Decision Guidance Analytics Language (DGAL)
Toward Reusable Knowledge Base Centric Modeling

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Abstract: Decision guidance systems are a class of decision support systems that are geared toward producing actionable recommendations, typically based on formal analytical models and techniques. This paper proposes the Decision Guidance Analytics Language (DGAL) for easy iterative development of decision guidance systems. DGAL allows the creation of modular, reusable and composable models that are stored in the analytical knowledge base independently of the tasks and tools that use them. Based on these unified models, DGAL supports declarative queries of (1) data manipulation and computation, (2) what-if prediction analysis, (3) deterministic and stochastic decision optimization, and (4) machine learning, all through formal reduction to specialized models and tools, and in the presence of uncertainty.

1 INTRODUCTION

Making decisions is prevalent in various domains including supply chain and logistics, manufacturing, power, energy and sustainability to name a few. To support decision making, enterprises turned to Decision Support Systems (DSS) (Shim et al., 2002) and, more recently, to Decision Guidance Systems (DGS) (Brodsky & Wang, 2008). Broadly, DSS support decision makers in general ways, including with useful well-organized information and visualization. DGS is a class of DSS that are geared toward producing actionable recommendations, typically based on formal analytical models and techniques. This paper proposes the Decision Guidance Analytics Language (DGAL) and Analytical Knowledge Base for easy iterative development and reuse of models and applications of DGS.

Consider an example of DGS for procurement and sourcing, e.g., for a manufacturing facility. Given databases and sources on possible suppliers, a procurement officer needs to monitor, make and execute procurement decisions, e.g., which items in what quantities and from which suppliers should be procured, as to satisfy business constraints (production schedule, risk mitigation, inventory capacity etc.) and minimize the total cost. The technical tasks to be used by DGS can be broadly divided into three categories of (1) descriptive, (2) predictive, and (3) prescriptive decision analytics. The descriptive analytics tasks resemble those of database management systems, dealing with data manipulation and transformation of data (especially temporal sequences) from multiple sources. For example, the procurement officer may want to monitor the status of orders, inventories, schedules, and DGS will need to generate different aggregated views of this information, continuously, over time. The predictive analytics may use the techniques of stochastic simulation and statistical learning for regression, classification and estimation (Shmueli & Koppius, 2011). For example, given the current inventory and orders status, as well as uncertainty in future pricing and supply, the procurement officer may want to estimate the level of inventories and financial status over the next month and identify risks. Prediction and estimation of uncertain outcomes, in turn, may involve regression analysis of functions (Montgomery, Peck & Vining, 2012) such as for cost, time, risk, or building classifiers for different categories of outcomes (e.g., normal operation, schedule delay, financial default, etc.) Prescriptive analytics involves optimization and sensitivity analysis (Haas et al., 2011). For example, the procurement officer may ask for a recommendation on procurement, e.g., which items in what quantities from which suppliers and at what time should be purchased, as to satisfy business rules and constraints and minimize the total procurement cost. The prescriptive analytics tasks would typically correspond to a deterministic or stochastic optimization problem, possibly using multiple criteria and under various business assumptions. The goal of the Decision Guidance Analytics Language (DGAL) and Analytical Knowledge Base (AKB), which we propose in this paper, is to support easy development and reuse of models of the descriptive, predictive and prescriptive analytical tasks in decision guidance systems.

As discussed in Section 2 (Related Work), due to the diversity of computational tools, each designed for a different task (such as data manipulation, predictive what-if analysis, decision optimization or statistical learning), modeling typically requires the use of different mathematical abstractions /languages. Essentially, the same underlying reality must often be modelled multiple times using different mathematical abstractions. Furthermore, the modeling expertise required for these abstractions/languages is typically not within the realm of business analysts and business end users.

Most problematic in decision guidance modeling today is the fact that it is task-centric: every analytical task is typically implemented from scratch, following a linear, non-reusable methodology of gathering requirements, identifying data sources, developing a model/algorithm using a range of modeling languages and tools, performing analysis (see upper part of Figure 1). Using this conventional approach, which we call *task-centric*, models and algorithms are difficult to develop, modify and extend. Furthermore, they typically are not modular or reusable, nor do they support compositionality.

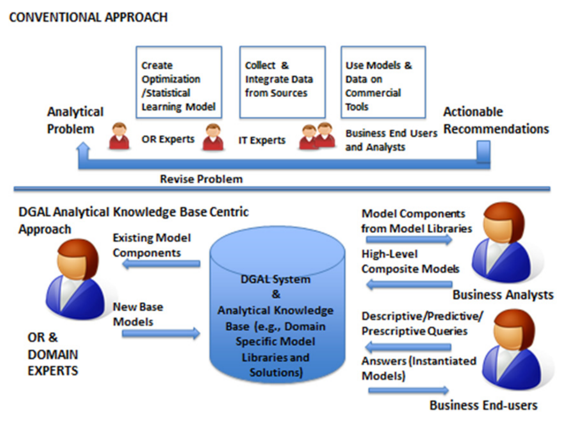

CONVENTIONAL APPROACH
Create Optimization /Statistical Learning Model
Collect & Integrate Data from Sources
Use Models & Data on Commercial Tools
Analytical Problem
OR Experts
IT Experts
Business End Users and Analysts
Actionable Recommendations
Revise Problem
DGAL Analytical Knowledge Base Centric Approach
Existing Model Components
New Base Models
OR & DOMAIN EXPERTS
DGAL System & Analytical Knowledge Base (e.g., Domain Specific Model Libraries and Solutions)
Model Components from Model Libraries
High-Level Composite Models
Business Analysts
Descriptive/Predictive/ Prescriptive Queries
Answers (Instantiated Models)
Business End-users

Figure 1: Conventional Approach vs. DGAL Approach.

Overcoming the outlined limitations of decision guidance modeling is the focus of this paper. More specifically, the contributions of this paper are twofold. First, we introduce the concept of and the design principles for the Decision Guidance Analytics Language (DGAL) and the Analytical Knowledge-Base (AKB). The key idea is a paradigm shift from the non-reusable task-centric modeling approach to reusable-AKB-centric approach (see lower part of Figure 1). In the latter approach, modular reusable and composable models, which we call analytical objects (AOs), are created and stored in the KB independently of the tasks and the tools that may use these models. Second, we provide a specification of DGAL and explain its features and semantics through examples. DGAL supports the creation, composition and easy modification of unified AO's which model (1) data and schema, (2) decision variables, (3) computation of functions, (4) constraints, and (5) uncertainty. Against the unified AO's in the AKB, DGAL users can pose declarative queries for (1) data manipulation and computation, (2) what-if prediction analysis, (3) deterministic and 1-stage stochastic decision optimization, and (4) machine learning. In DGAL the declarative queries are answered through a formal reduction to specialized models and tools.

The paper is organized as follows. In Section 2 we discuss the related work and also its limitations. Section 3 presents the design goals and key functionalities of DGAL. Section 4 overviews JSON and JSONiq for descriptive analytics, with its computational DGAL extensions. Section 5 describes the construction and compositio of Analytical Knowledge and explains how computation, optimization and learning queries can be posed against AOs and their semantics. Section 6 describes modeling of uncertainty in AO's and the corresponding declarative query operators. Section 7 concludes, provides more details on related work, and outlines future research questions.

2 RELATED WORK & ITS LIMITATIONS

To understand the current state of the art and its limitations, consider the six classes of tools/languages relevant to modelling analytical tasks in decision guidance systems: (1) closed domain-specific end-user oriented tools, e.g., strategic sourcing optimization modules within procurement applications (Katz et al, 2011) (Xu &

Howitt, 2009); (2) data manipulation languages, such as SQL, XQuery (Rys, Chamberlin & Florescu, 2005) and JSONiq (Florescu & Fourny, 2013); (3) simulation modelling languages, such as Modelica (Fritzson & Engelson, 1998); (4) simulation languages, such as Jmodelica and Simulink (Akesson et al., 2010); (5) optimization modelling languages, such as AMPL (Fourer, Gay & Kernighan, 1987), GAMS (Rosenthal, 2004), and OPL (Van Hentenryck et al, 1999) for Mathematical Programming (MP) and Constraint Programming (CP); and (6) statistical learning languages/interfaces, such as PMML (Guazzelli et al, 2009). Domain specific tools may be easy to use for a particular well-defined task, but are not extensible to reflect the diversity of emerging descriptive, predictive and prescriptive analytical tasks. Nor do they support compositionality, i.e., the ability to compose their (white-box) models to achieve global (system-wide) optimal predictions and/or prescriptions (e.g., actionable recommendations), rather than local (silo) optimal predictions and prescriptions. Data manipulation languages, obviously, do not support predictive what-if analysis, optimization or statistical learning.

Simulation languages and tools have the advantage of their modelling expressivity, flexibility, and OO modularity, which support reusability and interoperability of (black-box) simulation models. However, performing optimization using simulation models/tools is based on (heuristically-guided) trial and error. Because of that, simulation-based optimization is significantly inferior, in terms of optimality of results and computational complexity, to MP and CP tools/algorithms, for problems expressible in supported analytical forms (e.g., MILP) (Jain & Grossmann, 2001). Also, while sufficiently expressive, simulation languages were not designed for declarative and easy data manipulation provided by data manipulation languages such as SQL, XQuery and JSONiq.

Because optimization modelling languages such as AMPL (Fourer, Gay & Kernighan, 1987), GAMS (Rosenthal, 2004) or OPL (Van Hentenryck et al, 1999) are used with MP and CP solvers, which use a range of sophisticated algorithms that leverage the mathematical structure of optimization problems, they significantly outperform simulation-based optimization, in terms of optimality and running time. However, optimization modelling languages are not modular, extensible, reusable or support compositionality; nor do they support low-level granularity of simulation models (which is expressed through OO programs.) Statistical learning languages/tools have similar limitations and advantages, because most are based on optimization. Like the simulation tools, optimization and statistical learning tools were not designed for easy data manipulation, compared to data manipulation languages such as SQL, XQuery, and JSONiq.

The Modelica simulation modeling language was designed to reuse knowledge. It allows a detailed level of abstraction, including Object-Oriented code and differential equations (Fritzson & Engelson, 1998). However, Modelica by itself is not a language for performing optimization, learning, or prediction. But there are tools such as JModelica for simulation, and Optimica for simulation-based optimization (Akesson et al., 2010). However, because of the low level of abstraction allowed in Modelica, general Modelica models cannot be automatically reduced to MP/CP models and solved by MP/CP solvers.

3 DESIGN GOALS & KEY FUNCTIONALITIES OF DGAL

The following are the design principles we identified for the proposed DGAL language/system:

- Reusable-KB-centric approach: DGAL must support the paradigm shift from non-reusable task-centric modelling approach to reusable-KB-centric modelling approach.
- Task-independent representation of analytical knowledge: DGAL AOs must represent analytical knowledge (including data and its structure, parameters, control/decision variables, constraints and uncertainty) uniformly, regardless of the tasks (e.g., computation, prediction, optimization or learning) that may be using it.
- Unified language for analytical knowledge manipulation: DGAL must uniformly support (1) data manipulation (with the ease of data manipulation languages like SQL and JSONiq), (2) deterministic and stochastic computation/prediction, (3) decision optimization based on MP/CP, (4) statistical/machine learning, and (5) construction/composition of AOs.
- Flexible construction of analytical knowledge: DGAL must support modular AO composition, generalization, specialization and reuse.

- Algebra over analytical KB: DGAL operators must form an algebra over the set of well-defined AOs, that is, operators applied to AOs (in the AKB) must return an AO. Thus, the resulting AOs can be put back into the AKB, and then used by other operators. Note, this is analogous to data manipulation languages (such as SQL, XQuery and JSONiq), which are algebras over the corresponding data model (relational, XML or JSON).
- Declarative high-level language: DGAL analytical knowledge manipulation operators (compute, optimize, learn) must be declarative and simple for end users.
- Compact language core: It is desirable for DGAL to have a compact core, and allow additional functionality through built-in and user-developed libraries (in the knowledge-base).
- Ease of use by modellers: DGAL should be easy to use by mathematical modellers and software/DB developers.
- Ease of use by end users: DGAL should enable built-in KB libraries of AOs to raise the level of abstraction (obscure mathematical details, etc.), which would make it easy to use by end users (such as business analysts and managers)

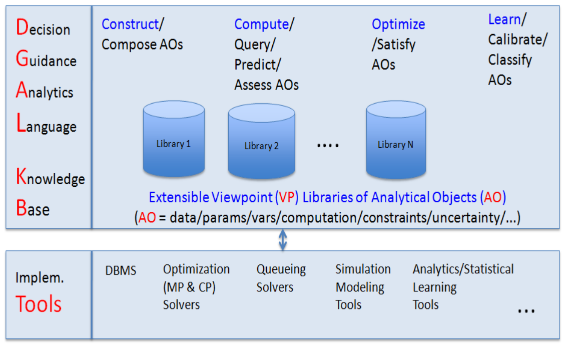

Decision
Guidance
Analytics
Language
Knowledge
Base
Construct/ Compose AOs
Compute/ Query/ Predict/ Assess AOs
Optimize /Satisfy AOs
Learn/ Calibrate/ Classify AOs
Library 1
Library 2
….
Library N
Extensible Viewpoint (VP) Libraries of Analytical Objects (AO)
(AO = data/params/vars/computation/constraints/uncertainty/...)
Implem.
Tools
DBMS
Optimization (MP & CP) Solvers
Queueing Solvers
Simulation Modeling Tools
Analytics/Statistical Learning Tools
…

Figure 2: DGAL Framework.

The high-level DGAL framework and functionality is depicted in Figure 2. Central to the framework is the Analytical Knowledge Base, which is a collection of Analytical Objects (AO), possibly organized in different Viewpoint Libraries. AO is the base component of analytical knowledge. Each AO can represent, uniformly:

- Data and typing: we adopted the Java Script Object Notation (JSON) as the data model, which is becoming a de-facto standard for data analytics.
- Within the data, decision/control variables and parameters over reals, integers and other domains.
- Computation of functions represented via JSON data manipulation language, JSONiq, extended with indexed access, and equation syntax of OPL.
- Constraints, using JSONiq syntax for Boolean expressions, including for universal quantification.
- Uncertainty, by adding distribution functions to expressions (in functions and constraints), which implicitly define random variables. All DGAL operators are applied to AO and return an AO, and so DGAL constitutes an algebra (like data manipulation languages SQL, XQuery and JSONiq). The DGAL operators are of four key types (Upper part of Figure 2).
- Construct: this class of operators allows to construct an AO from scratch, from another AO by specialization/generalization, or by composing an AO from previously defined AOs.
- Compute: this class of operators instantiate an AO by perform computation of functions (may involve uncertainty quantification).
- Optimize: this class of operators instantiate an AO by finding values of decision variables that optimize an objective, and then compute it with the optimal values.
- Learn: this class of operators instantiate an AO by funding values of its parameters, as to minimize an estimation error against a learning set.

The (deterministic) optimization and learning operators are performed by creating a formal mathematical or constraint programming model and solving it using an appropriate solvers (Lower part of Figure 2). The AO, AKB and DGAL operators are designed according to the designed principles outlined in this section.

4 OVERVIEW OF JSON AND JSONIQ

JavaScript Object Notation (JSON) is rapidly becoming a data model for descriptive (big data) analytics. For that reason, we would like to use JSONiq, a query language over JSON as a foundation of DGAL. Both JSON and JSONiq are reviewed in this section. We borrow here the description of JSONiq from (JavaScript Object Notation, 2014)(Fourny, 2013).

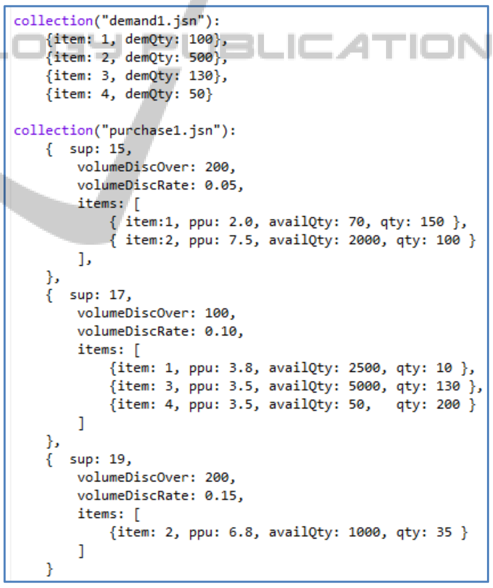
JSON data model is a "lightweight data-interchange format. It is easy for humans to read and write. It is easy for machines to parse and generate" (JavaScript Object Notation, 2014). JSON is an open standard format that uses human-readable text to transmit data objects consisting of attribute-value pairs. It is used primarily to transmit data between a server and web application, as an alternative to XML (Fourny, 2013). Similar to the fact that Xquery is a query and processing language designed for XML data model, JSON data model also have a specially designed powerful query language, JSONiq.

JSONiq is very similar to XQuery, adapted to JSON, including "the structure and semantics of the FLWR (FOR-LET-WHERE-RETURN) construct, the functional aspect of the language, the semantics of comparisons in the face of data heterogeneity, and the declarative, snapshot-based updates" (JavaScript Object Notation, 2014). However, Xquery is more complex and has more language constructs than JSONiq due to the fact that XML data model is more complex than that of JSON. For example, the element type of XML can be mixed with attributes, elements, or text. The order of children elements in XML is significant even with the same contents. The namespaces, QNames, and XML schema can be complicated to describe specific types of XML documents (Fourny, 2013).

The FLWR construct is an iteration structure for both JSONiq/Xquery. It is different from regular control structure of programming languages by considering the simplicity of JSON data model. FLWR makes the JSONiq a powerful, clean, and straightforward data processing language. In addition to querying collections of data in JSON format, JSONiq can extract, transform, clean, select, enrich, or join hierarchical or heterogeneous data sets (Fourny, 2013).

The main technical characteristics of JSONiq (and XQuery) are as follows:

- "It is a set-oriented language. While most programming languages are designed to manipulate one object at a time, JSONiq is designed to process sets (actually, sequences) of data objects" (Fourny, 2013).
- It falls into the category of the functional programming paradigm. Different from the procedural programming paradigm such as Object-Oriented programming languages, expression is the basic unit of the JSONiq programs. Every language construct is an expression and expressions are composed from one or more previous expressions (Fourny, 2013).
- It is a declarative language. It describes what computation will be performed, instead of how the computation (process) will be done. It does not consider the implementation details such as specific data structures, algorithms, memory allocations, and indexing in databases. The component of the declarative language usually has clear corresponding relationship to mathematical logic (Fourny, 2013).
- It is designed to process hierarchical or heterogeneous data which is sometimes semi-structured. The JSON data type does not need to follow any specific pattern but can be heterogeneous. It can be nested structures in multiple levels. Consequently it is hard to define a schema which can describe the JSON data well. Sometimes the schema may only be able to describe the data partially (Fourny, 2013).



```
collection("demand1.jsn"):
    {item: 1, demQty: 100},
    {item: 2, demQty: 500},
    {item: 3, demQty: 130},
    {item: 4, demQty: 50}

collection("purchase1.jsn"):
    {  sup: 15,
        volumeDiscOver: 200,
        volumeDiscRate: 0.05,
        items: [
            { item:1, ppu: 2.0, availQty: 70, qty: 150 },
            { item:2, ppu: 7.5, availQty: 2000, qty: 100 }
        ],
    },
    {  sup: 17,
        volumeDiscOver: 100,
        volumeDiscRate: 0.10,
        items: [
            {item: 1, ppu: 3.8, availQty: 2500, qty: 10 },
            {item: 3, ppu: 3.5, availQty: 5000, qty: 130 },
            {item: 4, ppu: 3.5, availQty: 50,   qty: 200 }
        ]
    },
    {  sup: 19,
        volumeDiscOver: 200,
        volumeDiscRate: 0.15,
        items: [
            {item: 2, ppu: 6.8, availQty: 1000, qty: 35 }
        ]
    }
```

Figure 3: Collections "demand1.jsn" and "purchase1.jsn".

To exemplify JSON and JSONiq, first consider JSON collections "demand1.jsn" and "purchase1.jsn" in Figure 3. JSON collections are sequences of objects, identified by a name which is a string, e.g. "demand1.jsn." In the example, the collection "demand1.jsn" is a sequence that contains four objects, {item: 1, demQty: 100}, {item: 2, demQty: 500}, {item: 3, demQty: 130}, and {item: 4, demQty: 50}. Objects are unordered sets of key/value pairs, separated by comma. A key is a string and a value can be any JSON building block. Each key/value pair is separated by a semicolon.

Four items are defined in this collection, with the quantity of demand specified.

Similarly the collection “purchase1.jsn” contains a sequence of three suppliers objects. For each supplier object, four sets of key/value pairs indicate supplier identifier, the overall supplier cost before and after volume discount, and the list of items being purchased from this supplier. Given “items” as the key of the key/value pair, a sequence of items is assigned as the value and of the type JSON Array. Array represents an ordered list of items (in any category) and can nest. For example, for the first supplier with key/value pair of “sup: 15”, two different items are purchased as {item:1, ppu: 2.0, availQty: 70, qty: 150} and {item:2, ppu: 7.5, availQty: 2000, qty: 100}. Each item is represented as a JSON object, with four key/value pairs. Each item has a unique identifier (e.g. 1), price per unit, available quantity from the specific supplier, and purchased quantity.

JSONiq is a language that makes computations of structures from the input collections easy. In the JSONiq example of Figure 4, a JSONiq function *orderAnalytics* is defined with variable $purchase_and_demand as argument and object as return type. The argument variable $purchase_and_demand is of the composite structure of both collections “purchase1.jsn” and “demand1.jsn” as follows:

```
collection("purchaseAndDemand1.jsn"):
{purchase: [collection("purchase1.jsn")],
 demand: [collection("demand1.jsn")]  }
```

The function implements a small supply chain with items, demand, and supplies. In the function body, the variable $supInfo is assigned as an array which contains the sequence of objects in the collection “purchase1.jsn”. The variable $suppliers is assigned as an array of all supplier identifiers such as 15 or 17. The variable $orderedItems is assigned as an array of all the items and demand quantities in the collection “demand1.jsn”.

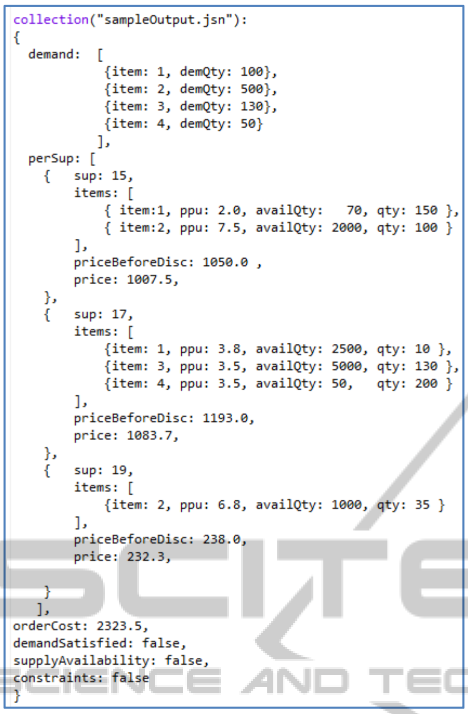
The variable $perSup is defined to calculate for each supplier, the total cost charged for all items supplied by this supplier. The “for” clauses is used to iterate each supplier of $suppliers. For all items supplied by that specific supplier, within the inner “for” clause of the second “let” statement, the variable $priceBeforeDisc represents the cost to purchase items, which are calculated from price per unit and item quantity. The variable $priceAfterDisc is defined to adjust the item cost by considering the volume discount rate. The cost after discount is calculated based on a volume discount formula. If the overall cost is more than the volumeDiscOver, it will be calculated at a discount rate volumeDiscRate. The variable $totalCost sums up the total cost of all suppliers. The variable $supAvailability is defined as a Boolean variable to enforce the business rule that the item quantity purchased must be less than or equal to the available quantity to each specific supplier. The variable $demandSatisfied is defined as another Boolean variable to enforce the business rule that the total market demand for each item cannot beyond the total item supply from all suppliers. The Boolean variable $constraint is the logical ‘&&’ of both $supAvailability and $demandSatisfied.

```
module namespace ns:= "www.gmu.edu/~brodsky/jsoniq_dga_example"
declare function ns:orderAnalytics($purchase_and_demand) as object { let
$supInfo := $purchase_and_demand.purchase[]
$suppliers := $supInfo[].sup
$orderedItems := $purchase_and_demand.demand[]
let $perSup := [
  for $s in $suppliers
    let $priceBeforeDisc := sum ( for $si in $supInfo, $i in $s.items[]
                                  where $si.sup = $s
                                  return $i.ppu * $i.qty
    )
    let $priceAfterDisc := (   let $bound := $s.volumeDiscOver
                 let $disc  := $s.volumeDiscRate
                 return  if $priceBeforeDisc <= $bound
                      then $priceBeforeDiscount
                      else $bound + ($priceBeforeDiscount - $bound) * $disc
    )
    return {sup:$s, items:$s.items,
                   priceBeforeDisc: $priceBeforeDisc, price:$priceAfterDisc}
]
let $totalCost := sum (for $s in $perSup[] return $s.price)
let $supAvailability as boolean :=
    every $i in $supInfo[].items[] satisfies $i.qty <= $i.availQty
let $demandSatisfied as boolean :=
    every ( $i in $orderedItems.item
            let $supQty := sum ($it in $supInfo[].items[]
                                  where $it.item = $i
                                  return $it.qty)
    )
    satisfies $i.demQty <= $supQty
let $constraints as boolean := $supAvailability && $demandSatisfied
return {
    demand: [$orderedItems],
    perSup: $perSup,
    orderCost: $totalCost,
    demandSatisfied: $demandSatisfied,
    supplyAvailability: $supAvailability,
    constraints: $constraints
    }
}
```

Figure 4: JSONiq Example.

The object returned by this function consists of six key-value pairs representing market demand, detailed information for each supplier (identifier, supplied items, price before volume discount, and price after volume discount), total cost of all purchases, if market demand has been satisfied, if the supply availability has been satisfied, and if both availability have been satisfied. The output of this JSONiq example is displayed in Figure 5, given the argument as composite collections in Figure 3.

The FLWOR expression is probably the most powerful JSONiq construct, which corresponds to the SQL Select-From-Where clause, but the JSONiq construct is more general and flexible. The FOR clause allows iteration over a sequence.



```
collection("sampleOutput.jsn"):
{
  demand:  [
            {item: 1, demQty: 100},
            {item: 2, demQty: 500},
            {item: 3, demQty: 130},
            {item: 4, demQty: 50}
           ],
  perSup: [
    {   sup: 15,
        items: [
            { item:1, ppu: 2.0, availQty:   70, qty: 150 },
            { item:2, ppu: 7.5, availQty: 2000, qty: 100 }
        ],
        priceBeforeDisc: 1050.0 ,
        price: 1007.5,
    },
    {   sup: 17,
        items: [
            {item: 1, ppu: 3.8, availQty: 2500, qty: 10 },
            {item: 3, ppu: 3.5, availQty: 5000, qty: 130 },
            {item: 4, ppu: 3.5, availQty: 50,   qty: 200 }
        ],
        priceBeforeDisc: 1193.0,
        price: 1083.7,
    },
    {   sup: 19,
        items: [
            {item: 2, ppu: 6.8, availQty: 1000, qty: 35 }
        ],
        priceBeforeDisc: 238.0,
        price: 232.3,

    }
   ],
orderCost: 2323.5,
demandSatisfied: false,
supplyAvailability: false,
constraints: false
}
```

Figure 5: Output of JSONiq Example.

5 DETERMINISTIC KNOWLEDGE MANIPULATION

5.1 Construction and Composition of Analytical Knowledge

To describe the concepts of DGAL knowledge-base, modules, and analytical object (AO) functions, we need the concept of an *indexable object.* Intuitively, a JSON object is *indexable* if, in every array it contains, the objects in that array are uniquely identified by the value of the first key, which serves as an index to the corresponding object. More formally, we say that a JSON object is *indexable* if, recursively, in every key-value pair, the value is one of the following:

- atomic (i.e., not object or array)
- an *indexable* object
- an array of *indexable* objects with the same keys, where there are no 2 objects with the same value for the first key.

For example, in the collection "*purchaseAndDemand1.jsn*", which serves as input to the *orderAnalytics* function, there are no two "supplier" objects with the same value for the key *sup;* similarly, there are no two *"item"* objects with the same value for the key *item.*

A DGAL knowledge-base comprises of a set of DGAL modules. A DGAL module is simply a JSONiq module that contains *analytical object (AO)* functions. An AO function is simply a JSONiq function with the following two properties:

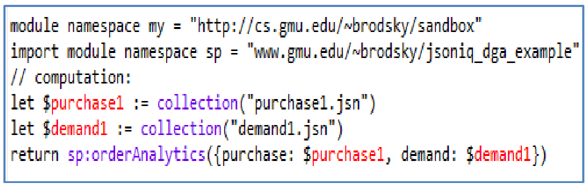
- The input and output of the function must be an *indexable object.*
- The output object must contain the key *constraints* with a value of type Boolean.

For example, the *orderAnalytics* function in the previous section satisfies both conditions, i.e., it is an AO function. Note that, while the *orderAnalytics* function, like any JSONiq function, can be viewed to describe data manipulation (of an input object into an output object), it can also be viewed as an *integrity constraint* on the input object. Namely, the integrity constraint is satisfied by the input object if and only if the Boolean value computed for the *key "constraint"* in the output object is true. Similarly, the *orderAnalytics* function, can be viewed as a mathematical function that gives numerical values (such as *orderCost* computed in the output object) from the input object (including numerical values in it).

Clearly, AO functions, like any JSONiq function, perform data manipulation. For example, to compute information about order analytics from the purchase and demand data, one can simply invoke the JSONiq function *orderAnalytics,* described in Figure 4. However, we will also use AO functions for optimization, statistical learning, simulation and prediction with the same ease as data manipulation. In the next subsections we explain, by examples, how to perform (1) computation/data manipulation, (2) deterministic optimization, and (3) learning in DGAL.

5.2 Computation and Data Manipulation

Performing computation/data manipulation with AO function is just an invocation of a JSONiq function, like in the example in Figure 6. First, we need to import the module in which the function *orderAnalytics* is defined. Then we assign values to the argument of the function. The function is then called to perform the computation. In the example, it returns an object representing demand, cost of each supplier, total cost, and a true/false Boolean value for demand being satisfied, the supply availability being satisfied, and both constraints being satisfied.



```
module namespace my = "http://cs.gmu.edu/~brodsky/sandbox"
import module namespace sp = "www.gmu.edu/~brodsky/jsoniq_dga_example"
// computation:
let $purchase1 := collection("purchase1.jsn")
let $demand1 := collection("demand1.jsn")
return sp:orderAnalytics({purchase: $purchase1, demand: $demand1})
```

Figure 6: Computation.

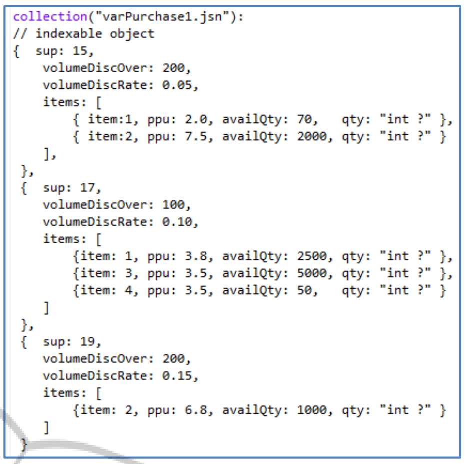
5.3 Optimization

If the quantities in the collection "purchase1.jsn" used as argument to JSONiq query in Figure 4 were not known, the procurement officer may want to find them as to make sure that the constraints of both Boolean expressions "$demandSatisfied" and "$supAvailability" are satisfied (i.e., computed to be true), and the total cost is minimized. Intuitively, the DGAL optimization operators are designed to do this kind of "reverse" computation.

To perform optimization in DGAL, we first need to annotate the input object to the AO function (*orderAnalytics* in the example) to indicate which values (*qty's* in the example) are not known but we would like the system to find (i.e., decision variables). Figure 7 gives an example of such annotation in the collection *"varPurchase1.jsn."* This collection is identical to the collection *"purchase1.jsn"* in Figure 3, with the exception that the key *qty* does not have a numerical value, but has instead a special annotation *"int ?"* to indicate that *qty* will now be a decision variable. The annotated collection must have the following structural limitation. Informally, the structure of the output object should not depend on the values of the annotated decision variables; only numerical values in the output object may depend on the decision variables.

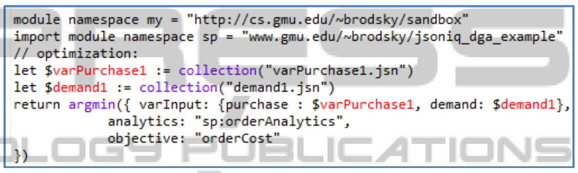
With the annotated input collection, performing optimization is very simple: it is performed by invoking the DGAL function *argmin* (or *argmax)* as exemplified in Figure 8.

The DGAL function *argmin* in Figure 8 is invoked with the following input object. The first key *varInput* has the value which is the annotated input to *orderAnalytics,* i.e., when we use the annotated collection *"varPurchase1.jsn"* from Figure 7 instead of the original collection *"purchase1.jsn"* from Figure 3. The second key *analytics* has the value that indicates the name of the AO function used, *orderAnalytics* in the example. The third key *objective* has the value that indicates the key of the numeric value in the output object of *orderAnalytics – orderCost* in the example - that we would like to use as the objective to be minimized.



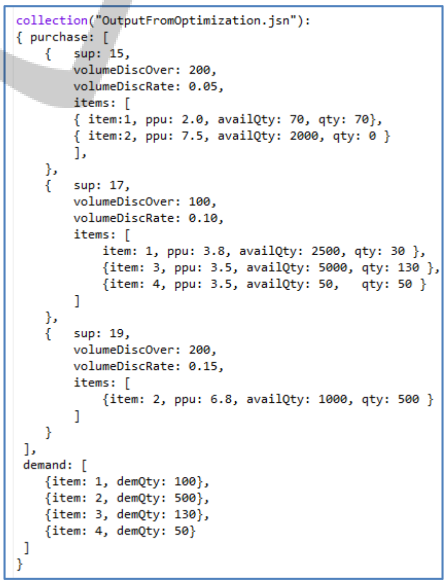
```
collection("varPurchase1.jsn"):
// indexable object
{  sup: 15,
    volumeDiscOver: 200,
    volumeDiscRate: 0.05,
    items: [
        { item:1, ppu: 2.0, availQty: 70,   qty: "int ?" },
        { item:2, ppu: 7.5, availQty: 2000, qty: "int ?" }
    ],
 },
 {  sup: 17,
    volumeDiscOver: 100,
    volumeDiscRate: 0.10,
    items: [
        {item: 1, ppu: 3.8, availQty: 2500, qty: "int ?" },
        {item: 3, ppu: 3.5, availQty: 5000, qty: "int ?" },
        {item: 4, ppu: 3.5, availQty: 50,   qty: "int ?" }
    ]
 },
 {  sup: 19,
    volumeDiscOver: 200,
    volumeDiscRate: 0.15,
    items: [
        {item: 2, ppu: 6.8, availQty: 1000, qty: "int ?" }
    ]
 }
```

Figure 7: varPurchase1.jsn.



```
module namespace my = "http://cs.gmu.edu/~brodsky/sandbox"
import module namespace sp = "www.gmu.edu/~brodsky/jsoniq_dga_example"
// optimization:
let $varPurchase1 := collection("varPurchase1.jsn")
let $demand1 := collection("demand1.jsn")
return argmin({ varInput: {purchase : $varPurchase1, demand: $demand1},
            analytics: "sp:orderAnalytics",
            objective: "orderCost"
})
```

Figure 8: Optimization.



```
collection("OutputFromOptimization.jsn"):
{ purchase: [
    {   sup: 15,
        volumeDiscOver: 200,
        volumeDiscRate: 0.05,
        items: [
        { item:1, ppu: 2.0, availQty: 70, qty: 70},
        { item:2, ppu: 7.5, availQty: 2000, qty: 0 }
        ],
    },
    {   sup: 17,
        volumeDiscOver: 100,
        volumeDiscRate: 0.10,
        items: [
            item: 1, ppu: 3.8, availQty: 2500, qty: 30 },
            {item: 3, ppu: 3.5, availQty: 5000, qty: 130 },
            {item: 4, ppu: 3.5, availQty: 50,   qty: 50 }
        ]
    },
    {   sup: 19,
        volumeDiscOver: 200,
        volumeDiscRate: 0.15,
        items: [
            {item: 2, ppu: 6.8, availQty: 1000, qty: 500 }
        ]
    }
 ],
 demand: [
    {item: 1, demQty: 100},
    {item: 2, demQty: 500},
    {item: 3, demQty: 130},
    {item: 4, demQty: 50}
 ]
}
```

Figure 9: Output of Optimization.

The output of the *argmin* function is an object identical to the annotated input object, with the exception that all annotations *"int ?"* (which denote decision variables) are now replaced with actual numerical values that (1) satisfy the *constraints,* i.e., result in the value true computed for the key *constraints* in the output object; and (2) minimize the numerical value computed for the key *orderCost,* which was designated as the objective in the invocation of the *argmin* function. The collection *"OutputFromOptimization.jsn"* in Figure 9 exemplifies the optimization output. More generally, given an invocation

argmin({varInput: I, analytics: A, objective:O})

let

- *X* be a vector of decision variables corresponding to annotations "int ?" and "float ?" in *I*
- *f: D* → *R* be a function from the domain *D* of X to Reals, that corresponds to the computation of the value of the key *O* (for *objective*) in the output object computed by the JSONiq function *A* applied to *I*
- *C(X)* be a set of constraints in variables X, that corresponds to the Boolean value for the key *constraints* in the output object computed by JSONiq function *A* applied to *I*

The semantics of the DGAL function *argmin* is as follows:

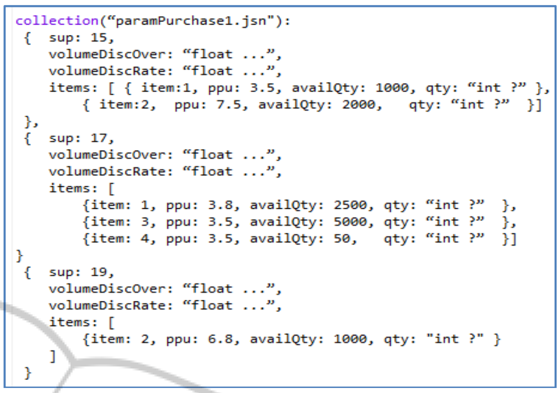
- If C(X) is infeasible, *argmin* returns the string *"infeasible."*
- If C(X) is feasible, but *f(X)* does not have a lower bound subject to C(X), *argmin* returns the string *"unbounded."*
- Otherwise, let Xo be a solution to the problem *min f(X) subject to C(X).* DGAL *argmin* function returns the annotated input object *I* in which all annotations "int ?" and "float ?" are replaced with the corresponding numerical values from Xo.

The semantics of *argmax* is similar.

5.4 Learning

If the volume discount parameters, *volumeDiscOver* and *volumeDiscRate,* were not known in the collection "*purchase1.jsn*", how can we learn them (via regression analysis) from historical data? The DGAL function *learn* is provided for this purpose.

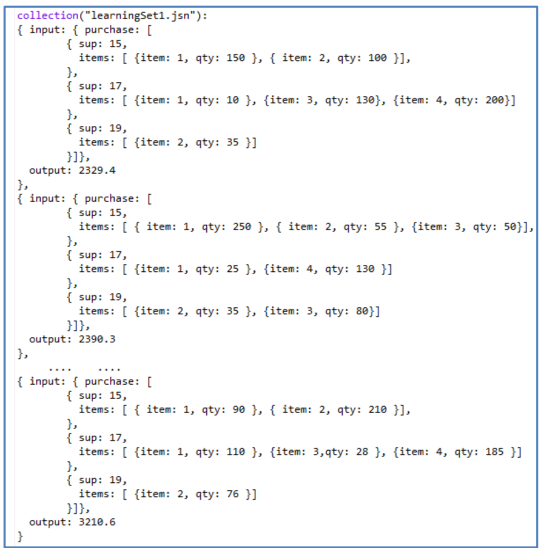
To use the *learn* function in DGAL, we first need to annotate the input object to the AO function (*orderAnalytics* in the example) to indicate which parameters we would like to learn. In the example these parameters are *volumeDiscOver* and *volumeDiscRate* for each supplier in collection *"varPurchase.jsn"*. Figure 10 gives an example of such annotation in the collection *"paramPurchase1.jsn"*. This collection is identical to the collection *"varPurchase1.jsn* in Figure 7, with the exception that the keys *volumeDiscOver* and *volumeDiscRate* do not have a numerical value, but have instead a special annotation *"float …"* to indicate that they will now be parameters to be learned using regression.



```
collection("paramPurchase1.jsn"):
 {  sup: 15,
    volumeDiscOver: "float ...",
    volumeDiscRate: "float ...",
    items: [ { item:1, ppu: 3.5, availQty: 1000, qty: "int ?" },
        { item:2,  ppu: 7.5, availQty: 2000,   qty: "int ?"  }]
 },
 {  sup: 17,
    volumeDiscOver: "float ...",
    volumeDiscRate: "float ...",
    items: [
        {item: 1, ppu: 3.8, availQty: 2500, qty: "int ?"  },
        {item: 3, ppu: 3.5, availQty: 5000, qty: "int ?"  },
        {item: 4, ppu: 3.5, availQty: 50,   qty: "int ?"  }]
}
 {  sup: 19,
    volumeDiscOver: "float ...",
    volumeDiscRate: "float ...",
    items: [
        {item: 2, ppu: 6.8, availQty: 1000, qty: "int ?" }
    ]
 }
```

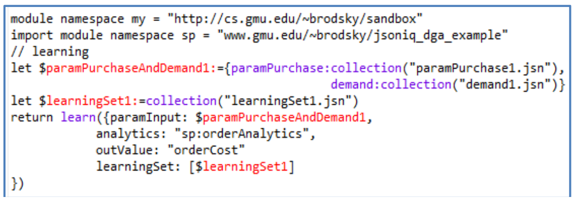
Figure 10: paramPurchase1.jsn.

We also need to create a learning set collection, which captures a set of (historical) input-output pairs of a function we are trying to regress. The collection "*learningSet1.jsn*" in Figure 11 exemplifies the learning set. It contains a sequence of JSON objects, each having keys input and output. The value for the key output in every object gives a historical value for *orderCost*. The value for the key input in every object gives a "partial" input object for the AO function orderAnalytics. It is "partial" because we only need information on the quantities for each item order from each supplier, but do not need any other information. With the "paramPurchase1.jsn" collection and the learning set object, performing regression learning is done by invocation of DGAL function *learn*, as shown in Figure 12.



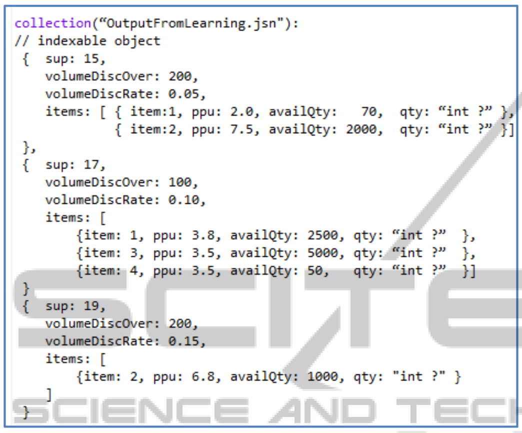
```
collection("learningSet1.jsn"):
{ input: { purchase: [
        { sup: 15,
          items: [ {item: 1, qty: 150 }, { item: 2, qty: 100 }],
        },
        { sup: 17,
          items: [ {item: 1, qty: 10 }, {item: 3, qty: 130}, {item: 4, qty: 200}]
        },
        { sup: 19,
          items: [ {item: 2, qty: 35 }]
        }]},
  output: 2329.4
},
{ input: { purchase: [
        { sup: 15,
          items: [ { item: 1, qty: 250 }, { item: 2, qty: 55 }, {item: 3, qty: 50}],
        },
        { sup: 17,
          items: [ {item: 1, qty: 25 }, {item: 4, qty: 130 }]
        },
        { sup: 19,
          items: [ {item: 2, qty: 35 }, {item: 3, qty: 80}]
        }]},
  output: 2390.3
},
     ....    ....
{ input: { purchase: [
        { sup: 15,
          items: [ { item: 1, qty: 90 }, { item: 2, qty: 210 }],
        },
        { sup: 17,
          items: [ {item: 1, qty: 110 }, {item: 3,qty: 28 }, {item: 4, qty: 185 }]
        },
        { sup: 19,
          items: [ {item: 2, qty: 76 }]
        }]},
  output: 3210.6
}
```

Figure 11: Learning Set.



```
module namespace my = "http://cs.gmu.edu/~brodsky/sandbox"
import module namespace sp = "www.gmu.edu/~brodsky/jsoniq_dga_example"
// learning
let $paramPurchaseAndDemand1:={paramPurchase:collection("paramPurchase1.jsn"),
                               demand:collection("demand1.jsn")}
let $learningSet1:=collection("learningSet1.jsn")
return learn({paramInput: $paramPurchaseAndDemand1,
          analytics: "sp:orderAnalytics",
          outValue: "orderCost"
          learningSet: [$learningSet1]
})
```

Figure 12: Learning.



```
collection("OutputFromLearning.jsn"):
// indexable object
 {  sup: 15,
    volumeDiscOver: 200,
    volumeDiscRate: 0.05,
    items: [ { item:1, ppu: 2.0, availQty:   70,  qty: "int ?" },
             { item:2, ppu: 7.5, availQty: 2000,  qty: "int ?" }]
 },
 {  sup: 17,
    volumeDiscOver: 100,
    volumeDiscRate: 0.10,
    items: [
       {item: 1, ppu: 3.8, availQty: 2500, qty: "int ?"  },
       {item: 3, ppu: 3.5, availQty: 5000, qty: "int ?"  },
       {item: 4, ppu: 3.5, availQty: 50,   qty: "int ?"  }]
 }
 {  sup: 19,
    volumeDiscOver: 200,
    volumeDiscRate: 0.15,
    items: [
       {item: 2, ppu: 6.8, availQty: 1000, qty: "int ?" }
    ]
 }
```

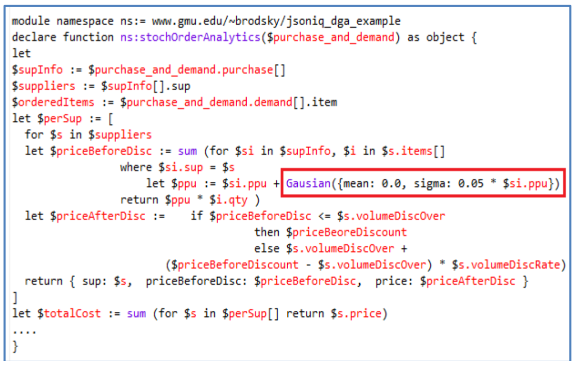
Figure 13: Output from Learning.

The function *learn* is invoked with an object with key-value pairs for *paramInput*; analytics to indicate the AO function to be learned (*orderAnalytics* in the example); *outValue*, to indicate which value the function computes (*orderCost* in the example), and the previously constructed *learningSet*.

The output of function *learn* is exemplified in Figure 13. The structure of output is exactly as the input collection "*paramPurchaseAndDemand1*" with the exception that parametric annotations "float …" replaced by values. However, the decision variables annotations "int ?" are still left as they were. Note that both parameters and decision variables can be either of type int or float. The values for parameters are constructed to minimize the summation of squares of learning errors for each input-output pair in the learning set.

6 UNCERTAINTY: SIMULATION, PREDICTION AND STOCHASTIC OPTIMIZATION

Consider Figure 14, which exemplifies how uncertainty is represented in AO functions.



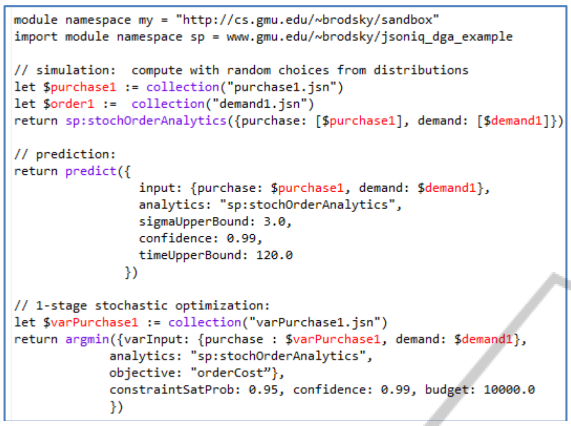
```
module namespace ns:= www.gmu.edu/~brodsky/jsoniq_dga_example
declare function ns:stochOrderAnalytics($purchase_and_demand) as object {
let
$supInfo := $purchase_and_demand.purchase[]
$suppliers := $supInfo[].sup
$orderedItems := $purchase_and_demand.demand[].item
let $perSup := [
  for $s in $suppliers
  let $priceBeforeDisc := sum (for $si in $supInfo, $i in $s.items[]
               where $si.sup = $s
                   let $ppu := $si.ppu + Gausian({mean: 0.0, sigma: 0.05 * $si.ppu})
               return $ppu * $i.qty )
  let $priceAfterDisc :=    if $priceBeforeDisc <= $s.volumeDiscOver
                             then $priceBeoreDiscount
                             else $s.volumeDiscOver +
                      ($priceBeforeDiscount - $s.volumeDiscOver) * $s.volumeDiscRate)
  return { sup: $s,  priceBeforeDisc: $priceBeforeDisc,  price: $priceAfterDisc }
]
let $totalCost := sum (for $s in $perSup[] return $s.price)
....
}
```

Figure 14: AO with uncertainty.

In the example, the *stochOrderAnalytics* function is identical to the (deterministic) *orderAnalytics* function in Figure 4 with the exception that, in the *$ppu* computation, a random value drawn from the Gaussian distribution is used (see a box in Figure 14). When this is done, the variable *$ppu* on the left of the assignment statement represents a random variable. Furthermore, all expressions that are dependent on it in the computation, directly or indirectly, also represent random variables.

In the context of uncertainty, we can talk about simulation, prediction and stochastic optimization (see example in Figure 15). Simulation is done exactly as computation, namely by invoking the AO function *stochOrderAnalytics*. Because a random value is drawn (see Figure 14), every invocation of *stochOrderAnalytics* may result in a different answer, which is a result of stochastic simulation.

We can also perform prediction, e.g., using the Monte-Carlo method, to estimate the expectation of the random variables (*$ppu* and all expressions that depend on it in Figure 14). This is done using the DGAL function *predict,* which is invoked with the input object that has key-value pairs *input, analytics, sigmaUpperBound, confidence* and *timeUpperBound* (see *prediction* part in Figure 15). The value for *input* is the same as in the computation (i.e., collection *purchaseAndDemand1*); *analytics* indicates the used AO function (*stochOrderAnalytics* in the example). The estimates of the random variables are done so that the standard deviation would not exceed the *sigmaUpperBound* with the indicated statistical *confidence,* unless computation time exceeds the indicated *timeUpperBound,* which is an optional parameter. The form of the output is the same as in the deterministic computation (the example in Figure 5), with annotation, with the exception that instead of the (deterministically) computed numeric values, the output object contains JSON objects that capture estimations of the expectation and standard deviation of random variables.



```
module namespace my = "http://cs.gmu.edu/~brodsky/sandbox"
import module namespace sp = www.gmu.edu/~brodsky/jsoniq_dga_example

// simulation:  compute with random choices from distributions
let $purchase1 := collection("purchase1.jsn")
let $order1 :=  collection("demand1.jsn")
return sp:stochOrderAnalytics({purchase: [$purchase1], demand: [$demand1]})

// prediction:
return predict({
                 input: {purchase: $purchase1, demand: $demand1},
                 analytics: "sp:stochOrderAnalytics",
                 sigmaUpperBound: 3.0,
                 confidence: 0.99,
                 timeUpperBound: 120.0
               })

// 1-stage stochastic optimization:
let $varPurchase1 := collection("varPurchase1.jsn")
return argmin({varInput: {purchase : $varPurchase1, demand: $demand1},
             analytics: "sp:stochOrderAnalytics",
             objective: "orderCost"},
             constraintSatProb: 0.95, confidence: 0.99, budget: 10000.0
             })
```

Figure 15: Simulation, prediction, 1-stage stochastic optimization.

Finally, a one-stage stochastic optimization is performed by invoking the DGAL function *argmin* (or *argmax*). The input object is the same as in the deterministic case, with the exception that it is extended with key-value pairs for the required *constraintSatProb* to indicate with what minimal probability the constraints must be satisfied and with what statistical *confidence*. Optionally, a maximum computation *budget* can be indicated.

The output of the *argmin* function has the same form as in the deterministic case. Semantically, *argmin* in this case is interpreted as a (one-stage) stochastic optimization to minimize the expectation of the indicated objective (*orderCost* in the example), which is now interpreted as a random variable.

7 MORE ON RELATED WORK AND CONCLUSIONS

In this paper we proposed the Decision Guidance Analytics Language (DGAL) for easy iterative development of decision guidance systems. The work on DGAL leverages our prior work on decision guidance and optimization languages. In particular, the unification of computation and equational syntax comes from CoJava (Brodsky & Nash, 2006), SC-CoJava (Brodsky, Al-Nory & Nash, 2012) and DGQL (Brodsky et al., 2011), CoReJava (Brodsky, Luo & Nash, 2008; Luo & Brodsky, 2011) on adding regression to DGAL, and DG-Query (Brodsky, Halder & Luo, 2014) which are designed to seamlessly add deterministic optimization and machine learning to Java, SQL and XQuery code, respectively, via automatic reduction to MP, CP or specialized algorithms. Also, DGAL fits into the framework of, but is significantly more general than, Decision Guidance Management Systems, proposed in (Brodsky and Wang, 2008). Finally, the concept of centralized AKB is borrowed from our work on the Process Analytics Formalism (Brodsky, Shao & Riddick, 2013; Alrazgan & Brodsky, 2014), which was limited to MP/CP optimization only.

Many research questions remain open. They include (1) specific reduction algorithms from DGAL queries and AO functions to specialized formal models of optimization, statistical learning, and uncertainty quantification; it may be promising to borrow from the work on constraint databases to support symbolic constraint (2) development specialized algorithms for stochastic optimization in DGAL that can leverage deterministic approximation encoded in DGAL analytical objects; (3) development of specialized algorithms that can utilize pre-processing of stored (and therefore, static) AO's, to speed up optimization, generalizing the results in (Egge, Brodsky, & Griva, 2013); and, (4) developing graphical user interfaces for domain specific languages based on DGAL.

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