi-domains simulation. Hilding Elmqvist (1992) proposed its main opinion firstly, which developed along with Differential Algebraic Equations (DAE). Modelica is now mostly used in mechanics design (Ferretti and Vigan, 2005) and aviation (Moormann and Looye, 2002) areas, but few researches can be seen in rocket flight simulation.

Object-orientation and non-causality are the main advantages of Modelica over other modeling languages (Alejandro and Perez, 2001). In consideration of two advantages above, Modelica models established through declarative modeling method, which is the process constructing the system out of assemblies in the way same as the topological structure, can always embody the multi-granularity of the physical system. Connectors and assemblies in the Modelica library can accomplish the declarative modeling, which reflect physical connection logics and similar hierarchical structure with the practical system.

Nowadays, researches about Modelica in rocket modeling areas still concerned about the flow direction and the corresponded libraries are

constructed under the subjects other than objects (Zhang et al., 2010), which conceal the superiority over traditional methods and lower the utilization of the standard Modelica library. Some of the researches used the six degree of freedoms model (Gertjan, 2008) which took few advantages of multi-body dynamics library (Schiavo et al., 2006; Zhang et al., 2011), others established specific defined environment functions (Tiziano and Marco, 2005) to calculate the gravity and atmosphere parameters. Moreover, most flight vehicle is concerned about the fixed structure with the same amount of variables, yet the variables in multi-stage rockets composed of segments are changed due to separations. It has become a new challenge for multi-body flight vehicle modeling.

For taking full advantages of non-causality and object-orientation characters, this paper proposes the Modelica modeling method in variable structured rocket and aims for establishing a multi-granularity rocket simulation library. Rocket modeling is a progress to build the rocket library as Figure 1. In order to decrease the complexity of the rocket modeling and enhance the universality, the rocket library and products are encapsulated in one package named 'RocketSim'. The simulation results of variable-structured rocket during the powered phase will be presented and discussed finally in the end.

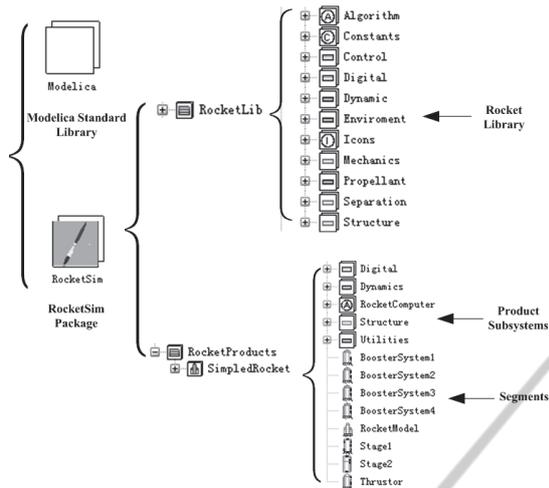


Figure 1: Level structure of the RocketSim package.

## 2 MULTI-GRANULARITY ROCKET LIBRARY

Rocket library includes: control system, dynamic system, mechanics, propellant, separation, digital, environment, algorithm, constants and icons, shown as Figure 2. Algorithm, constants and icons are assistant resources. Control system and digital package belong to electronics. Mechanics package,

structure, separation and dynamics belong to mechanics. Propellant belongs to thermotics.

Control system is composed of GNC packages, phase management block and servo systems. Digital package includes 458 and 1553B buses. Propellant includes the sensor in the tank, thrusters interpolating block and engines. Dynamics mainly contains all the components associated with the aerodynamics computation. Structure contains the main structure parts in rockets. Environment, separation and mechanics are explained in details in the following sections.

### 2.1 Environment Package Design

Original world model cannot provide J2 point gravity model and the atmosphere parameters, so it should be modified to gain more accurate environment. The gravity model considering J2 is expressed as

$$\mathbf{G} = mg'_r \mathbf{r}^0 + mg_{\omega e} \boldsymbol{\omega}_e^0 \quad (1)$$

where  $g'_r$  and  $g_{\omega e}$  are

$$\begin{cases} g'_r = -\frac{\mu}{r^2} \left( 1 + J_2 \left( \frac{a_e}{r} \right)^2 (1 - 5 \sin^2 \phi) \right) \\ g_{\omega e} = -2 \frac{\mu}{r^2} J_2 \left( \frac{a_e}{r} \right)^2 \sin \phi \end{cases}, \quad \sin^2 \phi = \left( \frac{r_y}{r} \right)^2 \quad (2)$$

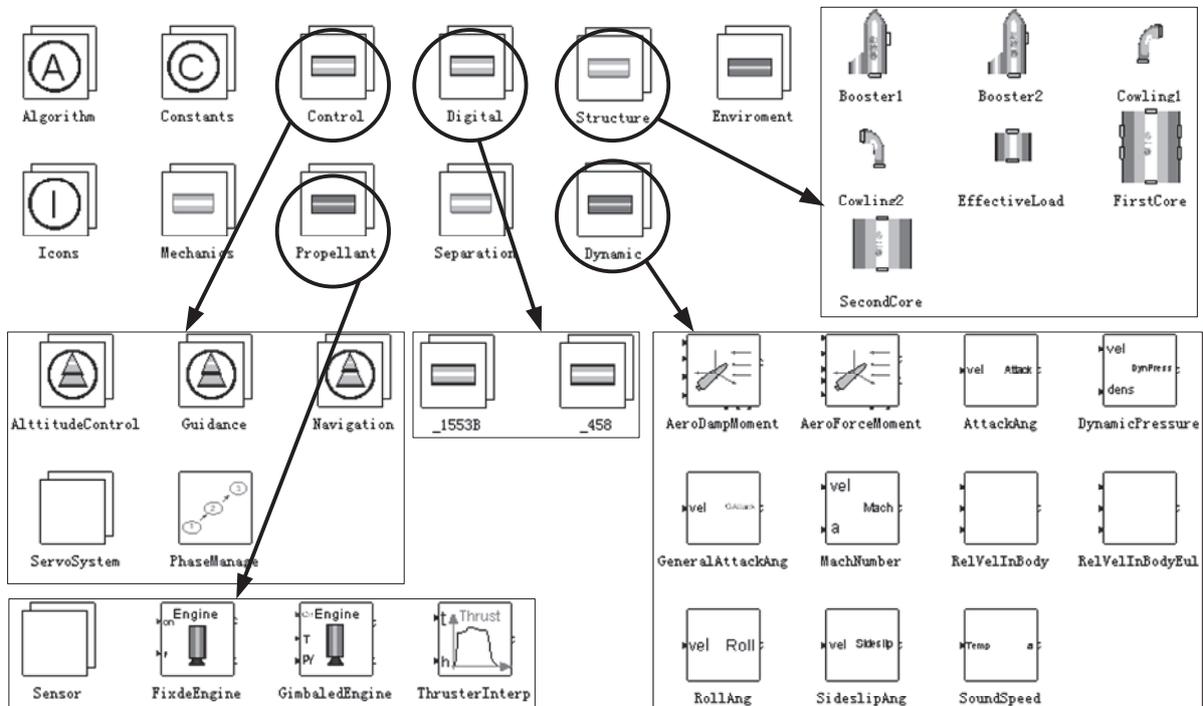


Figure 2: RocketLib components.

Models described above are applied in the world model and the J2 point gravity is defined as a new gravity type in the type class.

## 2.2 Separation Package

Separation devices shown in Figure 3 are the critical assemblies in variable structured multi-body dynamics; they can be classified according to the constraints modes. Separation package contains several common devices like separation bolts, separation guide rail and separation rocket, etc.

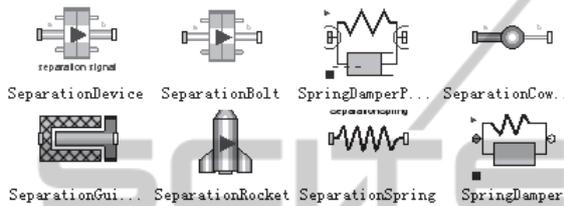


Figure 3: Mechanics package components.

The separation process is accomplished by switching constrained equations in mechanic frames. Due to the solving mechanism, the number of variables must be equal to that of equations, which means the quantity of equations in each judgment branch should be the same. Modelica codes are listed as:

```

r_rel = frame_b.r_0 - frame_a.r_0;
v_rel = der(r_rel);
angle = Frames.axesRotationsAngles(
  Frames.relativeRotation(
    frame_a.R, frame_b.R), {1,2,3}
);
w_rel = Frames.angularVelocity1(
  Frames.relativeRotation(
    frame_a.R, frame_b.R));
if separationsignal ==
  SeparationSignal then
  zeros(3) = frame_b.f;
  zeros(3) = frame_b.t;
else
  zeros(3) = der(v_rel);
  zeros(3) = der(w_rel);
end if;
zeros(3) = frame_a.f +
  Frames.resolveRelative(
    frame_b.f, frame_b.R,
    frame_a.R);
zeros(3) = frame_a.t +
  Frames.resolveRelative(
    frame_b.t, frame_b.R,
    frame_a.R) + cross(
  Frames.resolve2(frame_a.R, r_rel),
  Frames.resolveRelative(frame_b.f,
    frame_b.R, frame_a.R));

```

## 2.3 Mechanics Package

Mechanics library shown as Figure 4 is the directive manifestation of non-causality in rocket modeling. Mechanics package includes measurements, body, body shape, fixed rotation and mass interpolating block.

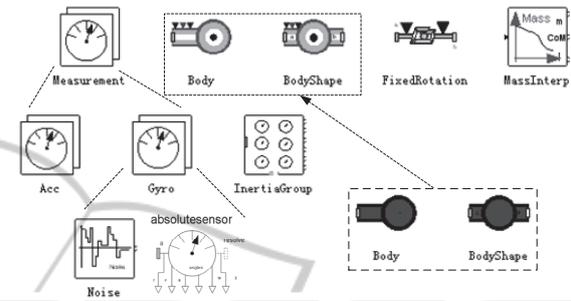


Figure 4: Mechanics package components.

All the mechanics components convey the physical quantities through the frames of the connectors. The body and body shape are derived from original body and body shape, whose centre of mass, inertia and mass become the input variables. Measurement is the package including accelerometer and gyroscope, which are the absolute sensors assembled with the measurement noise generator. The random seed and uniform distribution random generation functions are acquired by loading the external C functions through the head file "STDLIB.H" in the specified path.

## 3 ROCKET PRODUCT ASSEMBLING

### 3.1 Rocket Modeling and Simulation Flow

Before the whole rocket product is assembled, some of the segments and subsystems are required to be constructed from components in the rocket library and Modelica standard library. When the whole product is completed, the bottom platform will automatically determine the equations and values of the source codes. Then, the code generator will sort and trim the equations and translate the Modelica codes into executable codes. The flow from assembling to simulation is shown in Figure 5.

Most segments can be assembled directly with the rocket library, but aerodynamics and rocket computer still need further packaging. The details are elaborated below.

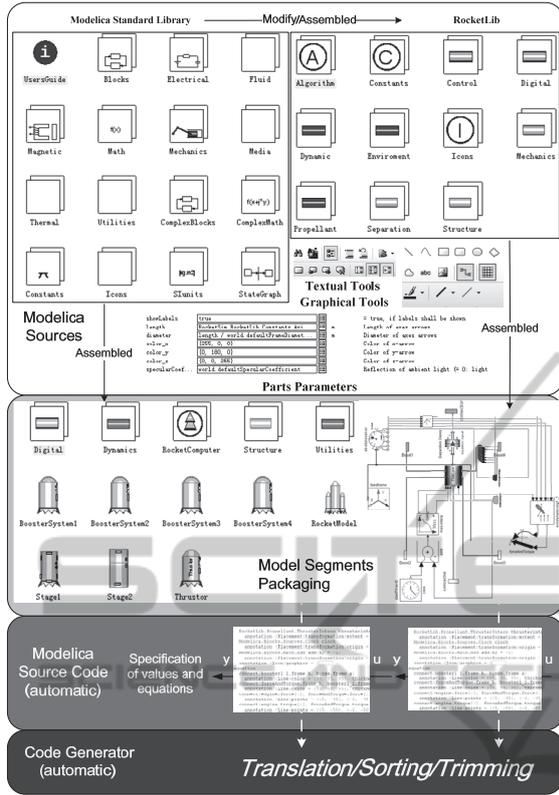


Figure 5: Modeling and simulation flow.

## 3.2 Main Subsystems Affirmation

### 3.2.1 Aerodynamics Subsystem

Aerodynamics force resolved in the body coordinate is

$$\mathbf{R}_b = \begin{bmatrix} R_x \\ R_y \\ R_z \end{bmatrix} = qS \begin{bmatrix} C_x \\ C_y \\ C_z \end{bmatrix} \quad (3)$$

where dynamic pressure  $q = \rho v^2 / 2$ ,  $\rho$  is the atmospheric density,  $S$  is the characterized area, and  $C_x$ ,  $C_y$ ,  $C_z$  are the air resistance coefficient, elevating force coefficient and lateral force coefficient. The coefficients are expressed as

$$\begin{cases} C_x = C_{x0} + C_x^{\alpha^2} + C_x^{\beta^2} \\ C_y = C_{y0} + C_y^{\alpha} \alpha + C_y^{\delta_z} \delta_z \\ C_z = C_z^{\beta} \beta + C_z^{\delta_y} \delta_y \end{cases} \quad (4)$$

where  $C_{x0}$ ,  $C_x^{\alpha^2}$  and  $C_x^{\beta^2}$  are resistance coefficients from unsymmetrical structure, unit attack angle and slide angle;  $C_{y0}$ ,  $C_y^{\alpha}$  and  $C_y^{\delta_z}$  are elevating

coefficients from unsymmetrical structure, unit attack angle and angle of rudder reflection;  $C_z^{\beta}$  and  $C_z^{\delta_y}$  are lateral force coefficients from unit slide angle and angle of rudder reflection;  $\delta_x$ ,  $\delta_y$  and  $\delta_z$  are actual angle of rudder reflection.

Aerodynamic moment resolved in body coordinate is

$$\mathbf{M}_b = \begin{bmatrix} M_{bx} \\ M_{by} \\ M_{bz} \end{bmatrix} = qSL \begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix} \quad (5)$$

where  $L$  is rocket characterized length;  $m_x$ ,  $m_y$  and  $m_z$  are moment coefficients, expressed as

$$\begin{cases} m_x = m_{x0} + m_x^{\beta} \beta + m_x^{\delta_x} \delta_x + m_x^{\delta_y} \delta_y + m_x^{\omega_x} \omega_x + m_x^{\omega_y} \omega_y \\ m_y = m_y^{\beta} \beta + m_y^{\delta_y} \delta_y + m_y^{\omega_y} \omega_y + m_y^{\omega_x} \omega_x \\ m_z = m_{z0} + m_z^{\alpha} \alpha + m_z^{\delta_z} \delta_z + m_z^{\omega_z} \omega_z \end{cases} \quad (6)$$

where  $m_{x0}$  and  $m_{z0}$  are coefficients caused by production errors;  $m_x^{\beta}$  and  $m_y^{\beta}$  are coefficients

caused by unit slide angle;  $m_z^{\alpha}$  is the moment coefficient from unit angle of attack;  $m_x^{\delta_y}$ ,  $m_x^{\delta_x}$ ,  $m_y^{\delta_y}$  and  $m_z^{\delta_z}$  are resistance coefficients from angle of velocity; the uniform unit angle of velocities  $\bar{\omega}_i = \omega_i \frac{L}{2v}$  ( $i = x, y, z$ ), where the velocity

$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$ . Angle of attack and slide are given by  $\alpha = \arctan\left(\frac{v_y}{v_x}\right)$ ,  $\beta = \arctan\left(\frac{v_z}{v_x}\right)$ .

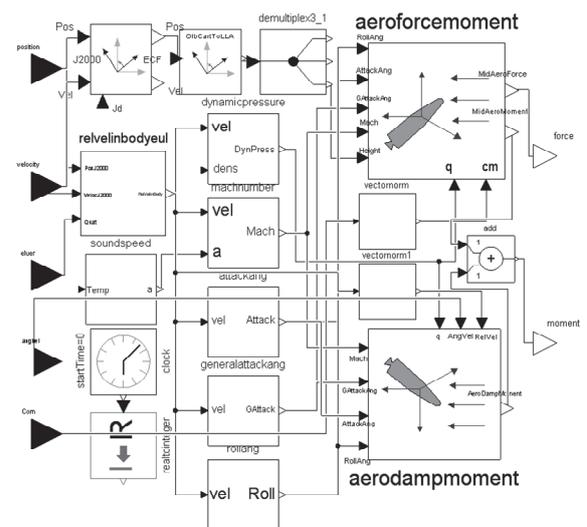


Figure 6: Aerodynamics block.

According to Eq. (3) to Eq. (6), the aerodynamics model is established in Figure 6, where the general angle of attack is the equivalent of the slide angle, and the coefficients above are obtained by interpolating.

### 3.2.2 Rocket Computer

Rocket computer is composed of phase management block, guidance and control blocks in first phase and second phase. The input signal of the phase switch function is obtained from digital buses, which include the measurements from inertia groups in first stage and second stage. The virtual function is designed to choose the correct signals for guidance and control blocks. The control signals are then chosen by the phase switch block.

### 3.3 Rocket Segment Packaging

#### 3.3.1 Booster Segment System

Inner structure and outer ports of booster model is shown as Figure 7. The values interpolated from the experimental data are utilized to replace the real working procedure. The swaying signals and the rocket height are obtained from bus 458, while the signal of starting up and separation is conveyed through 1553B.

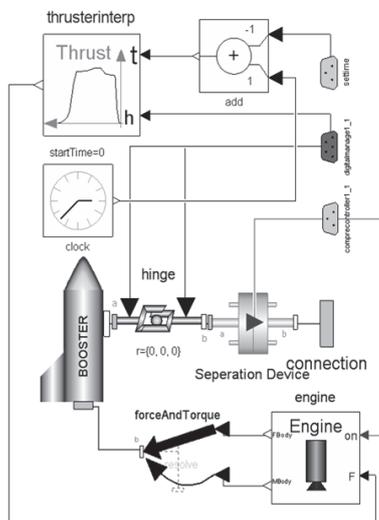


Figure 7: Booster system.

#### 3.3.2 Systems in the First and Second Cores

The mass and inertia of the rocket are focus in the core segments and mainly depend on the two interpolation blocks. When the first stage separation signal in 1553B is changed into high level and the

thrusters in the second core start up, the interpolating blocks in second stage begin to work and the rocket computer begins to receive the measurements from the second inertia group. The control signals are transmitted from the computer rocket to first stage system through the bus and transformed into the boosters' swaying angles by the synthetic controller. Furthermore, the outer equivalent aerodynamics is applied on first and second stages. The inner structures of two-stage systems are shown as Figure 8 and Figure 9.

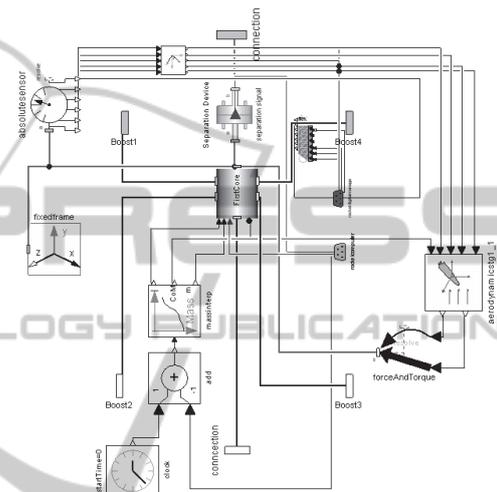


Figure 8: Inner structure in the first core.

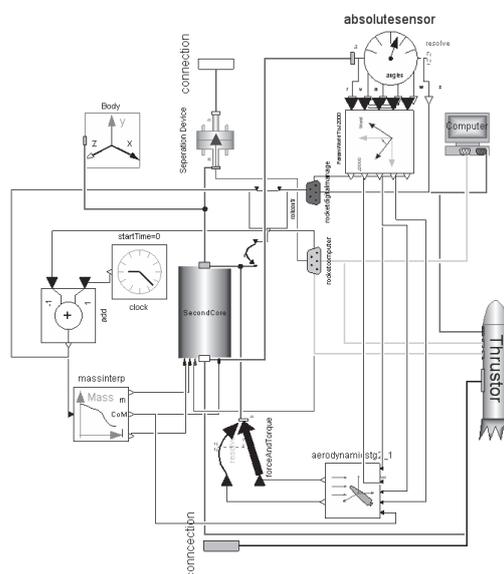


Figure 9: Inner structure in the second core.

#### 3.3.3 Rocket Product

The rocket product is established of seven segments

including a first stage, a second stage, an effective load and four boosters. The segments' physical structures are connected with mechanics connectors and the data transmitting is carried through 1553B and 458 digital buses. The whole structure of the rocket product is shown in Figure 10.

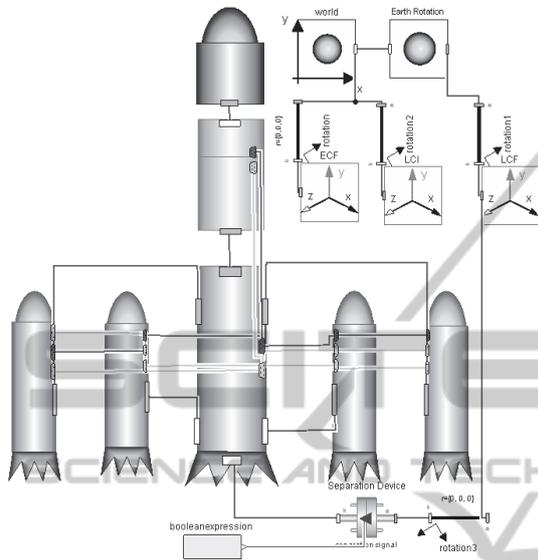


Figure 10: Top-level of simplified two-stage rocket.

#### 4 SIMULATION RESULTS AND ANALYSIS

A case study about two-stage rocket flight simulation in powered phase is described in this

section. The parameters setting blocks including rocket computer, body and interpolating blocks about the mass, aerodynamics and thrusters. The thrusters interpolating block and rocket computer of the rocket model are setting as Table 1.

Most animation choices are set as false, causing the quantities of variable decrease from 14000 to 9420, thus enhance the effectiveness of the simulation. Choose flight states at 30s, 60s, 90s, 120s as the characterized points. The standard and calculated ballistic parameters in the specific points are listed as below.

The rocket motion states and mass history in powered phase are shown in Figure 11 to Figure 14.

The first stage of the flight adapts exhausted cut off, the shutdown signal emerges when fuel level of tank under certain height at 62.73s. The second stage takes the perturbing guidance law, calculating the characterized shutting time accordance with the standard time 120.043s. The parameters of the shutdown point are  $x_k = 146099\text{m}$ ,  $y_k = 117480\text{m}$ ,  $z_k = 27577.1\text{m}$ ;  $v_{xk} = 2916.32\text{m/s}$ ,  $v_{yk} = 1905.97\text{m/s}$ ,  $v_{zk} = 216.970\text{m/s}$ . Compared with the standard parameters, one can conclude that the rocket model satisfies the rocket requirement of precisions. Rocket positions and velocities are continuously changing along with the time. The states in x and y tunnels alter more rapidly, but the position in z tunnel changes smoothly and the velocity remains around 230 m/s. It means the rocket flight is inside the launch plane. The nonzero values of initial velocity are due to the earth rotation relative to the launch inertial reference.

Table 1: Initial properties setting.

Name	Parameters		Parts
	Value	Unit	
fileName	"/Work/RocketSim/RocketProducts/SimplifiedRocket/Resources/Data/Stg1Thrust.dat"	—	booster.thrusterinterp
fileName	"/Work/RocketSim/RocketProducts/SimplifiedRocket/Resources/Data/FirstCoreMass.dat"	—	firstcore.massinterp
fileName	"/Work/RocketSim/RocketProducts/SimplifiedRocket/Resources/Data/SecondCoreMass.dat"	—	secondcore.massinterp
fileName	"/Work/RocketSim/RocketProducts/SimplifiedRocket/Resources/Data/AeroDynData.txt"	—	aerodynamic
iniDate	{2008, 6, 1, 12, 0, 0}	—	computer
launchAzi	53.292170909143	deg	computer
launchLLA	{30, 120, 0}	deg	computer
time_stage1_start	0	s	computer
time_stage1_shut	63.1	s	computer
time_stage1_sep	64.1	s	computer
time_stage2_start	65	s	computer
time_stage2_shut	132	s	computer
time_stage2_sep	133	s	computer

Table 2: Characterized points' simulation and standard results.

Parameters	30s		60s		90s		120s	
	Sim	Std	Sim	Std	Sim	Std	Sim	Std
X/m	12006.3	12008.7	35723.7	35718.8	76774.5	76761.2	145983	145972
Y/m	6668.08	6672.61	29276.9	29288.6	67396.1	67416.0	117404	117431
Z/m	7222.06	7223.91	14362.6	14363.7	21037.0	21023.2	27568.5	27554.9
Vx/(m/s)	535.235	535.094	1118.34	1117.84	1736.49	1736.54	2914.76	2914.86
Vy/(m/s)	449.369	449.174	1120.09	1119.20	1454.17	1453.46	1905.37	1904.70
Vz/(m/s)	241.253	241.210	236.467	236.420	218.595	218.45	216.972	216.896

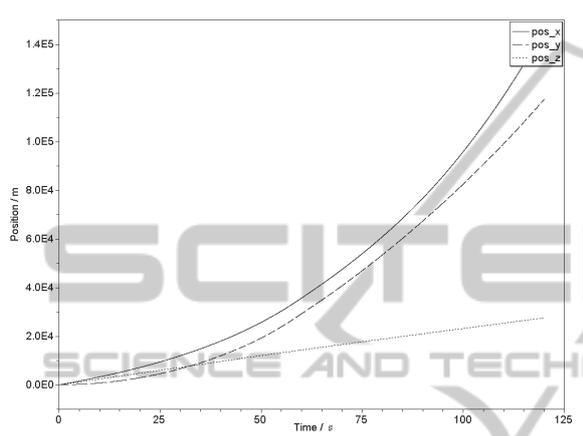


Figure 11: Rocket position in powered phase.

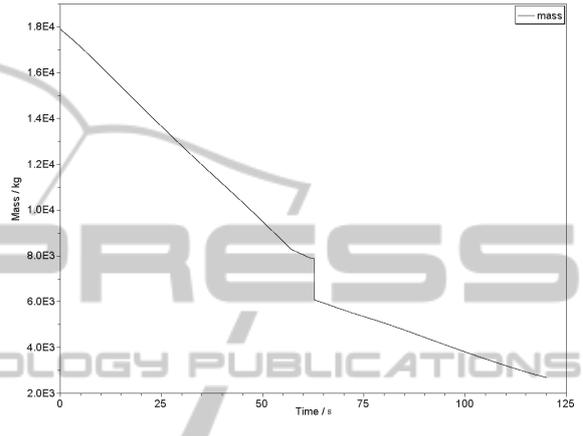


Figure 12: Rocket mass history in powered phase.

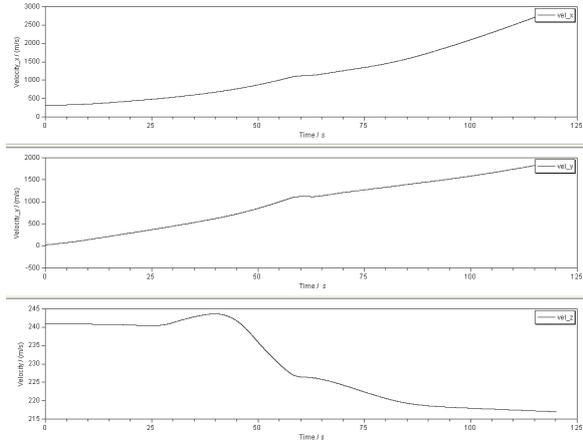


Figure 13: Rocket velocity in powered phase.

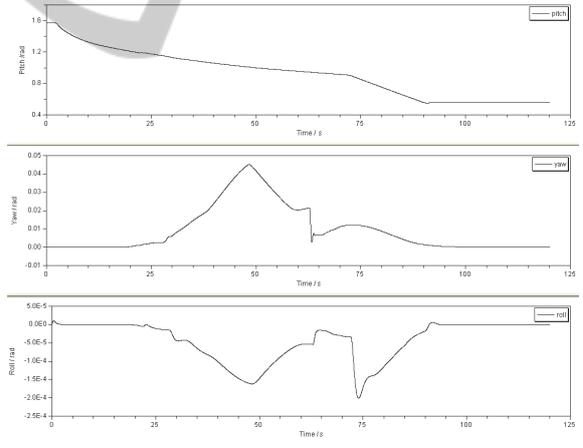


Figure 14: Rocket attitude in powered phase.

The rocket launches vertically, so the initial pitch angle is  $90^\circ$ . The gravity turning begins after 2s, and the rocket flies along the program angles. The slight changes of the pitch, yaw, roll and their angle velocities indicate the attitude stabilization of the rocket.

## 5 CONCLUSIONS

This paper studies the object-oriented and multi-granularity modeling method for the variable structured rocket. The rocket simulation library is composed of a rocket library package and a rocket product package. The rocket library is established by modifying Modelica standard library and packaging

some new assemblies. Then, the product package is constructed according to the topological structure of rockets. The simulation results verify that subsystems assembling the same objects in existence of rockets can be reapplied in different rocket design phases. Furthermore, as the development of techniques and models, the rocket library can complement itself continuously to enhance the modeling ability and applicability.

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