The LARES Mission: An Opportunity to Teach General Relativity
Frame Dragging and Lense-Thirring Effect

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Abstract: LARES is an Italian Space Agency mission devoted to test frame-dragging, a prediction of general relativity. On February 2012 the satellite has been successfully put in orbit with the qualification flight of VEGA, the new European Space Agency launcher. Basic concepts of general relativity are becoming more and more familiar because of the part they play in science fiction movies. But frame-dragging (more formally known as the Lense-Thirring effect), is so peculiar that it is a relatively unknown effect. The idea of this paper is to start from the description of the experiment and then to push some parameters of the experiment to extreme values in order to magnify the effects of relativity. This approach will provide not only the students and general people but also professionals not strictly specialized in general relativity, with increased interest in gravitational theories.

1 INTRODUCTION

Many concepts of special and general relativity, though difficult to understand in detail, are known to be true even by non-specialists. Deflection of light by a mass is an example. The phenomenon of gravitational waves is another. It is even possible, via analogies, to explain that space and time are indeed one single four-dimensional entity called spacetime. More difficult is the idea that time is not absolute but flows in different ways depending on the state of motion of the observer and on the strength of the gravitational field. Time, being relative to the observer, has been used in several science fiction movies so that many people are aware of this effect even though it remains an area for experts. Frame-dragging (Ciufolini, 2010) is another intriguing prediction of general relativity, but known only to specialists. The name “frame-dragging” was given by Einstein in a private communication with Ernst Mach (Einstein, 1913) before the general theory of relativity was published (Einstein, 1915). But the first mathematical derivation of this effect was performed few years later by Josef Lense and Hans Thirring, two Austrian physicists, that derived it in (Lense and Thirring, 1918). Later a generalization was performed by Roy Kerr that found an exact solution to the very complex non linear equations of general relativity in the case of a rotating mass (Kerr, 1963). However until few years ago the smallness of this effect made its direct measurement impossible. We need to come to the year 1997 to have the first measurement with the LAGEOS satellites (Ciufolini et al., 1997) that was later confirmed with an accuracy of about 10 % (Ciufolini and Pavlis, 2004), and, using a different spacecraft (Gravity Probe B), with an accuracy of 19 % (Everitt et al., 2011). There are also other proposed experiments to measure the frame-dragging such as GINGER (Bosi et al., 2011)(DiVirgilio et al., 2014) planned in the next few years. The LARES satellite, put in orbit with the new VEGA launcher on the 13th February 2012 (Paolozzi and Ciufolini, 2013) (Paolozzi et al., 2015), is expected to measure the Lense-Thirring effect with an unprecedented accuracy of about 1 % (Ciufolini et al., 2011)(Ciufolini et al., 2012a)(Ciufolini et al., 2012b). While the equivalence principle, at the foundation of general relativity, has a fundamental role also in classical Galilei-Newton mechanics, gravitational waves and frame-dragging have no counterparts in classical theories. The use of science fiction and relevant simulations could be seen as a way to involve students and non specialists in science and technology. In the following we will describe frame dragging, recalling some basic principles of relativity, and

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the LARES experiment. Then we will push some parameters of the experiment to extreme values so as to magnify the effects of relativity so well exploited in science fiction.

2 FRAME-DRAGGING AND LENSE-THIRRING EFFECT

We are so accustomed to very common physical phenomena that we do not realize the deep concepts they involve. The centrifugal force, or more generally any inertial force, is a good example. Everyone has felt those forces in a non-uniform motion. A car moving along a curve or accelerating induces an "apparent force": the inertial force or, in the first case, more specifically the centrifugal force. Those forces exist in the body fixed reference frame, which is not an inertial frame. But the next question is: what are the inertial reference frames? According to the standard definition they correspond to observers not subject to any force. General relativity generalizes that definition stating that all "freely falling" observers are equivalent to inertial observers, but the free fall depends on the mass-energy distribution in the universe. In fact masses distort spacetime, a fact mathematically embodied in the elements of the metric tensor, i.e. the mathematical tool expressing distances and, more generally, the geometry of spacetime. The warps in spacetime in turn determine the paths of freely falling objects in the gravitational field. These paths are called geodesics in space-time. A person ("an observer") falling along a geodesic path is locally indistinguishable from a person in an inertial reference frame. A massive body distorts spacetime, furthermore, a rotating massive body will produce an additional spacetime deformation. Since the Earth rotates, it will produce such an additional spacetime distortion, though very weakly because it rotates very slowly and is not very massive. In Newtonian mechanics a perfectly spherical and homogeneous body will generate a gravitational field in the same way as if the entire mass was concentrated in the centre of the sphere. The orbital plane of a satellite will remain fixed, in the simplified hypothesis of absence of non conservative forces, with respect to distant stars, which in classical mechanics form an inertial reference frame. But if the body rotates, general relativity predicts that also the orbital plane rotates very slowly. The classical angular momentum $\overrightarrow{J}$ of a body is a vector defined as $I\overrightarrow{\omega}$, where I represents the mass property of the body (more rigorously called moment of inertia) and $\overrightarrow{\omega}$ the angular velocity vector which is oriented as shown in Figure 1.

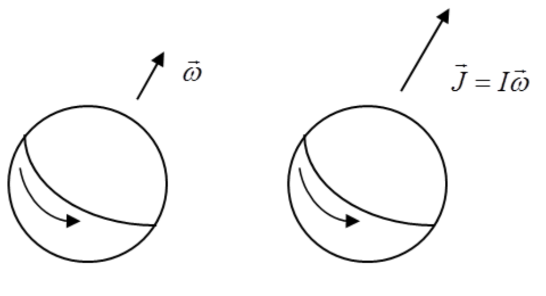

Figure 1: Representation of angular velocity (left) and of angular momentum (right).

2.1 Lense-Thirring Effect

A graphical representation of what happens to the orbital plane of a satellite in classical and in general relativity theory is represented in Figure 2.

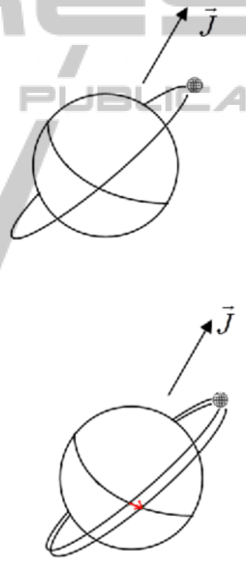

Figure 2: Orbital planes in a reference frame fixed with respect to distant stars. (Top) classical mechanics; (bottom) general relativity. The red arrow shows the Lense-Thirring nodal shift.

Mathematically, by representing the unit vector of $\overrightarrow{J}$ with $\hat{J}$, the angular velocity of the precession $\overrightarrow{\Omega}$ of the orbital plane of a satellite is (in the approximation of slow motion and weak field) (Chandrasekhar, 1983):

$$\overrightarrow{\Omega}_{LT} = \frac{2GJ}{c^2 a^3 (1-e^2)^{3/2}} \hat{J} \tag{1}$$

where $\overrightarrow{\Omega}_{LT}$ is the Lense-Thirring precession of an orbital plane, a and e the semimajor axis and the eccentricity of the orbit, c the speed of light and G the gravitational constant.

2.2 Frame-dragging

More generally frame-dragging can be evaluated, for a generic point of coordinates (a, θ), where θ is the colatitude. The longitude is not present because frame-dragging is identical for points of any longitude.

The precession is given by (Tartaglia, 2000):

$$\vec{\Omega} = \frac{R_S A C}{a^3 + aA^2 + R_S A^2 \sin^2\theta + aA^2\cos^2\theta(1+\frac{A^2}{a^2})}\hat{J} \quad (2)$$

where

$$A = \frac{J}{Mc} \quad (3)$$

$$J = I\omega \quad (4)$$

and

$$R_S = \frac{2GM}{c^2} \quad (5)$$

is the Schwarzschild radius and M the mass of the central body. The Schwarzschild radius can be considered a radius inside which a mass is so concentrated that nothing can escape from it, not even light.

3 LARES SPACE EXPERIMENT

LARES is a passive satellite of the Italian Space Agency (ASI) put in orbit on the 13 February 2012. The position of the satellite is obtained by means of about 50 laser stations belonging to the International Laser Ranging Service (Pearlman et al., 2002). The satellite was designed to minimize the effects of non gravitational perturbations (Paolozzi et al., 2011) and in particular thermal thrust (Ciufolini et al., 2014). Alternative designs were studied earlier in (Ciufolini et al., 2003) (Bosco et al., 2007). The main objective of the LARES mission is to measure the Lense-Thirring effect, and to improve its previous measurement (Ciufolini, 2010) (Ciufolini et al., 2012c) by one order of magnitude (Ciufolini et al., 2010). The feasibility of this goal has been demonstrated in (Ciufolini et al., 2013a) and by the recent data analysis performed in (Ciufolini et al., 2012b) (Ciufolini et al., 2013b).

The fact that the Earth is neither spherical nor homogeneous causes the orbital plane to rotate as shown in Figure 3 where the actual orbits of LARES, determined by laser ranging, are drawn. The experimental value obtained for LARES orbit precession is of 1.706 degrees per day, as can be approximately verified in Figure 3 using a protractor. An entire rotation of 360 degrees occurs in 211 days. The Lense-Thirring effect is instead much smaller: only about 0.118/year. That is, about 19 million times smaller than the classical effect.

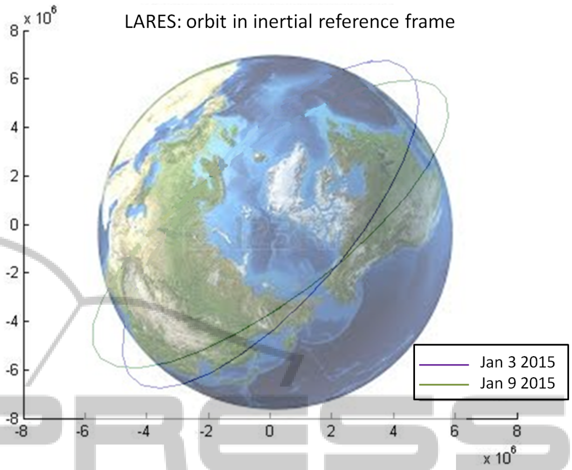

Figure 3: Real orbits of LARES six days apart one from the other as seen from the north pole in an inertial reference frame. Units in the axis are meters.

4 LIMIT CASES

In this section we will examine limit cases in which the relativity effects are magnified. We will suppose first the Earth rotating with a surface peripheral velocity not far from the speed of light. We will then consider LARES orbiting around a black hole with the same "size" of the Earth. We shall discover that LARES is too close to the event horizon of the black hole and it would spiral down. However the orbit of LARES will be considered stable also in this case.

4.1 Earth

Frame-dragging would be more pronounced if the Earth angular momentum would increase. Considering the Earth undeformable and unbreakable, let us imagine a limit case in which the Earth would rotate, so that the surface almost reaches the speed of light.

In Figure 4 equations 1 and 2 are compared. In particular the effect of the colatitude θ on frame dragging is small even at the speed of light. Values of θ are reported in the top left corner of the table.

So, without making significant errors, we can, for the sake of simplicity, assume θ = 90 degrees (i.e. a point of the equator). In this case equation 2 yields 19.4 degrees/year. So summarizing we see that frame dragging is very small around the Earth, no matter how fast it rotates: for an Earth equator rotating at

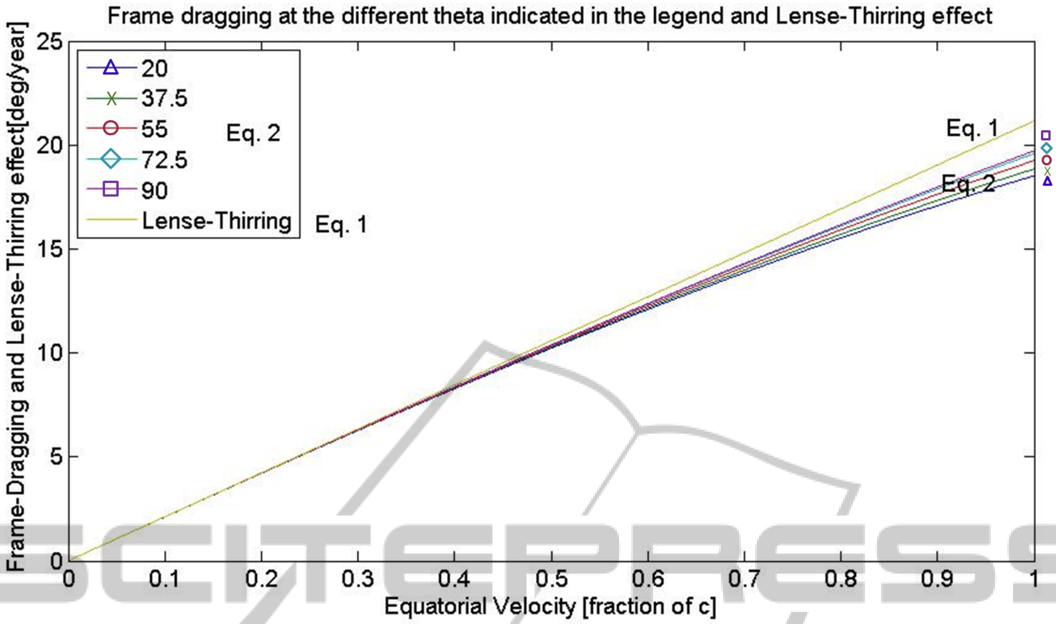

Figure 4: Frame-dragging for different values of θ and Lense-Thirring effect as a function of Earth peripheral speed (i.e. the speed of points located in the Earth equator).

almost the speed of light, the effect on the LARES orbit would be only 19.4 degrees/year i.e., smaller than the classical effect shown in Figure 3 due to the Earth oblateness, which amounts to 623 degrees/year.

4.2 Rotating Black Hole

Suppose now to have a rotating black hole (technically it is called a Kerr black hole) whose inner event horizon has the size of the Earth. This means that the mass of the black hole, M_{bh}, would be (using the Schwarzschild radius formula):

$$M_{bh} = \frac{R_s c^2}{2G} = \frac{R c^2}{2G} = 4.310^{33} kg \qquad (6)$$

i.e. about 2160 solar masses. If the black hole would not rotate, a small object (like LARES) could not revolve in a stable orbit at the same distance at which it rotates now. In fact the lowest possible circular orbit should have a radius of not less than 3 R_S (i.e. a semimajor axis of about 20000 km), whereas now LARES orbits at 7820 km from the center of the Earth. In practice LARES would fall into the black hole in a certain amount of time which is not the case to calculate here. Just to pursue our example this aspect will be neglected and LARES will be considered in a stable orbit around the black hole (Figure 5).

Let us first recall that for a black hole it is not possible to separate the angular velocity from the moment of inertia. In fact a black hole destroys all the information falling inside it; all that can be "felt", in the surroundings outside the event horizon, is the mass, the angular momentum and the charge. There is an upper limit for the total angular momentum i.e. the ratio A/R_S can be at most 1/2, which means that the highest possible angular momentum would be:

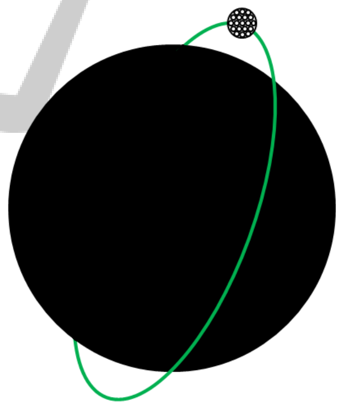

Figure 5: A LARES satellite orbiting a black hole of the "size" of the Earth.

$$J_{max} = G\frac{M^2}{c} \qquad (7)$$

The application of equation 2 in the case of the black hole under concern with a value of J ranging from zero to J_{max} provides the curves reported in Figure 6 parameterized with the value of the colatitudes θ listed in the box in the top left corner. The values of frame-dragging in this example are extraordinarily

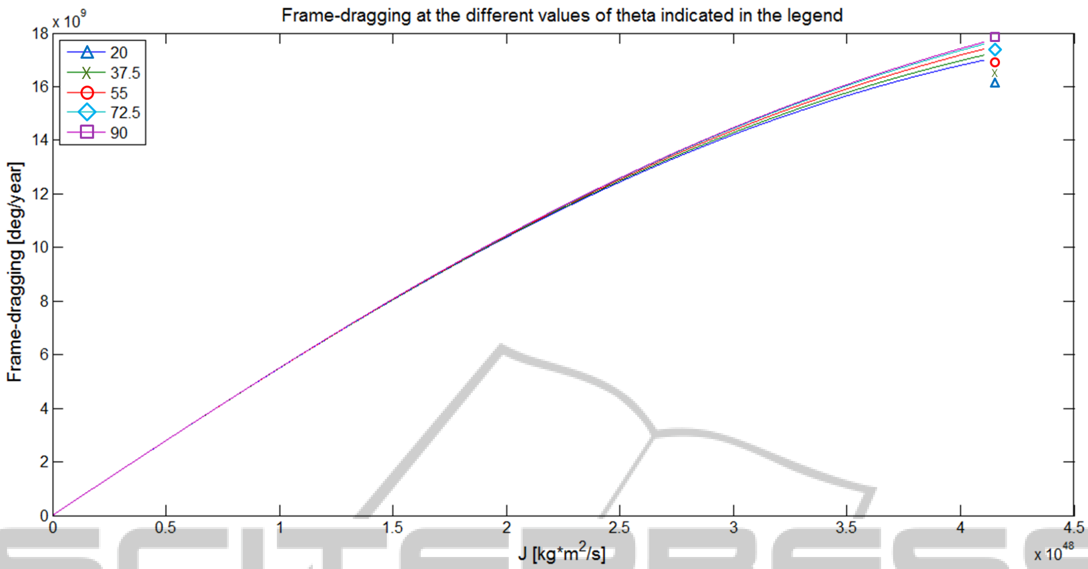

Figure 6: Frame-dragging for different values of θ (in the box on the upper left corner) as a function of the black hole angular momentum.

high. At almost the maximum value of J, frame dragging would be $16x10^9$ deg/year i.e. the nodes of the orbits would revolve at a rate of 1.4 rev/s.

5 CONCLUSIONS

The phenomenon of frame dragging has been described using the LARES mission. The parameters of the experiment have been pushed to unrealistic values, with the purpose of magnifying the effects of relativity. An hypothetical LARES orbiting a rotating black hole, with event horizon of the same size as the Earth has also been considered. The aim of the study is mainly to attract the attention and interest of non specialists to a tiny effect of general relativity, as the frame dragging is. It is shown that the dragging (or Lense-Thirring) effect becomes really important only in the extreme case of a central rotating black hole. Despite this fact the LARES mission will be able to measure it around the Earth. The reader can thus understand how delicate the experiment is and what level of accuracy is needed to bring it to success.

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