Synchronization of the Complex Dynamical Networks with a Gui Chaotic Strange Attractor

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Abstract: In this paper, impulsive neural networks with a Gui chaotic strange attractor is studied. By employing the Lyapunov-like stability theory of impulsive functional differential equations, some criteria for synchronization of impulsive neural networks are derived. An illustrative example is provided to show the effectiveness and feasibility of the proposed method and results.

1 INTRODUCTION

In recent years, the additive neural networks have been extensively studied, including both continuous time and discrete-time settings, and applied to associative memory, model identification, optimization problems, etc. Many essential features of these networks, such as qualitative properties of stability, oscillation, and convergence issues have been investigated (Cao, 1999; Hopfield, 1982; Song and Zhang, 2008; Subashini and Sahoo, 2014; Wang and Huang, 2014).

However, as we well know, nonautonomous phenomena often occur in many realistic systems. Particularly when we consider a long-term dynamical behaviors of a system, the parameters of the system usually will change with time. In addition, in many applications, the property of periodic oscillatory solutions of cellular neural networks also is of great interest. In fact, there has been considerable research on the nonautonomous neural networks(Gopalsamy and He, 1994; Subashini and Sahoo, 2014) . The cellular neural networks with impulse effect are studied, where the criteria on the existence, uniqueness and global stability of periodic solution are obtained.Further, a new chaos strange attractor was also found, known as Gui chaos strange attractor(Zhang and Gui, 2009a; Zhang and Gui, 2009b).

Another type of synchronization, impulsive synchronization, has been developed(Amritkar and Gupte, 1993; Anzo and Barajas-Ramłrez, 2014; Dörfler and Bullo, 2014; Wan and Cao, 2015; Xie and Xu, 2014). It allows synchronization of chaotic systems using only small impulses (Yang and Chua, 1999a) generated by samples of the state variables of the driving system at discrete time instances. These samples are called the synchronizing impulses and they drive the response system discretely at these instances. After a finite period of time, the two chaotic systems behave in accordance with each other and the synchronization of the two chaotic systems is achieved. In other words, the asymptotic stability property of the error dynamics between the driving and response systems is reached. The impulsive synchronization has been applied to a number of chaos-based communication systems which exhibit good performance for the synchronization purposes and for security purposes (Yang and Chua, 1997; Yang and Chua, 1999b) .

Motivated by the above discussions, the aim of this paper is to study the synchronization of impulsive neural networks with a Gui chaotic strange attractor. By employing the Lyapunov-like stability theory of impulsive functional differential equations, some criteria for synchronization of impulsive neural networks are derived.

The remainder of the paper is organized as follows: Section 2 describes the issue of synchronization of coupled impulsive systems with a Gui chaotic strange attractor. In Section 3, some sufficient conditions for the synchronization are derived by constructing suitable Lyapunov-like function. In Section 4, an illustrative example is given to show the effectiveness of the proposed method. Conclusions are given in Section 5.

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2 PRELIMINARIES AND PROBLEM FORMULATION

In this paper, we consider the following nonautonomous cellular neural networks model with impulses

$$\begin{cases} \dfrac{\mathrm{d}x_i}{\mathrm{d}t} = -a_i x_i(t) + \displaystyle\sum_{j=1}^{n} b_{ij} f_j(x_j(t)) + c_i(t), \\ \qquad t > 0, t \neq t_k, \\ \Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k^-) = d_{ik} x_i(t_k^-), \end{cases} \tag{1}$$

where $i = 1,2,\ldots,n; k = 1,2,\ldots; \Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k^-)$ are the impulses at moments t_k and $t_1 < t_2 < \ldots$is a strictly increasing sequence such that $\lim_{k\to\infty} t_k = +\infty$; $x_i(t)$ corresponds to the state of the ith unit at time t, $f_j(x_j(t))$ denotes the output of the jth unit at time t, b_{ij} denotes the strength of the jth unit on the ith unit at time t, $c_i(t)$ is the external bias on the ith at time t, a_i represents the rate with which the ith unit will reset its potential to the resting state when disconnected from the network and external inputs.

As usual in the theory of impulsive differential equations, at the points of discontinuity t_k of the solution $t \mapsto x_i(t)$we assume that $x_i(t_k) \equiv x_i(t_k^-)$. It is clear that, in general, the derivatives $x_i'(t_k)$ do not exist. On the other hand, according to the first equality of (1) there exist the limits $x_i'(t_k^{\mp})$. According to the above convention, we assume $x_i'(t_k) \equiv x_i'(t_k^-)$.

Throughout this paper, we assume that:

(H_1) Functions $f_j(u)$ $(j = 1,2,\ldots,n)$ are Lipschitz continuous and monotonically non-decreasing, i.e., for all $u_1, u_2 \in \mathbb{R} = (-\infty,\infty)$ there are constants $L_j > 0$ such that

$$0 \leqslant \frac{f_j(u_1) - f_j(u_2)}{u_1 - u_2} \leqslant L_j.$$

(H_2) There exists a positive integer T , such that

$$t_{k+T} = t_k + \omega, d_{i(k+T)} = d_{ik},$$

where $k = 1,2,\ldots,$ $i = 1,2,\ldots,n.$

In (Gui and Ge, 2006) , the system (1) were found to have a Gui chaotic strange attractor. Now we consider the drive system in the form of the neural networks (1). For the purpose of synchronization, we introduce the response system that is driven by (1) via a set of signals

$$\begin{cases} \dfrac{\mathrm{d}y_i}{\mathrm{d}t} = -a_i y_i(t) + \displaystyle\sum_{j=1}^{n} b_{ij} f_j(y_j(t)) + c_i(t), \\ \qquad t > 0, \quad t \neq t_k, \\ y_i(t_k^+) = 2d_{ik} x_i(t_k^-) + (1-d_{ik}) y_i(t_k^-), \end{cases} \tag{2}$$

where $i = 1,2,\ldots,n,$ $k = 1,2,\ldots.$ Letting $e(t) = y_i - x_i$ be synchronization error, where $e(t) = (e_1(t), e_2(t), \ldots, e_n(t))^T$, $x_i(t)$ and $y_i(t)$ are the state variables of drive system (1) and response system (2). Thus,we can derive the error dynamical system as follows:

$$\begin{cases} \dot{e} = -De(t) + WG(e(t)), \quad t \neq t_k, \\ e(t_k^+) = (I - D_k)e(t_k^-), \quad k = 1,2,\ldots, \end{cases} \tag{3}$$

where

$$g_j = f_j(e_j + x_j) - f_j(x_j),$$
$$G(e) = [g_1, g_2, \ldots, g_n]^T.$$

I is identity matrix, and

$$D_k = \begin{pmatrix} d_{1k} & 0 & \cdots & 0 \\ 0 & d_{2k} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & d_{nk} \end{pmatrix}.$$

In fact, from the analysis above, we can see that (1) and (2) are synchronized if and only if the equilibrium point of (3) is asymptotically stable for any initial condition.So the global impulsive synchronization problem can be solved if the controller gain matrices d_{ik} are suitably designed such that the zero solution of (3) is globally asymptotically stable.

3 MAIN RESULTS

In this section, we will derive some sufficient conditions for synchronization in the sense of the fact that the error system (3)

Theorem 1. *If there exist a positive constant* $\varepsilon > 0, \alpha > 0$, *positive definite diagonal matrix* $P > 0$, *such that*

1. Linear matrix Inequality

$$\begin{bmatrix} -PD - D^T P + \varepsilon L_M^2 \lambda_M(W^T W) - \alpha P & P \\ P & -\varepsilon \end{bmatrix} < 0 \tag{4}$$

where $L_M = \max\{L_j\}, \lambda_M$ *denote the largest eigenvalue of the matrix* $(*)$.

2. $\tau < \inf_{k\in N}\{t_k - t_{k-1}\} < 1$;

3. There exists a constant α *such that*

$$\ln(\eta\beta_k) + \alpha(t_k - t_{k-1}) < 0, \tag{5}$$

where $\beta_k = \lambda_M^2(I - D_k)$*; then, the origin of system* (3) *is globally asymptotically stable, which implies that* (1) *and* (2) *are completely synchronized.*

Proof. Construct a Lyapunov function in the form of

$$V(e(t)) = e^T(t)Pe(t).$$

When $t \in (t_k - t_{k-1}]$, the total derivative of $V(e)$ with respect to (3) is

$$\begin{aligned}\dot{V}(e) =& \dot{e}^T Pe + e^T P\dot{e},\\ =& (-De + WG(e))^T Pe\\ &+ e^T P(-De + WG(e))\\ =& -e^T(PD + D^T P)e\\ &+ 2e^T PWG(e).\end{aligned}$$

Also, by the well-known inequality

$$2a^T b \leqslant \varepsilon^{-1} a^t a + \varepsilon b^T b,$$

for $\forall \varepsilon > 0$, we obtain

$$\begin{aligned}\dot{V}(e) \quad \leqslant \quad & -e^T(PD + D^T P)e + \varepsilon^{-1} e^T PPe\\ & + \varepsilon L_M^2 \lambda_M (W^T W) G(e).\end{aligned}$$

From the Assumption (H_1), we can obtain

$$||G(e)||^2 \leqslant L_M^2 ||e||^2$$

This leads to

$$\begin{aligned}\dot{V}(e) \quad \leqslant \quad & -e^T(PD + D^T P)e + \varepsilon^{-1} e^T PPe\\ & + \varepsilon L_M^2 \lambda_M (W^T W) e^T e. \end{aligned} \tag{6}$$

From Linear matrix inequality (4) and (6), we have

$$\dot{V}(e) < \alpha e^T Pe = \alpha V(e).$$

Let $V(t) = V(e(t))$, then

$$V(t) \leqslant V(t_{k-1}^+) \exp[\alpha(t - t_{k-1})], \tag{7}$$

where $t \in (t_{k-1}, t_k], k \in N$. From the second equation in (3), we have

$$\begin{aligned}V(t_k^+) &= e^T(t_k^+) Pe(t_k^+)\\ &= [(I - D_k)e(t_k)]^T P[(I - D_k)e(t_k)]\\ &\leqslant \lambda_M^2 (I - D_k) V(t_k)\\ &= \beta_k V(t_k).\end{aligned} \tag{8}$$

For $t \in (t_0, t_1]$, from (7) and (8), we have

$$V(t) \leqslant V(t_0) \exp[\alpha(t - t_0)], \;\; t \in (t_0, t_1],$$

which leads to

$$V(t_1) \leqslant V(t_0) \exp[\alpha(t_1 - t_0)], \;\; t \in (t_0, t_1],$$

and

$$V(t_1^+) \leqslant V(t_0) \beta_1 \exp[\alpha(t_1 - t_0)], \;\; t \in (t_0, t_1],$$

Similarly, for $t \in (t_1, t_2]$,

$$\begin{aligned}V(t) \quad &\leqslant \quad V(t_1^+) \exp[\alpha(t - t_1)],\\ &\leqslant \quad V(t_0) \beta_1 \beta_2 \exp[\alpha(t - t_0)]\end{aligned}$$

In general, for $t \in (t_{k-1}, t_k]$,

$$\begin{aligned}V(t) \quad &\leqslant \quad V(t_1^+) \exp[\alpha(t - t_1)],\\ &\leqslant \quad V(t_0) \beta_1 \beta_2 \cdots \beta_k \exp[\alpha(t - t_0)]\end{aligned} \tag{9}$$

For $t \in (t_k, t_{k+1})]$, it follows from (5) and (9) that,

$$\begin{aligned}V(e(t)) \quad &\leqslant \quad V(e(t_0)) \beta_1 \beta_2 \cdots \beta_k \exp[\alpha(t - t_0)]\\ &\leqslant \quad V(e(t_0)) \{\beta_1 \exp[\alpha\tau_1]\} \{\beta_2 \exp[\alpha\tau_2]\}\\ &\qquad \cdots \{\beta_k \exp[\alpha\tau_k]\} \exp[\alpha(t - t_k)]\\ &\leqslant \quad V(e(t_0)) \frac{\exp[\alpha(t - t_k)]}{\eta^k}\end{aligned} \tag{10}$$

From (10) , we can see that the trivial solution of system (3) is globally asymptotically stable.This completes the proof.

4 AN ILLUSTRATIVE EXAMPLE

In order to demonstrate and verify the performance of the proposed method, some numerical simulations are presented in this section.

As is known to all that (1) can exhibit Gui chaotic strange attractor (Zhang and Gui, 2009a; Zhang and Gui, 2009b).In order to show it clearly, we give the following example:

$$\begin{cases}\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = -\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}\\ \qquad + \begin{bmatrix}1.2 & -1.6 & 0\\ 1.2 & 1.0 & 0.9\\ 0 & 2.2 & 0.15\end{bmatrix}\begin{bmatrix}f_1(x_1)\\ f_2(x_2)\\ f_3(x_3)\end{bmatrix}\\ \qquad + \begin{bmatrix}c_1(t)\\ c_2(t)\\ c_3(t)\end{bmatrix}\\ x_i(t_k^+) = x_i(t_k) + d_{ik} x_i(t_k), \;\; i = 1, 2, 3 \;\; k \in \mathbb{Z}^+,\end{cases} \tag{11}$$

where

$$d_{1k} = 0.35, d_{2k} = 0.4, d_{3k} = 0.5,$$
$$f_j(x_j) = 0.5(|x_j + 1| - |x_j - 1|), \;\; j = 1, 2.$$

Obviously, $f_j(x)$ satisfy (H_1).

Now we investigate the influence of the period T of impulsive effect on the system (11). Set

$$\begin{bmatrix}c_1(t)\\ c_2(t)\\ c_3(t)\end{bmatrix} = \begin{bmatrix}2 - 2\cos t\\ 2 - 2\sin t\\ 1 + \cos t\end{bmatrix}.$$

For $T = 1$, then (H_2) isn't satisfied. Periodic oscillation of system (11) will be destroyed by impulses effect. Numeric results show that system (11) still has

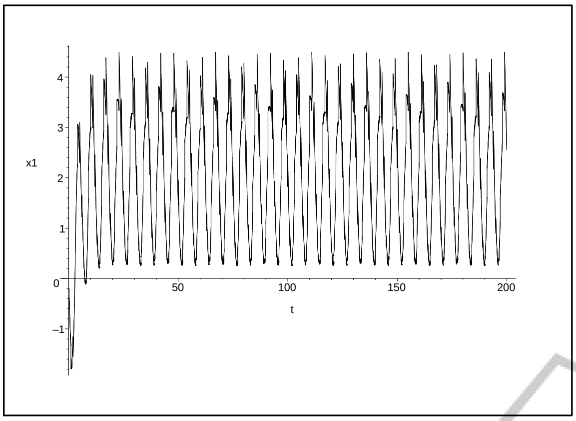

Figure 1: Time-series of the $x_1(t)$of system (11).

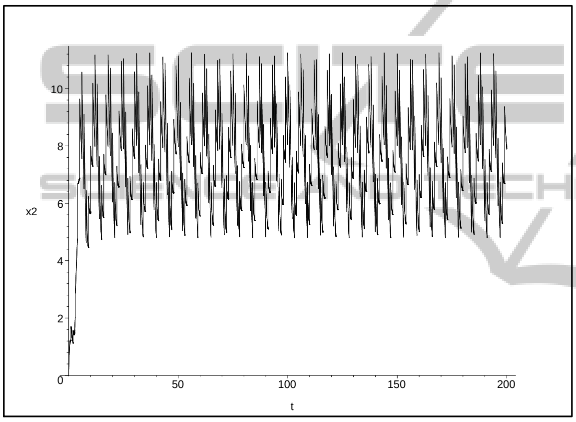

Figure 2: Time-series of the $x_2(t)$ of system (11).

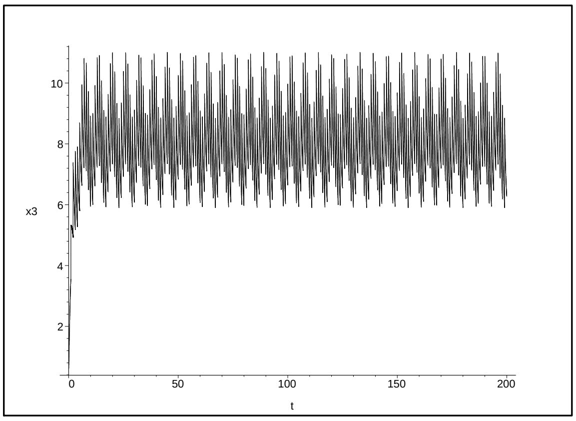

Figure 3: Time-series of the $x_3(t)$ of system (11).

a global attractor which can be a Gui chaotic strange attractor(see Figs.1-4).

Every solutions of system (11) will finally tend to the Gui chaotic strange attractor. As shown in Figs. 1-4, the system (11) possesses a Gui chaotic strange attractor.

Now the response chaotic cellular neural network

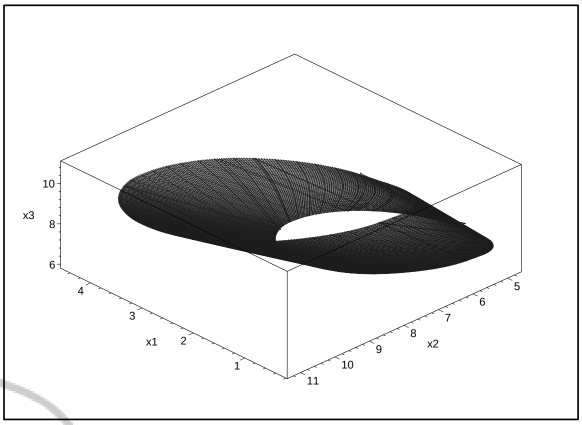

Figure 4: Phase portrait of Gui chaotic strange attractor of system (11) with $T = 1$.

is designed as follows:

$$\begin{cases} \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \end{bmatrix} = - \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \\ \quad + \begin{bmatrix} 1.2 & -1.6 & 0 \\ 1.2 & 1.0 & 0.9 \\ 0 & 2.2 & 0.15 \end{bmatrix} \begin{bmatrix} f_1(y_1) \\ f_2(y_2) \\ f_3(y_3) \end{bmatrix} \\ \quad + \begin{bmatrix} c_1(t) \\ c_2(t) \\ c_3(t) \end{bmatrix} \\ y_i(t_k^+) = 2d_{ik}x_i(t_k^-) + (1 - d_{ik})y_i(t_k^-), \end{cases} \qquad (12)$$

Remark 1. In (Zhang and Gui, 2009a; Zhang and Gui, 2009b), the authors investigate the influence of the period T of impulsive effect on the system (11). If $T = \frac{2}{5}\pi$ or $T = 0.1\pi$, then $q = 5$ or $q = 20$, respectively, in (H_2). According to Theorem 1 and Theorem 2, the cellular neural networks model (11) has a unique 2π-periodic solution which is globally asymptotically stable. For $T = \frac{2}{5}\pi, \gamma_1 = 0.6, \gamma_2 = 0.85$, each positive solution tends to a unique positive 2π-periodic solution with 5-impulses in a period. For $T = 0.1\pi, \gamma_1 = 0.4, \gamma_2 = 0.3$, each positive solution tends to a unique positive 2π-periodic solution with 20-impulses in a period.

Remark 2. If $T = 1$, then (H_2) isn't satisfied. Periodic oscillation of system (11) will be destroyed by impulses effect. Numeric results show that system (11) still has a Gui chaotic strange attractor(Zhang and Gui, 2009a; Zhang and Gui, 2009b).

Let $e(t) = y_i - x_i$, then the error system (13) of drive system (11) and respond system (12) is constructed as follow

$$\begin{cases} \begin{bmatrix} \dot{e}_1(t) \\ \dot{e}_2(t) \\ \dot{e}_3(t) \end{bmatrix} = -\begin{bmatrix} e_1(t) \\ e_2(t) \\ e_3(t) \end{bmatrix} \\ \qquad + \begin{bmatrix} 1.2 & -1.6 & 0 \\ 1.2 & 1.0 & 0.9 \\ 0 & 2.2 & 0.15 \end{bmatrix} \\ \qquad \cdot \begin{bmatrix} f_1(e_1(t)+x_1) - f_1(x_1) \\ f_2(e_2(t)+x_1) - f_2(x_2) \\ f_3(e_3(t)+x_3) - f_3(x_3) \end{bmatrix} \\ e_1(t_k^+) = (1-d_{1k})e_1(t_k^-), \\ e_2(t_k^+) = (1-d_{2k})e_2(t_k^-), \\ e_3(t_k^+) = (1-d_{3k})e_3(t_k^-). \end{cases} \tag{13}$$

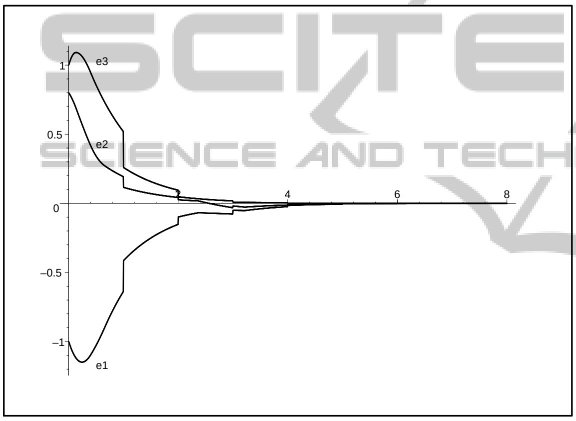

Figure 5: Synchronization errors between drive system (11) and response system (12) with $T = 1$.

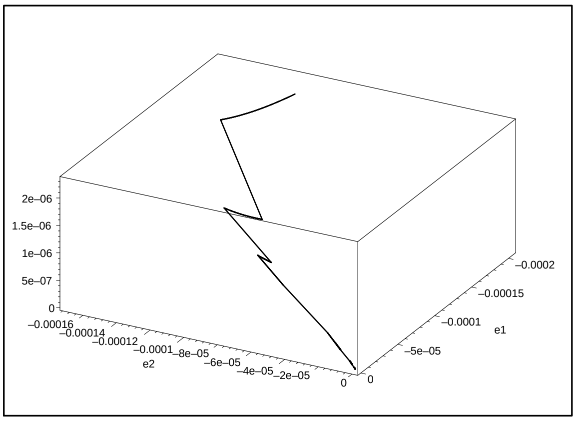

Figure 6: Phase portrait of Synchronization errors with $T = 1$.

If one choose $L_1 = L_2 = L_3 = 1, \eta = 1.1, d_{1k} = 0.35, d_{2k} = 0.4, d_{3k} = 0.5$, It is easy to check the conditions in Theorem 1 are satisfied. So, the system (11) and (12) is synchronized. By Theorem 1, synchronization can be obtained. The synchronization performance is illustrated by Figs. 5,6. The numerical simulations show that synchronization could be quickly achieved.

Furthermore, if $T = 0.4$, set

$$\begin{bmatrix} c_1(t) \\ c_2(t) \\ c_3(t) \end{bmatrix} = \begin{bmatrix} 2-2\sin t \\ 3-3\cos t \\ 1-1\sin t \end{bmatrix}.$$

then, periodic oscillation of system (11) will be destroyed by impulses effect. Numeric results show that system (11) still has a Gui chaotic strange attractor(see Fig.7).

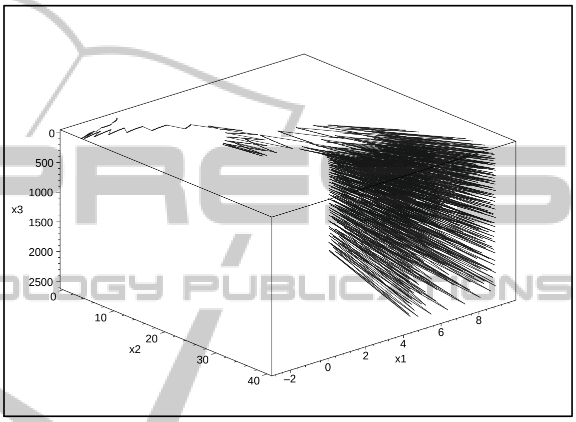

Figure 7: Phase portrait of Gui chaotic strange attractor of system (11) with $T = 0.4$.

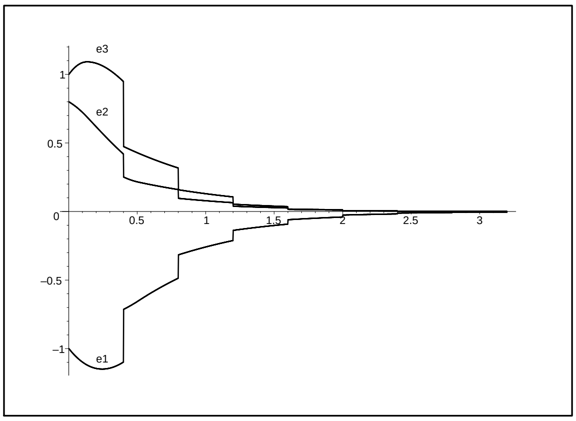

Figure 8: Synchronization errors between drive system (11) and response system (12) with $T = 0.4$.

It follows from Theorem 1 that systems (11) and (12) are impulsively synchronized. Figs. 8 and 9 depict the synchronization error of the state variables between the drive system and the response system.

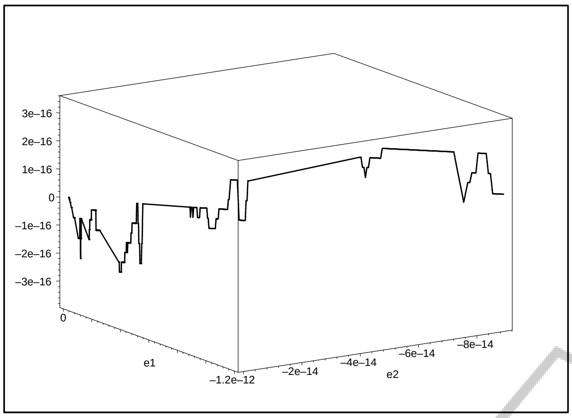

Figure 9: Phase portrait of Synchronization errors with $T = 0.4$.

5 CONCLUSIONS

In the paper, the synchronization of the impulses complex dynamical network with a Gui chaotic strange attractor and has been investigated based on the stability analysis of impulsive functional differential equation. The criteria for the synchronization are derived. An illustrative example is finally included to visualize the effectiveness and feasibility of the developed methods. Compared with the correspondingly previous works(Luo, 2008; Yang and Cao, 2007; Yang and Cao, 2010; Zhang, 2009), our model of research is new. As far as we know, There is no paper to deal with such a problem.

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