A Fault Detection Scheme for Time-delay Systems using Minimum-order Functional Observers

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Abstract: This paper presents a method for designing residual generators using minimum-order functional observers to detect actuator and component faults in time-delay systems. Existence conditions of the residual generators and functional observers are first derived, and then based on a parametric approach to the solution of a generalized Sylvester matrix equation, we develop systematic procedures for designing minimum-order functional observers to detect faults in the system. The advantages of having minimum-order observers are obvious from the economical and practical points of view as cost saving and simplicity can be achieved, particularly when dealing with high-order complex systems. Extensive numerical examples are given to illustrate the proposed fault detection scheme. In all the numerical examples, we design minimum-order residual generators and functional observers to detect faults in the system.

1 INTRODUCTION

Time-delay systems are commonly encountered in various engineering complex systems. As cited in (Duan and Patton, 2002; Fu et al., 2004; Wu and Duan, 2007) time delays appear in practical processes such as aviation industries, chemical processes, long transmission lines and rolling mill systems. In fact, when time delays appear in high complex systems, they can cause the systems to be more vulnerable to unexpected faults. Faults can enter the systems via input or state delay. Normally, faults can cause malfunctions of the system operations such as partly break down or even whole system shutdown (Teh and Trinh, 2013; Chen and Patton, 2012). Hence, the problems of fault detection have been extensively considered over the last several decades to improve reliability and safety of system performance.

Accounting from the last several decades, there is a wide range of approaches which are based on the foundation of mathematics to build models in order to detect system faults. One of the theories is applying Kalman filter method to generate a residual based on the difference between the ideal output and real output of the system, this method can be seen in (Wang et al., 2006; Zhong et al., 2003). Another trend in this field is using geometric approach which can be seen in (Meskin and Khorasani, 2009). Meanwhile, a very common approach, which is used for fault detection, is observer-based strategy. The basic idea behind the observer-based approach is to estimate the state and the output of the system from the measurements by using some types of state observers, and then construct a residual by a properly weighted output estimation error. The residual is then examined for the likelihood of faults (Duan and Patton, 1998; Duan and Patton, 2001; Huong et al., 2014).

In this paper, the work on reduced-order functional observers for dynamical systems (Darouach et al., 1999; Darouach, 2001; Trinh and Ha, 2007; Fernando et al., 2010; Trinh et al., 2004; Trinh and Fernando, 2012; Fernando and Trinh, 2013) and (Fernando and Trinh, 2014) are used to design a simple and effective scheme to detect faults for time-delay systems. We construct residual generators based on the system outputs and minimum-order functional observers to trigger faults occurring in the systems. The salient feature of our work is that the order of our designed residual generators and observers is very low and hence our fault detection scheme is very practical and easy to implement. In the next section, we present system description and problem statement. This is then followed by preliminaries results and our proposed fault detection scheme for time-delay systems. Finally, three examples in Numerical Examples section to illustrate the proposed theory can be seen.

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2 SYSTEM DESCRIPTION AND PROBLEM STATEMENT

We consider the following time-delay system

$$\begin{cases} \dot{x}(t) = Ax(t) + A_d x(t-\tau) + Bu(t) \\ \qquad\quad + Ef(t), \quad t \geq 0, \\ y(t) = Cx(t), \\ x(t) = \phi(t), \quad t \in [-\tau, 0), \end{cases} \tag{1}$$

where τ is a positive real number presenting the known time delay in the state, $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$ and $y(t) \in \mathbb{R}^p$ are the state, input and output vector, respectively. Unpredictable fault signals $f(t) \in \mathbb{R}^l$ enter from the system input. $A \in \mathbb{R}^{n\times n}$, $A_d \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$ and $E \in \mathbb{R}^{n\times l}$ are known constant matrices. We assume that the pair (A, C) is observable, $\mathrm{rank}(C) = p$, $\mathrm{rank}(E) = l$. Furthermore, the faults $f(t)$ are assumed to be linear independent to avoid vagueness situations which may appear when some faults occur simultaneously. Hence, the residual generator may not detect faults because of zero overall effects of faults.

Let us introduce a functional observer $\omega(t)$ of a general q-order, $1 \leq q \leq n$. Here, $\omega(t)$ is an estimation of a function $Lx(t)$, $L \in \mathbb{R}^{q\times n}$, where

$$\begin{cases} \dot{\omega}(t) = N\omega(t) + Gy(t) + G_d y(t-\tau) \\ \qquad\quad + Hu(t), \quad t \geq 0, \\ \omega(t) = \varphi(t), \quad t \in [-\tau, 0), \end{cases} \tag{2}$$

$\omega(t) \in \mathbb{R}^q$, $N \in \mathbb{R}^{q\times q}$, $G \in \mathbb{R}^{q\times p}$, $G_d \in \mathbb{R}^{q\times p}$ and $H \in \mathbb{R}^{q\times m}$ are constant observer parameters which will be determined such that $\omega(t)$ is an asymptotic estimation of the function $Lx(t)$ when there is no fault appearing in the system, i.e., $f(t) = 0$. Matrix L will be determined for the purpose of fault detection.

Let us consider a residual generator $r(t)$ which is used to trigger faults in system (1) whenever the faults appear, i.e., $f(t) \neq 0$,

$$r(t) = T\omega(t) + Fy(t), \tag{3}$$

where $T \in \mathbb{R}^{1\times q}$ and $F \in \mathbb{R}^{1\times p}$ are constant matrices which will be detemined to satisfy the residual function such that

$$\lim_{t\to\infty} r(t) = \begin{cases} 0 \text{ if } f(t) = 0 \\ c \text{ or undefined if } f(t) \neq 0, \end{cases} \tag{4}$$

where $c \neq 0$, and $f(t) = 0$ implies a faultless condition, $f(t) \neq 0$ implies a faulty condition (Trinh et al., 2013).

3 PRELIMINARIES RESULTS

Obviously, the reduced-order functional observers and the residual generator can detect faults in systems (1) if all the unknown parameters satisfy the following conditions in Theorem 1.

Theorem 1. *Under faultless conditions, $\omega(t)$ is an asymptotic estimate of $Lx(t)$ and residual generator $r(t)$ function as (4) for any initial condition $\phi(t), x(0), \omega(0)$ and any $u(t)$ if and only if*

$$N \text{ is Hurwitz}, \tag{5}$$

$$NL + GC - LA = 0, \tag{6}$$

$$G_d C - LA_d = 0, \tag{7}$$

$$H - LB = 0, \tag{8}$$

$$TL + FC = 0. \tag{9}$$

Under fault conditions, residual $r(t)$ satisfies (4) *if and only if all the parameters satisfy the conditions* (5)-(9) *and*

$$LE \neq 0. \tag{10}$$

Proof. Let us define an error vector $e(t) \in \mathbb{R}^q$ which is the difference between the estimate $\omega(t)$ and the function $Lx(t)$ as follows

$$e(t) = \omega(t) - Lx(t), \quad t \geq -\tau. \tag{11}$$

It follows from (1), (2) and (11), we obtain

$$\begin{aligned} \dot{e}(t) = {} & Ne(t) + (NL + GC - LA)x(t) \\ & + (G_d C - LA_d)x(t-\tau) \\ & + (H - LB)u(t) - LEf(t), \quad t \geq -\tau. \end{aligned} \tag{12}$$

Sufficiency: In the case of faultless conditions, i.e., $f(t) = 0$, if conditions (6)-(8) are satisfied then equation (12) is reduced to $\dot{e}(t) = Ne(t)$. As a result, if condition (5) is satisfied then the error $e(t)$ asymptotically tends to zero. Hence, the reduced-order functional observer $\omega(t)$ is an estimation of the functional $Lx(t)$ as expected. Furthermore, by (1), (3) and (11), residual $r(t)$ can be expressed as follows

$$r(t) = Te(t) + (TL + FC)x(t). \tag{13}$$

It is clear that under no fault condition, the error $e(t)$ is expected to asymptotically tends to zero, thus the residual is proposed to be zero and it happens if condition (9) holds.

Necessity: Under no fault conditions, if condition (5) is unsatisfied then even conditions (6)-(8) hold, the error $e(t) \not\to 0$ with any initial condition of $\phi(t)$ and $x(0)$. Contrarily, if one of the conditions (6)-(8) is not satisfied then even (5) holds, it is possible to find $x(0)$ to make $e(t) \not\to 0$.

Under fault conditions, if conditions (5)-(9) hold, by (12) and (13), the residual is then governed by the following equations

$$\begin{cases} \dot{e}(t) = Ne(t) - LEf(t), \\ r(t) = Te(t). \end{cases} \tag{14}$$

It follows from (14), $r(t)$ can detect faults $f(t)$ if condition (10) is satisfied. This completes the proof of Theorem 1. □

The design of the functional observers and the residual generator is now reduced to finding matrices L, N, G, G_d, H, T and F which satisfy Theorem 1.

4 FAULT DETECTION SCHEMES

This section is to determine the possible order of the observer and the necessary parameters for designing reduced-order functional observers and residual generator to detect faults in system (1). Note that, based on the conditions in Theorem 1, whenever matrix L is found, matrix H is determined from (8) and condition (10) is checked. Moreover, other unknown parameters are solutions of equations (6), (7), (9) and condition (5) holds. In order to simplify the three matrix equations (6), (7) and (9), we employ the partition method introduced in (Trinh and Fernando, 2012).

Let $P \in \mathbb{R}^{n\times n}$ be defined by

$$P = \begin{bmatrix} C^+ & C^\perp \end{bmatrix}, \tag{15}$$

where $C^+ \in \mathbb{R}^{n\times p}$ is the Moore-Penrose inverse of matrix C, $CC^+ = I_p$, and $C^\perp \in \mathbb{R}^{n\times n-p}$ is an orthogonal basis for the null space of matrix C, $CC^\perp = 0$. Then P is an invertible matrix (see, (Trinh and Fernando, 2012)).

Now we define the following partitions

$$CP = \begin{bmatrix} I_p & 0 \end{bmatrix}, \tag{16}$$
$$LP = \begin{bmatrix} L_1 & L_2 \end{bmatrix}, \tag{17}$$
$$P^{-1}AP = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \tag{18}$$
$$P^{-1}A_dP = \begin{bmatrix} A_{d11} & A_{d12} \\ A_{d21} & A_{d22} \end{bmatrix}, \tag{19}$$

where submatrices $L_1 \in \mathbb{R}^{q\times p}$, $L_2 \in \mathbb{R}^{q\times(n-p)}$, $A_{11} \in \mathbb{R}^{p\times p}$, $A_{12} \in \mathbb{R}^{p\times(n-p)}$, $A_{21} \in \mathbb{R}^{(n-p)\times p}$, $A_{22} \in \mathbb{R}^{(n-p)\times(n-p)}$, $A_{d11} \in \mathbb{R}^{p\times p}$, $A_{d12} \in \mathbb{R}^{p\times(n-p)}$, $A_{d21} \in \mathbb{R}^{(n-p)\times p}$ and $A_{d22} \in \mathbb{R}^{(n-p)\times(n-p)}$.

Post-multiply both sides of equations (6), (7) and (9) by matrix P, we have following equations

$$\begin{aligned} NLP + GCP - LPP^{-1}AP &= 0, & (20)\\ G_dCP - LPP^{-1}A_dP &= 0, & (21)\\ TLP + FCP &= 0. & (22) \end{aligned}$$

Substitute equations (15)-(19) into (20)-(22), we obtain

$$\begin{aligned} L_1A_{11} + L_2A_{21} - NL_1 &= G, & (23)\\ L_1A_{d11} + L_2A_{d21} &= G_d, & (24)\\ F + TL_1 &= 0, & (25)\\ NL_2 - L_2A_{22} - L_1A_{12} &= 0, & (26)\\ L_1A_{d12} + L_2A_{d22} &= 0, & (27)\\ TL_2 &= 0. & (28) \end{aligned}$$

It is clear from equations (23), (24) and (25) that matrices G, G_d and F are computed when matrices N, L_1, L_2 and T are found. Consequently, the design of the observers and residual generator is now reduced to determine matrices N, L_1, L_2 and T such that three conditions (26)-(28) are satisfied and condition (5) holds.

4.0.1 Case 1: $p > \frac{2n}{3}$

In this case, we consider the design of only first-order functional observers and residual generators to detect faults in the systems (1). We show that indeed it is possible provided that the number of outputs satisfying $p > \frac{2n}{3}$ and that condition (10) is satisfied.

Theorem 2. *If $p > \frac{2n}{3}$, first-order observers and residual generators always exist to detect faults in the system if the condition* (10) *is satisfied.*

Proof. To design a first-order functional observer, i.e., $q = 1$, $N \in \mathbb{R}^{1\times 1}$ can be chosen to be any negative real number, i.e.,

$$N = s, s < 0. \tag{29}$$

Note that $s < 0$ ensures the satisfaction of the condition (5), i.e., N is Hurwitz. By letting $L_2 = 0$, equation (26) and (27) are reduced to

$$\begin{aligned} L_1A_{12} &= 0, & (30)\\ L_1A_{d12} &= 0. & (31) \end{aligned}$$

We can express (30) and (31) as

$$L_1 \begin{bmatrix} A_{12} & A_{d12} \end{bmatrix} = 0. \tag{32}$$

Since $p > \frac{2n}{3}$, $\begin{bmatrix} A_{12} & A_{d12} \end{bmatrix} \in \mathbb{R}^{p\times 2(n-p)}$ is a column matrix, i.e., $p > 2(n-p)$, thus a solution to (32) where $L_1 \neq 0$ always exists. Let $\mathcal{N}(X)$ be a matrix of row basis vectors for the row-nullspace of X, i.e., $\mathcal{N}(X)X = 0$. The solution to $L_1 \neq 0$ according to (32) exists when $\mathcal{N}(X) \neq 0$. Therefore, solutions for L_1 can be computed by first finding $\hat{L}_1$ according to

$$\hat{L}_1 = \mathcal{N}\begin{bmatrix} A_{12} & A_{d12} \end{bmatrix}, \tag{33}$$

and then choosing any row of $\hat{L}_1$ as L_1.

In (28) and since $L_2 = 0$, $T \neq 0$ can be arbitrarily chosen to be any scalar, say, α where $\alpha \neq 0$. Finally, if the condition (10), $LE \neq 0$, is satisfied, where $L = \begin{bmatrix} L_1 & 0 \end{bmatrix} P^{-1}$ is obtained according to (17), matrices H, G, G_d and F can then be obtained from (8), (23), (24) and (25), respectively. This completes the proof of Theorem 2. □

The effectiveness and the simplicity of this scheme can be seen in Example 1 of the Numerical Examples section, in which a system with $n = 4$, $p = 3$, $m = 1$ and $l = 1$ is taken under consideration. For this system, according to Theorem 2, we only need to design first-order observer and residual generator to detect faults in the system.

4.0.2 Case 2: $1 \leq p \leq \frac{2n}{3}$

In this case, first-order functional observers do not exist and we have to consider observers of higher order. We present a solution to the three matrix equations (26)-(28) with the requirement that N has prescribed distinct eigenvalues. For completeness, let us first present a parametric solution (Duan, 1993) to the generalized Sylvester matrix equation (26).

Let $N \in \mathbb{R}^{q \times q}$ with q distinct eigenvalues be defined as follows

$$N = Q^{-1} \Lambda Q, \tag{34}$$

where $Q \in \mathbb{R}^{q \times q}$ is any freely chosen invertible matrix, $\Lambda = \mathtt{diag}(s_1, s_2, ..., s_q)$, $s_i \neq s_j$ for $i \neq j$ and $\mathtt{Re}(s_i) < 0$ for all $i = 1, 2, ..., q$.

With N as defined in (34), L_1 and L_2 satisfying (26) are given in the following parametric forms (Duan, 1993)

$$L_1 = Q \begin{bmatrix} U(s_1)b_1 & U(s_2)b_2 & \dots & U(s_q)b_q \end{bmatrix}^\top, \tag{35}$$

$$L_2 = Q \begin{bmatrix} Z(s_1)b_1 & Z(s_2)b_2 & \dots & Z(s_q)b_q \end{bmatrix}^\top, \tag{36}$$

where $b_i \in \mathbb{C}^p (i = 1, 2, ..., q)$ are free parameters satisfying $b_i = \bar{b}_j$ if $s_i = \bar{s}_j$, $\bar{s}_j$ denotes the complex conjugate of s_i. Matrices $U(s) \in \mathbb{R}^{p \times p}$ and $Z(s) \in \mathbb{R}^{(n-p) \times p}$ are coprime polynomial matrices satisfying the following coprime factorization

$$(sI_{n-p} - A_{22}^\top)^{-1} A_{12}^\top = Z(s) U^{-1}(s). \tag{37}$$

The reader can refer to (Duan, 1993) for a numerically reliable algorithm to compute $Z(s)$ and $U(s)$. Also, as suggested in (Trinh and Fernando, 2012), $U(s)$ and $Z(s)$ can be conveniently computed according to the following equations

$$U(s) = \mathtt{det}(sI_{n-p} - A_{22}) I_p, \tag{38}$$

$$Z(s) = \mathtt{adj}(sI_{n-p} - A_{22}^\top) A_{12}^\top, \tag{39}$$

where $\mathtt{det}(.)$ and $\mathtt{adj}(.)$ denote the determinant and the adjugate matrix of matrix $(.)$, respectively. For any given A_{22}, the characteristic polynomial can be obtained as follows

$$\begin{aligned} a(s) &\triangleq \mathtt{det}(sI_{n-p} - A_{22}) \\ &= s^{n-p} + a_1 s^{n-p-1} \\ &\quad + a_2 s^{n-p-2} + \dots + a_{n-p}, \end{aligned} \tag{40}$$

where the coefficients $a_i, i = 1, 2, ..., (n-p)$, are real constants.

The adjugate matrix $\mathtt{adj}(.)$ is then obtained as follows

$$\begin{aligned} \mathtt{adj}(sI_{n-p} - A_{22}^\top) &= \Upsilon_1 s^{n-p-1} + \Upsilon_2 s^{n-p-2} \\ &\quad + \Upsilon_3 s^{n-p-3} + \dots + \Upsilon_{n-p}, \end{aligned} \tag{41}$$

where Υ_i, $i = 1, 2, ..., n-p$, are computed by using the coefficients of $a(s)$ and matrix A_{22} as given below

$$\begin{cases} \Upsilon_1 = I_{n-p}, \\ \Upsilon_2 = \Upsilon_1 A_{22}^\top + a_1 I_{n-p}, \\ \Upsilon_3 = \Upsilon_2 A_{22}^\top + a_2 I_{n-p}, \\ \vdots \\ \Upsilon_{n-p} = \Upsilon_{n-p-1} A_{22}^\top + a_{n-p-1} I_{n-p}. \end{cases} \tag{42}$$

Theorem 3. *A functional observer always exists with q-order where q is the lowest order that matrix M (M is defined in equation* (45)*) has row basis vectors for the row-nullspace of M, $\mathcal{N}(M) \neq 0$. Furthermore, the proposed residual generator* (3) *can detect the faults in systems if condition* (10) *holds.*

Proof. Now, by substitute L_1 and L_2 from (35) and (36) into the transpose of (27) and (28), we obtain

$$\begin{bmatrix} A_{d12} \\ A_{d22} \end{bmatrix}^\top \begin{bmatrix} U(s_1)b_1 & U(s_2)b_2 & \dots & U(s_q)b_q \\ Z(s_1)b_1 & Z(s_2)b_2 & \dots & Z(s_q)b_q \end{bmatrix} Q^\top = 0, \tag{43}$$

$$\begin{bmatrix} Z(s_1)b_1 & Z(s_2)b_2 & \dots & Z(s_q)b_q \end{bmatrix} (QT)^\top = 0. \tag{44}$$

Since Q is an invertible matrix, let $(QT)^\top = \begin{bmatrix} t_1 & t_2 & \dots & t_q \end{bmatrix}$, $t_i \neq 0$, $i = 1, 2, ..., q$, are arbitrarily chosen real numbers. It follows (43) and (44), we obtain

$$M\beta = 0, \tag{45}$$

where $M \in \mathbb{R}^{(q+1)(n-p) \times pq}$, $\beta \in \mathbb{R}^{pq \times 1}$ and

$$M = \begin{bmatrix} \bar{A}_1 & 0 & \dots & 0 \\ 0 & \bar{A}_2 & \dots & 0 \\ & \vdots & & \\ 0 & 0 & \dots & \bar{A}_q \\ Z(s_1)t_1 & Z(s_2)t_2 & \dots & Z(s_q)t_q \end{bmatrix},$$

$$\bar{A}_i = A_{d12}^\top U(s_i) + A_{d22}^\top Z(s_i), i = 1, 2, \dots, q,$$

$$\beta = \begin{bmatrix} b_1^\top & b_2^\top & \dots & b_q^\top \end{bmatrix}^\top.$$

Let $\mathcal{N}(M)$ be a matrix of row basis vectors for the row-nullspace of M, i.e., $M\mathcal{N}(M) = 0$. Therefore, the solutions for $\beta \neq 0$ in (45) exists iff $\mathcal{N}(M) \neq 0$, and β can be selected as any column of $\hat{\beta}$, where

$$\hat{\beta} = \mathcal{N}(M). \quad (46)$$

This completes the proof of Theorem 3. $\square$

Remark 1: For the case $\frac{n}{2} < p \leq \frac{2n}{3}$, if M is a row matrix, $\mathcal{N}(M) \neq 0$ always exists, thus implies

$$q > \left[\frac{n-p}{2p-n}\right]. \quad (47)$$

This leads to a method for searching for the order q, we only need to search for the lowest q, which satisfies Theorem 3, in the range of

$$2 \leq q \leq \left[\frac{n-p}{2p-n} + 1\right]. \quad (48)$$

Remark 2: For the case $1 \leq p \leq \frac{n}{2}$, M in (45) is always a column matrix, thus its row basis vectors for the row-nullspace, $\mathcal{N}(M) \neq 0$, exists if M is not a full rank matrix, that implies

$$\operatorname{rank}(M) < qp. \quad (49)$$

Based on Remark 2, q-order can be selected as the smallest order that M satisfies condition (49).

It is concluded that since matrices T is arbitrarily chosen, matrices L_1, L_2 and N are determined through this section, the parameters H, G, G_d and F are calculated based on equations (8), (23), (24) and (25), respectively. Matrix L is then achieved from the equation (17), we can check if condition (10) holds, i.e., $LE \neq 0$. Thus, all the conditions in the Theorem 1 are satisfied and the design of the reduced-order functional observers and the first-order residual generator to detect the faults in system (1) is completed. Examples 2 and 3 in the Numerical Examples section illustrate the theory of this section.

5 NUMERICAL EXAMPLES

Example 1: In this example, we take consideration of timely fault detection in a time-delay system with $n = 4$, $p = 3$, $m = 1$, and $l = 1$. Since we have the case where $p > \frac{2n}{3}$, and as discussed in Case 1 of Section 4, we only need to design a residual generator based on first-order observer to detect the faults in the system. For this example, $C = \begin{bmatrix} I_3 & 0 \end{bmatrix}$, A, A_d, B and E are as given below

$$A = \begin{bmatrix} -5 & 0 & 1 & 2 \\ 1 & -1 & 0 & -2 \\ 0 & 0 & -3 & -1 \\ -2 & 2 & 0 & -2 \end{bmatrix}, B = \begin{bmatrix} 1 \\ -2 \\ 4 \\ -1 \end{bmatrix},$$

$$A_d = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & -2 \\ 1 & 0 & 0 & -1 \end{bmatrix}, E = \begin{bmatrix} 1 \\ -1 \\ 2 \\ -3 \end{bmatrix}.$$

Now, the design of a first-order functional observer and residual generator can be readily carried out.

Since C is already in the desired form, i.e., $C = \begin{bmatrix} I_3 & 0 \end{bmatrix}$, so P is an identity matrix, i.e., $P = I_4$. According to the partitions (18) and (19), submatrices A_{11}, A_{12}, A_{21}, A_{22}, A_{d11}, A_{d12}, A_{d21} and A_{d22} are obtained, where

$$\left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right] = \left[\begin{array}{ccc|c} -5 & 0 & 1 & 2 \\ 1 & -1 & 0 & -2 \\ 0 & 0 & -3 & -1 \\ \hline -2 & 2 & 0 & -2 \end{array}\right],$$

$$\left[\begin{array}{c|c} A_{d11} & A_{d12} \\ \hline A_{d21} & A_{d22} \end{array}\right] = \left[\begin{array}{ccc|c} 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & -2 \\ \hline 1 & 0 & 0 & -1 \end{array}\right].$$

It is clear that $\begin{bmatrix} A_{12} & A_{d12} \end{bmatrix}$ is a column matrix and thus its matrix of row basis vectors for the row-nullspace exists, i.e., $\mathcal{N}\begin{bmatrix} A_{12} & A_{d12} \end{bmatrix} \neq 0$. As a result, a first-order functional observer exists.

For the design of first-order observer and first-order residual generator, let us assign $N = -3$ and L_1 is computed according to (33), we obtain

$$L_1 = \begin{bmatrix} -0.5571 & -0.7428 & 0.3714 \end{bmatrix}.$$

Since $L_2 = 0$ and according to (17), matrix L is obtained as

$$L = \begin{bmatrix} -0.5571 & -0.7428 & 0.3714 & 0 \end{bmatrix}.$$

With L as obtained above, condition (10) is found to be satisfied since

$$LE = 0.9285 \neq 0.$$

Hence, a first-order residual generator exists and can be constructed to detect faults in the system. By choosing $T = -5$, matrices H, G, G_d and F are obtained according to equations (8), (23), (24) and (25), respectively, where

$$\begin{aligned} H &= 2.4140, \\ G &= \begin{bmatrix} 0.3714 & -1.4856 & -0.5571 \end{bmatrix}, \\ G_d &= \begin{bmatrix} 0 & 0.9285 & 0 \end{bmatrix}, \\ F &= \begin{bmatrix} -2.7854 & -3.7139 & 1.8570 \end{bmatrix}. \end{aligned}$$

Figure 1 shows that the residual generator can effectively detect the faults $f(t)$ in the system. Fault $f(t)$ appears at time $t = 20s$ and clears from $t = 30s$. During the time the fault happens, the residual generator triggers the fault, when there are no faults, the residual generator converges to zero as expected. Note also that the residual is insensitive to the input $u(t)$ as expected. It is clear in this example that, a significantly lower order (only first-order) residual generator is designed using a first-order functional observer. In contrast, existing fault detection schemes using full-order or reduced-order state observers would give higher order schemes.

Example 2: This example is given to demonstrate Case 2 (Section 4), where $\frac{n}{2} < p \leq \frac{2n}{3}$. Let us consider a system which has $C = \begin{bmatrix} I_3 & 0 \end{bmatrix}$ and matrices A, A_d, B and E given as

$$A = \begin{bmatrix} -1 & 0 & 0 & 1 & -2 \\ 0 & -5 & 3 & 4 & 0 \\ 1 & 1 & -8 & 3 & 0 \\ -4 & 0 & 2 & -6 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & -1 \\ 0 & 0 \\ 0 & 1 \end{bmatrix},$$

$$A_d = \begin{bmatrix} 0 & 0 & 2 & -1 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 3 & 0 \\ -1 & 0 & 1 & -2 & 0 \\ 0 & 1 & 0 & 0 & -3 \end{bmatrix}, E = \begin{bmatrix} 1 & 3 \\ -1 & 1 \\ 2 & -2 \\ -3 & 2 \\ 2 & -3 \end{bmatrix}.$$

Thus, for this example, we have $n = 5$, $p = 3$, $m = 2$ and $l = 2$. Since $\frac{n}{2} < p \leq \frac{2n}{3}$, this falls into the Case 2 (Section 4) and therefore we can carry out the design of a reduced-order observer and a residual generator to detect faults in the system.

Since $C = \begin{bmatrix} I_3 & 0 \end{bmatrix}$, thus $P = I_5$ and according to (18) and (19), sub-matrices A_{11},A_{12},A_{21}, A_{22}, A_{d11},A_{d12},A_{d21} and A_{d22} are obtained, where

$$\left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right] = \left[\begin{array}{ccc|cc} -1 & 0 & 0 & 1 & -2 \\ 0 & -5 & 3 & 4 & 0 \\ 1 & 1 & -8 & 3 & 0 \\ \hline -4 & 0 & 2 & -6 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right],$$

$$\left[\begin{array}{c|c} A_{d11} & A_{d12} \\ \hline A_{d21} & A_{d22} \end{array}\right] = \left[\begin{array}{ccc|cc} 0 & 0 & 2 & -1 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 3 & 0 \\ \hline -1 & 0 & 1 & -2 & 0 \\ 0 & 1 & 0 & 0 & -3 \end{array}\right].$$

According to (40)-(42), we obtain the characteristic polynomial and adjugate matrix as

$$a(s) = s^2 + 7s + 6,$$
$$\mathtt{adj}(sI_{n-p} - A_{22}^\top) = \Upsilon_1 s + \Upsilon_2,$$

where

$$\Upsilon_1 = I_2, \;\; \Upsilon_2 = \begin{bmatrix} 1 & 1 \\ 0 & 6 \end{bmatrix}.$$

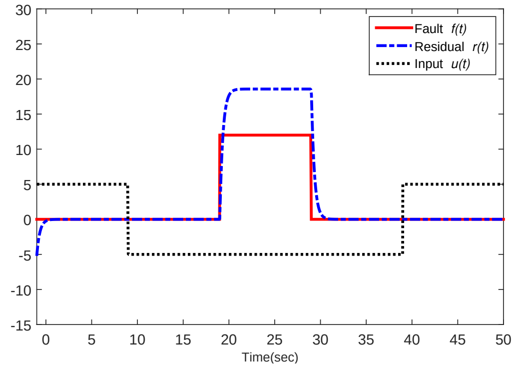

Figure 1: Residual generator using first-order observer effectively triggers fault in the system.

The pair of coprime polynomial matrices $U(s)$ and $Z(s)$ are then calculated based on (38) and (39)

$$U(s) = (s^2 + 7s + 6)I_3,$$
$$Z(s) = (\Upsilon_1 s + \Upsilon_2)A_{12}^\top.$$

As in Theorem 3 and Remark 1, now we search for the lowest possible order q of the observers $\omega(t)$. It follows equation (48) we have

$$2 \leq q \leq 3.$$

For the case that $q = 2$, let us assign the poles of N to be at $s_1 = -3$, $s_2 = -5$ and choose Q to be $Q = I_2$ and $TQ = \begin{bmatrix} 1 & 1 \end{bmatrix}$.

From (45), matrix M and the matrix of row basis vectors for the row-nullspace of M are obtained

$$M = \begin{bmatrix} 14 & 10 & -6 & 0 & 0 & 0 \\ 18 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 16 & 28 & 12 \\ 0 & 0 & 0 & 6 & 0 & 0 \\ -4 & -8 & -6 & -6 & -16 & -12 \\ -6 & 0 & 0 & -2 & 0 & 0 \end{bmatrix},$$

$$\mathcal{N}(M) = \begin{bmatrix} 0 & 0.2339 & 0.3898 & 0 & 0.3509 & -0.8187 \end{bmatrix}^\top.$$

Since $\mathcal{N}(M) \neq 0$, the second-order observers exist for the system. Accordingly, $\beta \neq 0$ exists and is obtained by taking any column of $\mathcal{N}(\mathcal{M})$. Matrices b_1 and b_2 are then obtained based on (45), where

$$b_1 = \begin{bmatrix} 0 & 0.2339 & 0.3898 \end{bmatrix}^\top,$$
$$b_2 = \begin{bmatrix} 0 & 0.3509 & -0.8187 \end{bmatrix}^\top.$$

From (35) and (36), L_1 and L_2 are obtained

$$L_1 = \begin{bmatrix} 0 & -1.4034 & -2.3390 \\ 0 & -1.4034 & 3.2747 \end{bmatrix},$$

$$L_2 = \begin{bmatrix} -4.2103 & 0 \\ 4.2103 & 0 \end{bmatrix}.$$

It follows (17) and $P = I_5$, $L = \begin{bmatrix} L_1 & L_2 \end{bmatrix}$ and it is easy to verify condition (10) that

$$LE = \begin{bmatrix} 9.3562 & -5.1459 \\ -4.6781 & 0.4678 \end{bmatrix} \neq 0.$$

Thus, all the conditions for Theorem 3 are satisfied and hence a second-order observer and first-order residual generator exist and can be constructed to detect faults in the system by determining all other unknown parameters, where

$$\begin{aligned} N &= \begin{bmatrix} -3 & 0 \\ 0 & -5 \end{bmatrix}, \; T = \begin{bmatrix} 1 & 1 \end{bmatrix}, \\ G &= \begin{bmatrix} 14.5021 & 0.4678 & -0.9356 \\ -13.5665 & 3.2747 & -5.6137 \end{bmatrix}, \\ G_d &= \begin{bmatrix} 4.2103 & 1.4034 & -1.8712 \\ -4.2103 & 1.4034 & 0.9356 \end{bmatrix}, \\ H &= \begin{bmatrix} -2.3390 & 0.9356 \\ 3.2747 & -4.6781 \end{bmatrix}, \\ F &= \begin{bmatrix} 0 & 2.8069 & -0.9356 \end{bmatrix}. \end{aligned}$$

Figure 2 indicates that the residual generator can detect the faults $f_1(t)$ and $f_2(t)$ in the system. It is clear in this example that the design of residual generator is very easy and systematic. Furthermore, the order of the functional observer is very low comparing to conventional FD schemes using full-order or reduced-order state observers. This example thus further highlights the attractiveness of the FD scheme proposed.

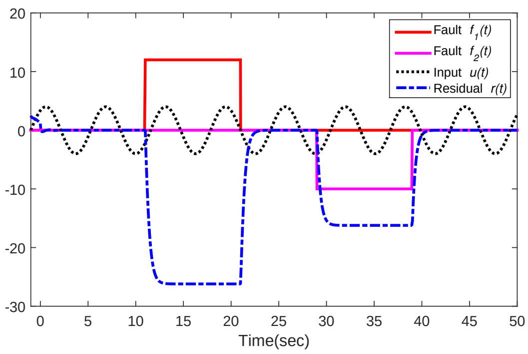

Figure 2: Residual generator based on second-order fuctional observers detects faults in the system.

Example 3: This example is given to demonstrate Case 2 (Section 4), where $1 \leq p \leq \frac{2n}{2}$. Let us consider a system which has matrices A, B, and E as same as in Example 2, however, matrices A_d and C given as

$$A_d = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 2 & -1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}^\top.$$

Thus, for this example, we have $n = 5$, $p = 2$, $m = 2$ and $l = 2$. Since $C = \begin{bmatrix} I_2 & 0 \end{bmatrix}$, implies $P = I_5$ and according to (18) and (19), sub-matrices A_{11}, A_{12}, A_{21}, A_{22}, $A_{d11}, A_{d12}, A_{d21}$ and A_{d22} are obtained, where

$$\left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right] = \left[\begin{array}{cc|ccc} -1 & 0 & 0 & 1 & -2 \\ 0 & -5 & 3 & 4 & 0 \\ \hline 1 & 1 & -8 & 3 & 0 \\ -4 & 0 & 2 & -6 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right],$$

$$\left[\begin{array}{c|c} A_{d11} & A_{d12} \\ \hline A_{d21} & A_{d22} \end{array}\right] = \left[\begin{array}{cc|ccc} -1 & 0 & 0 & 0 & 0 \\ 2 & -1 & 0 & 0 & 0 \\ \hline 1 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{array}\right].$$

Let us assign $q = 2$, the poles of N to be at $s_1 = -7$, $s_2 = -9$, $Q = I_2$ and $TQ = \begin{bmatrix} 1 & 1 \end{bmatrix}$. It follows the same line as in the calculations of Example 2, the matrix M in (45) is obtained as

$$M = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & -16 & -8 & 14 \\ 0 & 0 & 0 & 0 & 0 & 0 & -30 & -78 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -20 & 10 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 8 & -40 & 0 \end{bmatrix}^\top.$$

Since $\text{rank}(M) = 3 < qp = 4$, the condition (49) holds and $\mathcal{N}(M) \neq 0$ exists, where

$$\mathcal{N}(M) = \begin{bmatrix} 0.3152 & 0.1686 & -0.7354 & -0.5757 \end{bmatrix}^\top.$$

Accordingly, b_1 and b_2 are obtained, where

$$b_1 = [0.3152 \quad 0.1686]^\top, b_2 = [-0.7354 \quad -0.5757]^\top.$$

From (35) and (36), L_1 and L_2 are obtained

$$\begin{aligned} L_1 &= \begin{bmatrix} 13.2368 & 7.0828 \\ -17.6490 & -13.8173 \end{bmatrix}, \\ L_2 &= \begin{bmatrix} -10.1017 & -15.6751 & 4.4123 \\ 10.1017 & 15.6751 & -4.4123 \end{bmatrix}. \end{aligned}$$

It follows (17) and $P = I_5$, $L = \begin{bmatrix} L_1 & L_2 \end{bmatrix}$ and it is easy to verify that

$$LE = \begin{bmatrix} 41.8003 & 22.4096 \\ -39.4781 & -42.3809 \end{bmatrix} \neq 0.$$

Thus, all the conditions for Theorem 3 are satisfied and hence a second-order observer and a residual generator exist and can be constructed to detect faults in the system by determining all other unknown parameters, where

$$\begin{aligned} N &= \begin{bmatrix} -7 & 0 \\ 0 & -9 \end{bmatrix}, \; T = \begin{bmatrix} 1 & 1 \end{bmatrix}, \\ G &= \begin{bmatrix} 132.019 & 4.064 \\ -193.791 & -45.168 \end{bmatrix}, \\ G_d &= \begin{bmatrix} 6.502 & 7.431 \\ -15.559 & -0.697 \end{bmatrix}, \\ H &= \begin{bmatrix} 3.135 & 21.597 \\ -7.547 & -28.331 \end{bmatrix}, F = \begin{bmatrix} 4.412 \\ 6.734 \end{bmatrix}^\top. \end{aligned}$$

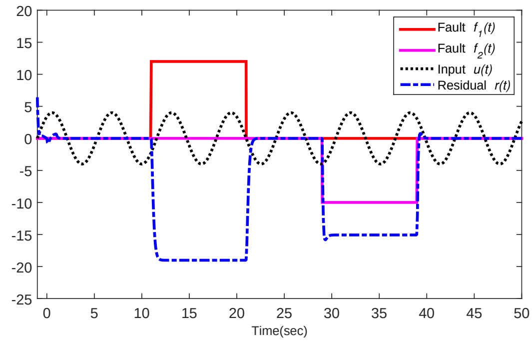

Figure 3: Residual generator based on second-order functional observers detects faults in the system.

Figure 3 indicates that a residual generator based on a second-order observer can effectively detects the faults $f_1(t)$ and $f_2(t)$ in the system. It clearly illustrates the Remark 2.

6 CONCLUSION

This paper has proposed a new fault detection scheme using minimum-order functional observers to construct residual generators to timely trigger actuator faults in time-delay systems. The proposed approach is based on solving a generalized Sylvester matrix equation via a parametric approach. Existence conditions and systematic procedures for designing the proposed fault detection scheme have been presented. The lowest possible order and the simplicity of the approach are the hallmark of the proposed novel fault detection scheme. Three examples have been constructed to prove the theory of the scheme.

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