Stochastic Resonances in Photon Number Resolving Detectors

Shree Krishnamoorthy[1], Harish Ravishankar[2], Pradeep K. Kumar[3] and Anil Prabhakar [1]

[1]*Indian Institute of Technology, Madras, Chennai, India*

[2]*Indian Institute of Science, Bangalore, India*

[3]*Indian Institute of Technology, Kanpur, India*

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Abstract: The photon number statistics of a coherent optical pulse will typically follow a Poissonian distribution. At low photon numbers, a gated avalanche photo-detector (GAPD) is used to detect the presence of photons in each optical pulse. GAPDs use a thresholding logic, but suffer from after-pulsing effects. The efficiency of a GAPD was characterized and its after-pulses were analyzed by looking at the detection patterns obtained. The GAPD was found to show evidence of stochastic resonance which affected the dark noise of the detector. We post-process the detected bit patterns to eliminate the resonances and estimate the true dark count of the detector. The GAPD was then used with a recirculating optical loop to build a multi-photon resolving detector (MPRD). In the MPRD, the probability of detection at consecutive loop round trip times were used to estimate the mean photon number. We quantify these statistics and establish a reliable measure of photon number at an optical power of -94 dBm. The digital electronics was able to store data for 2^{24} optical pulses, making the statistical analysis meaningful.

1 INTRODUCTION

Quantum cryptography holds the promise of unconditional security (Bennett et al., 1984; Lo and Chau, 1999). However, practical implementations of quantum key distribution, such as DPS-QKD or FC-QKD, use coherent optical states that are susceptible to photon number splitting attacks (Norbert and Mika, 2002; Inoue et al., 2003; Bloch et al., 2007; Valerio et al., 2009). In the absence of purely single photon sources, development of QKD systems need photon number resolution (Hadfield, 2009). While, optical fibers are easily adapted for quantum communication, the detectors needed for photon number resolution are not easily available and the development of photon counting detectors continues to be of current interest (Blasej et al., 2014). In the telecommunication band, around $1.55\,\mu$m, PMTs have very low efficiencies of about 2% where as InGaAs avalanche photodiodes (APDs) provide an efficiency of 20% with lower dark counts (Hadfield, 2009). APDs operated in Geiger mode indicate only the presence of or absence of photons and cannot resolve the number of incident photons, making their output binary in nature. The Geiger mode enhances the sensitivity, but is accompanied by a degradation in the signal to noise ratio (Kolb, 2014). For photon number resolution, the incoming photons are redistributed to multiple detection slots, using spatial or temporal multiplexing methods (Fitch et al., 2003; Mogilevtsev, 2010). Temporal multiplexing can be achieved by an optical fiber loop connected to the coupler (Haderka et al., 2004; Ravi and Prabhakar, 2011), and the timing information can be used to gate the APD.

APDs are based on the avalanche process which dictates the noise characteristics in them (McIntyre, 1966). Gated APD (GAPD) systems suffer from dark counts and after-pulsing (Cova et al., 2004; Tosi et al., 2009). Other than the known noises, our implementation of the GAPD shows evidence of stochastic resonance at low photon numbers. In this article we analyze the probabilities of occurrence of different bit patterns to extract the photon number statistics in the presence of stochastic resonance. We first characterize the GAPD, and estimate the efficiency of detection. We then measure the photon arrival statistics and compare them to theoretical predictions for a GAPD in the presence of noise and underlying stochastic resonance.

We implement a MPRD with a recirculating optical loop followed by the GAPD. When a coherent state is input to the MPRD, the bit pattern that ensues should follow a statistical distribution. The roundtrip time of the circulating loop was less than one-eighth

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of the time between coherent optical pulses input to the loop. This allowed us to use digital electronics to store and analyze the 8-bit pattern of 1's and 0's following each input pulse, and to repeat the exercise for $N = 2^{24}$ input pulses. Our experiments validate the Poissonian statistics of the coherent state down to an average photon number of 0.024.

2 DETECTOR CHARACTERIZATION

When counting photons, we often assume that the sources of noise (typically shot and thermal noise) are independent of the photon counts. In cases where there are other low amplitude stationary processes that look like noise, the assumption of independent noise statistics breaks down. Stochastic resonance is one such phenomenon that can cause higher null detections at periodic intervals in a system using gated detection, like in our system. We find a suitable operating point for the GAPD and extract actual photon detection probabilities with proper data analysis in the presence of stochastic resonances.

2.1 Efficiency

Dark counts are a result of thermally triggered electron currents. We describe how to find the optimal operating point for the detector and find average contribution of the underlying noise. A null detection must occur in the absence of any electron currents, triggered either thermally or by incident photons. We assume that dark counts occur, with a probability $p_d(1)$ and an efficiency η for the detector. When a pulse with mean photon number n is incident on the detector from a coherent laser source, we could register a '1' either due to a photo-generated avalanche process, or due to the dark noise of the detector. The probability of no detection, $p(0)$, occurs when there are no photons and there is also no dark count, i.e.,

$$p(0) = e^{-\eta n}(1 - p_d(1)). \tag{1}$$

To arrive at the probability of a positive detection, $p(1)$, we recognize that a detection occurs either due to the presence of a photons, with a probability $(1 - e^{-\eta n})$, or due to the dark count, with a probability $p_d(1)$. However, since the two events are independent, we must subtract the probability of both events occurring together, i.e., $p_d(1)(1 - e^{-\eta n})$. Thus,

$$\begin{aligned} p(1) &= (1 - e^{-\eta n}) + p_d(1) - p_d(1)(1 - e^{-\eta n}) \\ &= 1 - p(0). \end{aligned} \tag{2}$$

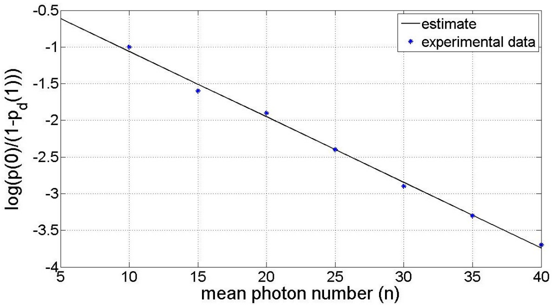

Figure 1: Null detections with increasing mean photon numbers, used to find detector efficiency η.

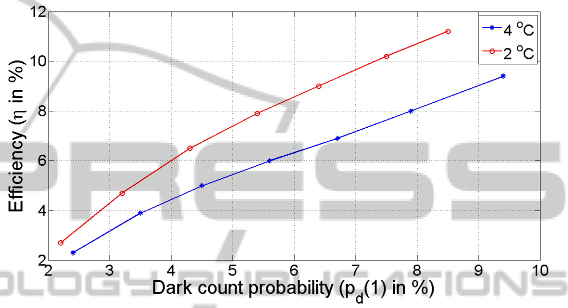

Figure 2: Detection efficiency for different case temperatures and dark counts at constant bias. The APD was cooled by an internal Peltier, to about 20 °C below the case temperature.

For our experiments, we have used an InGaAs APD (model NR8300FP from NEC) with an internal Peltier cooler. The detector case was further cooled by an external Peltier stage. The breakdown voltage of the APD was about 71 V at a case temperature of 4 °C. To operate the APD in Geiger mode, we bias the APD below 71 V and apply a gating voltage of 3.3 V for 5.2 ns. The GAPD followed by digital counting electronics (FPGA based), acted as one photon counter. The digital electronics had 2 GB RAM, or 16 Gbits, to easily collect and store obtained statistics.

Fig. 1 shows the dependence of $p(0)/(1 - p_d(1))$ on n, from which we extract the efficiency η of the detector. At a bias of 69.7 V and a detector case temperature of 4 °C, we found that the efficiency η was 0.0893 ± 0.006. We also confirm that a change in temperature and bias conditions change the dark count and the efficiency of the detector, as shown in Fig. 2.Thus, we find the optimal operating point for the GAPD at a case temperature of 2 °C with a bias voltage of 69.7 V, where $\eta = 0.11$, and the dark count $p_d(1) = 0.085$. For a 10% higher detection rate than the dark count the null detection probability is $p(0) = 0.9065$. Substituting $p(0)$ in (1) and using $\eta = 0.11$, we estimate the average photons per pulse to be $n = 0.0848$, or one true detection for every 12 gate pulses at the GAPD. The average photon number

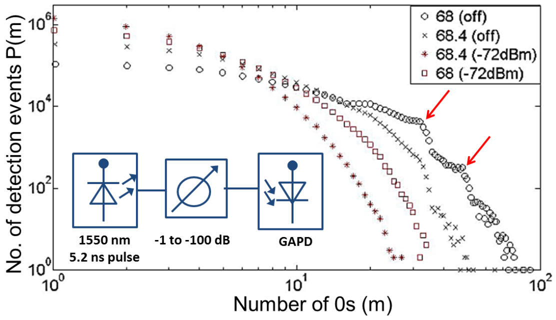

Figure 3: Effect of bias voltage and optical input, on the time between consecutive detections. Each 0 bit corresponds to a delay of $1\,\mu$s.

resolution can be further improved by using this detector after a recirculating loop as described in Sec.3.

2.2 Detection Probabilities and Stochastic Resonance

In the previous section, we obtained the optimal operating point for the GAPD by studying the average noise statistics. However, we are in a position to look at the statistics of how the detector responds in Geiger mode, both with and without an incident optical pulse.

Consider a coherent optical pulse train, with an average of n photons in each pulse, incident on the detector. For the experimental setup in the inset of Fig. 3, we send N such pulses, and collect statistics on the number of times the detector is triggered. For each received pulse, if a null detection occurs then a "0" is recorded and when a positive detection occurs a "1" is recorded. We thus obtain binary statistics of $(0,1)$ for each received pulse.

For N incident optical pulses, positive detections will follow a binomial distribution with a mean $\mu = Np$ and a variance of $N(1-p)p$, $p \triangleq p(1)$ is the probability of detection for each incident pulse. Averaging over the number of detection events, we get an estimate $\bar{p} = \mu/N = p$, with the standard deviation

$$\sigma_N = \sqrt{\frac{p(1-p)}{N}}$$

decreasing with increasing N.

There is a $1\,\mu$s gap between successive gate pulses. Analyzing the digital bit pattern at the output of the GAPD is akin to obtaining a digital frequency spectrum, with a resolution of 1 MHz. In Fig. 3, we plot the frequency of occurrence of the m consecutive 0-bits for $N = 2^{24}$ bits collected in each experiment.We observe an increased occurrence of some patterns when compared to others, as indicated by the arrows. These "resonances" are quenched when optical pulses of power of -72 dBm are provided. The

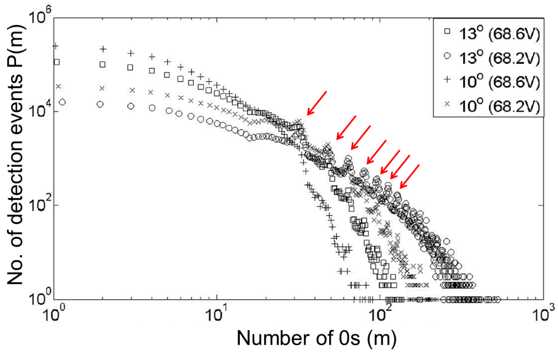

Figure 4: GAPD behavior in terms of bit pattern occurances, with change in bias and temperature. The arrows mark anomalous increases in event detections, and are attributed to a stochastic resonance.

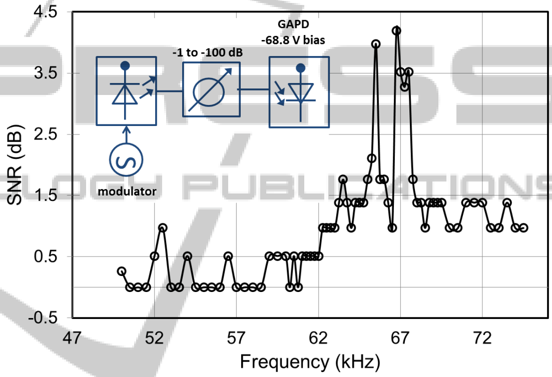

Figure 5: Frequency of stochastic resonance of the GAPD, with inset showing the experimental setup used.

occurrence of patterns at periodic intervals and its reduction due to the incidence of a few photons is suggestive of a stochastic resonance in the system (Gammaitoni, 1995).

In Fig. 4, we see that these resonances persist, and are even magnified, as we change the bias and increase the temperature. Both bias voltage and the temperature signal change the noise in the system and affect the intensity of the resonance. To further confirm the stochastic resonance behavior, we perform a separate experiment with the same detection system. We used a laser modulated by a sinusoidal signal, as shown in the inset of Fig. 5. We found that the GAPD shows a four fold increase in signal to noise ratio (SNR) close to 66 kHz. This enhancement in SNR is related to the Kramer's rate of switching between the 0 and 1 bit for the binary output detector system (McDonnell et al., 2008).

Previous experiments on the GAPD at a lower bias of 67 V, had established that a $1\ \mu$s gap between gate pulses would be sufficient to release any trapped energy (Kumar et al., 2009). With increasing ON time of the laser pulses (20 ns), longer dead times of up to

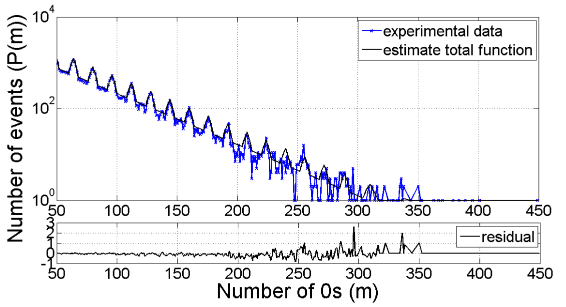

Figure 6: After pulsing statistics in the presence of stochastic resonances. The APD was gated every 1 μs.

10 μs in the GAPD has been reported in other studies (Ben-Michael et al., 2006; Tosi et al., 2009).

Longer dead times are attributed to after-pulsing effects in the avalanche photodetector. We take a closer look at one of the data sets in Fig. 4, with optics off, and estimate the probability of dark count detection in the presence of stochastic resonances. Since $p_\mathrm{d} < 0.1$, the probability of detecting more than two after-pulses becomes negligible. Hence, we restrict our analysis to understanding the probability of observing two 1s separated by m 0s. This is equivalent to the probability of obtaining m 0s followed by a 1 in the state diagram model for the gated Geiger mode APD, developed by Kolb, for obtaining SNR with afterpulsing (Kolb, 2014). Assuming that our first bit is detected with a probability p_d, followed by m 0s, the next bit is detected with a probability $p_\mathrm{d}^2(1-p_\mathrm{d})^m$. Repeating this experiment N times, we obtain

$$P(m) = Np_\mathrm{d}^2(1-p_\mathrm{d})^m. \tag{3}$$

To estimate the probability p from the data we ignore the regions affected by the resonance and fit the remaining data using (3). In all the collected data-sets, we found that resonance peaks occur every $m = 16$ pulses, consistent with one of the stochastic resonance peaks in Fig. 5. The width of each resonance is 4 consistently for all bias and temperature conditions. With this information we exclude the data around the local maxima, fit the rest of the data to a semilog function, find the slope and extract $p_\mathrm{d} = 0.024$. Since the data was obtained in the absence of any optical signal, we found that removing the resonance peak in the collected data, while post-processing, decreased the effective dark counts by a factor of four. In Fig. 6, the residuals were scaled with respect to the number of events found experimentally for each point. Fig. 6 also shows that we can observe stochastic resonances till approximately 0.2 ms, corresponding to $m = 200$, beyond which the residual error in our fit builds up.

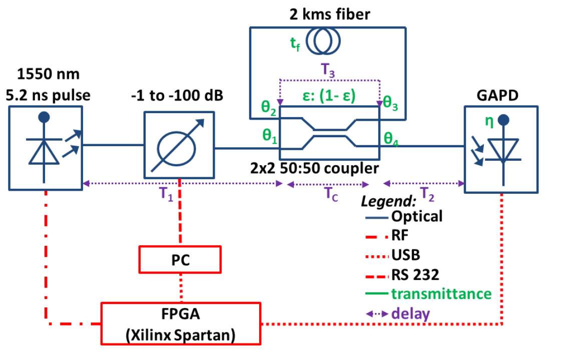

Figure 7: The MPRD setup showing timing information (T_i),coupler splitting ratio (ε),and transmittances (θ_i).

3 RESOLVING PHOTON NUMBERS

The MPRD setup consists of a coherent, pulsed laser source, a variable optical attenuator (VOA) followed by an optical recirculating loop and GAPD as shown in Fig. 7. The laser pulses, attenuation of the VOA and the gating of GAPD are computer controlled. To analyze the working of the MPRD, we use the GAPD characteristics obtained in Sec. 2, and estimate the decrease in detection probability for each recirculation of the optical pulse. Finally, we analyze the bit patterns obtained at the detector for consecutive detections and compare them to theoretical probability estimates. With 2 GB RAM, we were able to collect over a million instances of the experiment.

3.1 Experimental Setup

The MPRD experiment was first reported in (Ravi and Prabhakar, 2011). A laser pulse of 5.2 ns pulse width is sent through the MPRD. We chose a delay fiber length of 2 km, which gave the detector 10 μs to recover between successive measurements. The recirculation repeats until the pulses are sufficiently attenuated in the system. We used a 3 dB splitter, connectors with about 0.5 dB loss and a 2 km long spool with 0.4 dB loss. The optical path lengths were adjusted so that the return pulses arrived in synchronization with the system clock. For this purpose, we used an optical fiber array with length increasing in steps of 20 cms (1 ns delay) in series with our fiber spool until our pulses were correctly synchronized and the detection probability was maximized.

All the elements in the MPRD were controlled electronically, by a XILINX (SPARTAN XC3S400) field programmable gate array (FPGA). The FPGA was clocked at 24 MHz and two digital clock managers (DCMs) available on the FPGA were used to

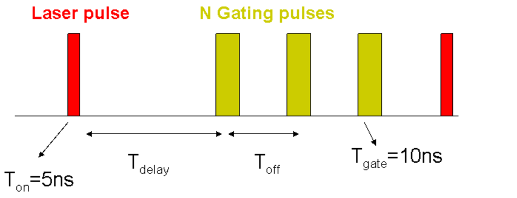

Figure 8: Pulse travel time from the laser to the detector after multiple round trips in the MPRD (Number of gating pulses, N=8).

multiply the clock frequency to 48 MHz and to provide a 90 degree phase shifted signal. These were used to generate laser pulses of width 5.2 ns and gating pulses of width 10.4 ns. The FPGA was used for pulsing the laser, gating the APD, setting of bias and threshold. The variable optical attenuator was used to change the incident power or photon number in the optical pulse. At the output of the comparator in the GAPD, eight detections are recorded for each laser pulse transmitted through the MPRD, giving us a byte of data for each laser pulse, with each bit synchronized to the round trip time of the recirculating loop.

3.2 Timing Synchronization in MPRD

To synchronize the detector gates to the incoming optical pulses from the recirculating loop, we look at the pulse arrival times in Fig. 8. The pulses arrive at the GAPD with a fixed delay T_{delay} from the laser. The detector has a gating period of 10.4 ns twice that of the optical pulse. The off time (T_{off}) of the detector needs to be precisely matched to the subsequent pulse arrival. If the pulse takes time T_1 to travel up to the coupler, time T_2 to travel from the edge of the coupler to the GAPD, time T_3 to loop once in the fiber and time T_c to travel through the coupler. Thus, the arrival time for the k^{th}pulse at the detector is found to be $T_1 + T_c + (T_3 + T_c)k + T_2$. We set $T_{\text{delay}} = T_1 + T_c + T_2$ using multiple sections of optical fiber, while $T_{\text{off}} = T_3 + T_c$ is set electronically.

3.3 Photon Redistribution Statistics

The coupler has a division ratio of $\varepsilon : 1-\varepsilon$, the optical delay fiber has a transmittance t_f, the connectors have a transmittance of θ_i=θ and the detector has an efficiency of detection η as shown in Fig. 8. We calculate the transmission of the k^{th} pulse (Ravi and Prabhakar, 2011)

$$T_{\text{eff}_k} = \theta_2 \varepsilon^{k-1}(1-\varepsilon)^2 (t_f \theta_3 \theta_1)^k \theta_4 \eta. \qquad (4)$$

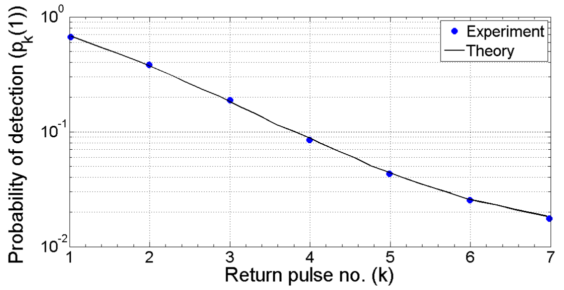

Figure 9: Average detection probability of successive return pulse in MPRD.

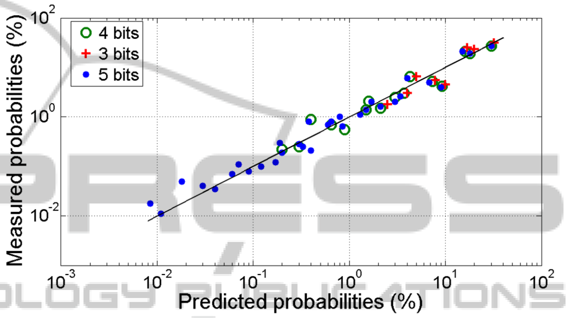

Figure 10: Measured versus predicted bit pattern probabilities of detection patterns obtained in MPRD at -63 dBm. Modified from (Ravi and Prabhakar, 2011).

For a coherent input with an average photon number n, the k^{th} detection pulse corresponds to an average photon number of nT_{eff_k}.

Starting from an average photon number n, the theoretical prediction for the probability of detection $p_k(1)$ for the k^{th} pulse is,

$$p_k(1) = 1 - \underbrace{e^{-\eta n T_{\text{eff}_k}}}_{\text{no photon}} \underbrace{(1-p_{\text{d}})}_{\text{no noise}}. \qquad (5)$$

For n = 30, η = 0.15, $T_{\text{eff}_k} = 0.398$ (corresponding to 4 dB loss), and $p_{\text{d}}(1) = 0.014$, Fig. 9 shows the decreasing probability of a detection for each round trip and we observe a good match between our theoretical estimates and our experimental observations.

For each byte recorded, we analyzed bit patterns for 3, 4 and 5 bits as shown in Fig. 10. An input pulse with average power of -63 dBm at the input of the 3 dB splitter in the MPRD, goes through seven round trips through the optical loop, with a loss of 4 dB per recirculation, to produce the last detection bit. The last bit out of the recirculating loop will, now, have an average power of -94 dBm, assuming a 5.2 ns pulse of optical wavelength 1.55 μm, corresponding to an average photon number of 0.024. Even so, the Poissonian statistics of a coherent pulse are well preserved and the occurrence of a bit pattern follows the predicted probabilities.

4 SUMMARY

This article describes the characterization of a gated avalanche photo-detector, used subsequently in a multi-photon resolving detector. We first experimentally characterize the GAPD and find its efficiency. We found that the GAPD was susceptible to stochastic resonance, which was affected by temperature and bias. The stochastic resonance was further investigated, and we saw that it had characteristic frequency around 66 kHz. When we used gated detection, with a delay of $1\,\mu$s between gate pulses, we were in effect digitally sampling the stochastic resonance. Discounting these resonances in our experimentally measured probabilities, we were able to characterize the GAPD for very low average photon numbers.

A recirculating loop is a simple way of splitting a Poissonian state into a series of temporal pulses. We described the statistical properties of such a multi-photon resolving detector. In our experiments, the recirculating loop was set up to achieve a delay of $10\,\mu$s between successive optical pulses, while we gated the avalanche photodiode every $1\,\mu$s. After each input coherent optical pulse we recorded the output for 8 gate pulses as one byte, and did this for $N = 2^{24}$ optical pulses. We then searched for correlations between successive gate pulses in the data bytes by looking at the bit patterns for 3, 4 and 5 bits. The probability of occurrence of a bit pattern followed the predicted probabilities, within the noise bounds of the system. Thus, we conclude that the MPRD could be used for photon number resolution, for average photon numbers as low as $n = 0.024$ per pulse.

We observed that the stochastic resonance depends on both APD bias and temperature. Consequently, we were able to adjust these parameters and avoid the resonance, as we set up our photon number resolving experiments. We believe that we are the first authors to report on stochastic resonances in a GAPD system. Further investigations about the origins of the resonace, along with an appropriate statistical noise model, will help improve the performance of single photon detectors.

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