# A Closed-form Approximated Expression for the Residual ISI Obtained by Blind Adaptive Equalizers Applicable for the Non-Square QAM Constellation Input and Noisy Case

Roy Shevach and Monika Pinchas
*Dept. of Electrical and Electronic Engineering, Ariel University of Samaria, Ariel 40700, Israel*

Keywords: Blind Equalization, Residual ISI.

Abstract: Recently, closed-form approximated expressions were obtained for the residual Inter-Symbol Interference (ISI) obtained by blind adaptive equalizers valid for the real or two independent quadrature carrier case such as the 16 Quadrature Amplitude Modulation (QAM) input. In this paper we propose for the complex and dependent quadrature carrier case (such as the 32QAM source), a closed-form approximated expression for the achievable residual ISI which depends on the step-size parameter, equalizer's tap length, input signal statistics, channel power and SNR. This approximated expression is applicable for blind adaptive equalizers where the error is fed into the adaptive mechanism, which updates the equalizer's taps and can be expressed as a polynomial function up to order five of the equalized output. Godard's algorithm for example, applies a third order polynomial function to the adaptation mechanism of the equalizer thus belongs to the above-mentioned type of equalizers. Since the channel power is measurable, or can be calculated if the channel coefficients are given, there is no need to perform any simulation with various step-size parameters and different values of SNR to reach the required residual ISI for the dependent quadrature carrier input case.

## 1 INTRODUCTION

Intersymbol interference (ISI) is a well known phenomenon in which subsequent symbols at the receiver are overlapping due to channel characteristics such as bandwidth limitation or multipath effects. This distortion makes it difficult for the decision device at the receiver to recover the transmitted data. Thus, for bandwidth-efficient communication systems, operating in high inter-symbol interference (ISI) environments, adaptive equalizers have become a necessary component of the receiver architecture. An accurate estimate of the amplitude and phase distortion introduced by the channel is essential to achieve high data rates with low error probabilities (Abrar, Zerguine and Nandi 2012).

Modern digital communication systems are both band limited and used to transmit high data rate, therefore the adaptive equalization method which relies on training phase is either impractical or very costly in terms of data throughput. Hence, a blind adaptive equalization algorithm is the preferable choice between the three types of equalization methods (non-blind, semi-blind and blind). Using these blind algorithms, individual receivers can begin self-adaptation without transmitter assistance. This ability of blind startup also enables a blind equalizer to self-recover from system breakdowns. This self-recovery ability is critical in broadcast and multicast systems where channel variation often occurs (Zhi Ding 2009). The algorithm itself generates an estimate of the desired response by applying a non-linear transformation to sequences involved in the adaptation process (Nikias, Petropulu, 1993). Since the equalizer performance depends on the above-mentioned transformation, equalizer's tap length, step-size parameter, channel characteristics, added noise (SNR) and input signal statistics, therefore tailoring an equalizer for a given channel (application dependent) was involved with a long process of simulation to assure the equalizer will meet the system requirements. For example, choosing a "big" step-size may lead to fast convergence time at the expense of a high residual ISI where the eye diagram is considered to be close but on the other hand, "small" step-size may improve the equalizer performance in terms of residual ISI at the expense of long convergence time. This expansive time can be spared by a closed-form

Shevach R. and Pinchas M..
A Closed-form Approximated Expression for the Residual ISI Obtained by Blind Adaptive Equalizers Applicable for the Non-Square QAM Constellation Input and Noisy Case.
DOI: 10.5220/0005322202170223
In *Proceedings of the 5th International Conference on Pervasive and Embedded Computing and Communication Systems* (PECCS-2015), pages 217-223
ISBN: 978-989-758-084-0

expression for the residual ISI taking into account all the above mentioned parameters.

Up to now, such an expression was obtained for real and independent input for the noiseless (Pinchas, 2010a) and noisy (Pinchas, 2010b) cases by Pinchas, and for blind adaptive equalizers with equalized output gain lower or equal to one (Kupchan and Pinchas, 2014). However none of those expressions are applicable for the non-square QAM constellations (such as 32QAM).

Recently, an expression for the non-square QAM constellations was developed by Pinchas (Pinchas, 2012). But, this expression is only applicable for the noiseless case and for blind adaptive equalizers where the error is fed into the adaptive mechanism, which updates the equalizer‘s taps is expressed as a polynomial function up to order three of the equalized output.

In this paper we propose a new closed-form approximated expression for the residual ISI for the real as well as for the general case of complex and dependent input signals such as the 32QAM constellation applicable also for the noisy case. This new proposed expression is valid for type of blind equalizers where the error that is fed into the adaptive mechanism which updates the equalizer’s taps can be expressed as a polynomial function of order five of the equalized output.

This paper is organized as follows: the system under consideration is depicted in Section 2, the closed-form approximated expression for the achievable residual ISI is introduced in Section 3. In Section 4 simulation results are presented and the conclusion is presented in Section 5.

## 2 SYSTEM DESCRIPTION

The system under consideration is similar to the system presented by Pinchas (Pinchas, 2010b) and illustrated here in Figure 1.

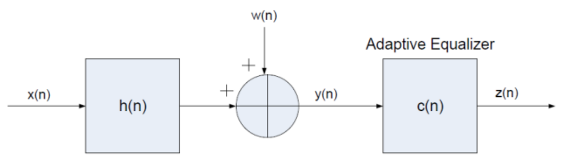


Figure 1: Block Diagram of a baseband communication system.

The following assumptions have been taken:

1. The input signal $x[n]$ is a real or complex quadrature amplitude modulated (QAM) signal with variance $\sigma_x^2$, where for the complex case the real part and the imaginary part are dependent.
2. The unknown channel $h[n]$ is a possibly non-minimum phase linear time-invariant (LTI) filter in which the transfer function has no ”deep zeros”, namely, the zeros lie sufficiently far from the unit circle.
3. The equalizer $c[n]$ is a tap-delay line.
4. The noise $w[n]$ is an additive Gaussian white noise with variance $\sigma_w^2$.

After that the input signal has been transmitted through the channel $h[n]$, it is corrupted with an additive Gaussian white noise $w[n]$. Therefore, the equalizer input may be expressed as:

$$y[n] = x[n] * h[n] + w[n] \quad (1)$$

where the notation "*" refers to the convolution operation. Following (Pinchas, 2010a), the equalizer output can be expressed as:

$$z[n] = x[n] + p[n] + \tilde{w}[n] \quad (2)$$

where $p[n]$ is the convolutional noise produced due to the error between the actual derived or initial given value for $c[n]$ and the ideal value for $c[n]$ and $\tilde{w}[n] = w[n] * c[n]$, namely, the noise passing through the equalizer. The ISI is defined by:

$$ISI = \frac{\sum_{\tilde{m}} |\tilde{s}(\tilde{m})|^2 - |\tilde{s}|^2_{\max}}{|\tilde{s}|^2_{\max}} \quad (3)$$

where $|\tilde{s}|_{\max}$ is the component of $\tilde{s}$, given by:

$$\tilde{s}[n] = c[n] * h[n] \quad (4)$$

having the maximal absolute value (Pinchas, 2010a).

The equalizer adaptive mechanism responsible for minimizing the convolutional error can be expressed as:

$$\underline{c}_{eq}[n+1] = \underline{c}_{eq}[n] - \mu \cdot \frac{\partial F[n]}{\partial z[n]} \underline{y}^*[n] \quad (5)$$

where $\underline{c}_{eq}[n+1]$ and $\underline{c}_{eq}[n]$ are the equalizer coefficients at the next and current iteration respectively, $\mu$ is the equalizer’s step size, $F[n]$ is the cost-function that characterized the ISI and $\underline{y}[n]$ is the input vector $\underline{y}[n] = [y[n] \ldots y[n-N+1]]^T$, where $N$ is the equalizer's tap length. Note that the operator $(\ )^T$ represents the transpose function and $(\ )^*$ is the conjugate operator.

# 3 THE RESIDUAL ISI

In this section we educe a new closed-form approximated expression for the achievable residual ISI which depends on the step-size parameter, equalizer's tap length, input signal statistics, channel power and SNR.

**Theorem.**

Under the following assumptions:

1. The convolutional noise $p[n]$, is a zero mean, white Gaussian process with variance $\sigma_p^2 = E[p[n](p[\mathrm{n}])^*]$, where $E[\ ]$ represents the expectation operator.
2. The input signal $x[n]$ is a complex and dependent quadrature amplitude modulated constellation with known variance and higher moments (e.g. 32QAM).
3. The convolutional noise $p[n]$ and the input signal are independent.
4. $\left|\tilde{s}\right|^2_{\max} = 1$, where $\tilde{s}$ is defined in (4).
5. $\frac{\partial F[n]}{\partial z[n]}$ can be expressed as a polynomial function of order five of the equalized output namely as $P(z)$.

The residual ISI expressed in dB units is defined as:

$$ISI = 10\log_{10}\left(m_p\right) - 10\log_{10}\left(\sigma_x^2\right) \qquad (6)$$

where $\sigma_x^2$ is the variance of the input signal and $m_p$ is defined by:

$$m_p = \min\left[Sol_1^{mp}, Sol_2^{mp}\right] \text{ for } Sol_1^{mp} > 0 \text{ and } Sol_2^{mp} > 0$$

or

$$m_p = \max\left[Sol_1^{mp}, Sol_2^{mp}\right] \text{ for } Sol_1^{mp} \cdot Sol_2^{mp} < 0 \text{ where}$$

$$Sol_1^{mp} = \frac{-B_1 + \sqrt{B_1^2 - 4A_1C_1B}}{2A_1}$$

$$Sol_2^{mp} = \frac{-B_1 - \sqrt{B_1^2 - 4A_1C_1B}}{2A_1}$$

$$\begin{aligned}
A_1 = {} & B(6a_1a_3 + 27a_3^2(\sigma_x^2+\sigma_w^2) + 54a_1a_5(\sigma_x^2+\sigma_w^2) \\
& + 216a_3a_5(E[|x|^4] + E[|w|^4]) + 864a_3a_5\sigma_x^2\sigma_w^2 \\
& + 300a_5^2(E[|x|^6] + E[|w|^6]) \\
& + 2700a_5^2(E[|x|^4]\sigma_w^2 + E[|w|^4]\sigma_x^2)) \\
& - (6a_3 + 36a_5(\sigma_x^2+\sigma_w^2))
\end{aligned}$$

$$\begin{aligned}
B_1 = {} & B(a_1^2 + 8a_1a_3(\sigma_x^2+\sigma_w^2) \\
& + 9a_3^2(E[|x|^4] + E[|w|^4]) \\
& + 36a_3^2\sigma_x^2\sigma_w^2 + 18a_1a_5(E[|x|^4] + E[|w|^4]) \\
& + 32a_3a_5(E[|x|^6] + E[|w|^6]) \\
& + 288a_3a_5(E[|x|^4]\sigma_w^2 + E[|w|^4]\sigma_x^2) \\
& + 25a_5^2(E[|x|^8] + E[|w|^8]) + 72a_1a_5\sigma_x^2\sigma_w^2 \\
& + 400a_5^2(E[|x|^6]\sigma_w^2 + E[|w|^6]\sigma_x^2) \\
& + 900a_5^2E[|x|^4]E[|w|^4]) \\
& - (2a_1 + 4a_3(\sigma_x^2+\sigma_w^2) + 24a_5\sigma_x^2\sigma_w^2 \\
& + 6a_5(E[|x|^4] + E[|w|^4]))
\end{aligned}$$

$$\begin{aligned}
C_1 = {} & a_1^2(\sigma_x^2+\sigma_w^2) + 2a_1a_3(E[|x|^4] + E[|w|^4]) \\
& + a_3^2(E[|x|^6] + E[|w|^6]) \\
& + 2a_1a_5(E[|x|^6] + E[|w|^6]) \\
& + 9a_3^2(E[|x|^4]\sigma_w^2 + E[|w|^4]\sigma_x^2) \\
& + 2a_3a_5(E[|x|^8] + E[|w|^8]) + 8a_1a_3\sigma_x^2\sigma_w^2 \\
& + 18a_1a_5(E[|x|^4]\sigma_w^2 + E[|w|^4]\sigma_x^2) \\
& + 32a_3a_5(E[|x|^6]\sigma_w^2 + E[|w|^6]\sigma_x^2) \\
& + 72a_3a_5E[|x|^4]E[|w|^4] \\
& + a_5^2(E[|x|^{10}] + E[|w|^{10}]) \\
& + 25a_5^2(E[|x|^8]\sigma_w^2 + E[|w|^8]\sigma_x^2) \\
& + 100a_5^2\left(E[|x|^6]E[|w|^4] + E[|w|^6]E[|x|^4]\right)
\end{aligned}$$

$$B = \mu N\sigma_x^2\left(\sum_{k=0}^{k=R-1}\left|h(k)\right|^2 + SNR^{-1}\right)$$

$$\left(SNR = \frac{\sigma_x^2}{\sigma_w^2}\right) \qquad (7)$$

$R$ is the channel length, $N$ is the equalizer's tap length $x = x[n]$ and $a_1, \mathrm{a}_3, a_5$ are the coefficients of the above-mentioned polynomial function $P(z)$, defined as:

$$\frac{\partial F[n]}{\partial z[n]} = P(z) = a_1 z[n] + a_3 \left|z[n]\right|^2 z[n] + a_5 \left(\left|z[n]\right|^2\right)^2 z[n] \tag{8}$$

**Proof.**

We begin our proof by first recalling from (Pinchas, 2012) the expression for $E\left[\Delta\left(p[n]\left(p[n]\right)^*\right)\right]$:

$$\begin{aligned} &E\left[\Delta\left(p[n]\left(p[n]\right)^*\right)\right] \cong \\ &-\mu E\left[\left(p[n]\left(P\left(z[n]\right)\right)^* + p[n]^* P\left(z[n]\right)\right)\right] \cdot \\ &E\left[\sum_{m=0}^{m=N-1} y[n-m] y^*[n-m]\right] \\ &+\mu^2 E\left[P\left(z[n]\right)\left(P\left(z[n]\right)\right)^*\right] \cdot \\ &E\left[\sum_{m=0}^{m=N-1} y[n-m] y^*[n-m] \sum_{m=0}^{m=N-1} y[n-m] y^*[n-m]\right] \end{aligned} \tag{9}$$

where $p[n]$ is the convolutional noise and $P\left(z[n]\right)$ is the polynomial function defined at (8). Substituting (2) and (8) into (9) yields:

$$\begin{aligned} &E\left[\Delta\left(p[n]\left(p[n]\right)^*\right)\right] \cong \\ &B \cdot \left(F_1 m_p^5 + E_1 m_p^4 + D_1 m_p^2 + A_1 m_p^2 + B_1 m_p + BC_1\right) \end{aligned} \tag{10}$$

where $m_p = E\left[p[n]\left(p[n]\right)^*\right]$, $A_1, B_1, C_1, B$ are given in (7), and $D_1, E_1, F_1$ are given by:

$$\begin{aligned} &F_1 = 945 a_5^2 B \\ &E_1 = (210 a_3 a_5 + 2625 a_5^2 (\sigma_x^2 + \sigma_w^2))B \\ &D_1 = (30 a_1 a_5 + 15 a_3^2 + 480 a_3 a_5 (\sigma_x^2 + \sigma_w^2) \\ &\quad 4500 a_5^2 \left((\sigma_x^2)^2 + (\sigma_w^2)^2\right) \\ &\quad + 6000 a_5^2 \sigma_x^2 \sigma_w^2)B - 15 a_5 \end{aligned} \tag{11}$$

As was shown in (Pinchas, 2012), at the latter stages where the algorithm has converged we may write that $E[\Delta(p[n]\left(p[n]\right)^*)] \cong 0$. For an easy channel (where the ISI is relatively low but the eye diagram is still closed) we can neglect the products of $D_1 m_p^3$, $E_1 m_p^4$ and $F_1 m_p^5$, thus denoting (10) as:

$$A_1 m_p^2 + B_1 m_p + BC_1 \cong 0 \tag{12}$$

The solution for this second order equation with respect to $m_p$ is given in (7). The relation between the convolutional noise power $m_p$ and ISI was developed in (Pinchas, 2010a) noted in (6). This completes our proof.

# 4 SIMULATION RESULTS

In this section we test the new closed-form expression for the residual ISI via simulation. In the simulation we use two equalizers to examine the benefit of the new expression. The equalizer was initialized by setting the central equalizer‘s tap to one and all others to zero. The first equalizer is based on Godard algorithm (Godard, 1980) (third order polynomial function) which equalizer's taps are updated according to:

$$c_m[n+1] = c_m[n] - \mu_G \left(\left|z[n]\right|^2 - \frac{E[\left|x\right|^4]}{E[\left|x\right|^2]}\right) z[n] y^*[n-m] \tag{13}$$

where $\mu_G$ is the step-size and $a_1, a_3$ are given by:

$$a_1^G = 1, \quad a_3^G = -\frac{E[\left|x\right|^4]}{E[\left|x\right|^2]} \tag{14}$$

In order to examine the new expression with an equalizer of fifth order polynomial function, we are using an ad hoc equalizer which equalizer's taps are updated according to:

$$\begin{aligned} c_m[n+1] = c_m[n] - \mu_{new} &\left(a_1 + a_3 \left|z[n]\right|^2 + a_5 \left(\left|z[n]\right|^2\right)^2\right) \cdot \\ &z[n] y^*[n-m] \end{aligned} \tag{15}$$

where $\mu_{new}$ is the step-size and $a_1, a_3, a_5$ are given by:

$$\begin{aligned} &a_1^{new} = -1, \quad a_3^{new} = 4 \cdot A, \quad a_5^{new} = -2A/15 \\ &A = \frac{E[\left|x\right|^4] E[\left|x\right|^{10}] - E[\left|x\right|^6] E[\left|x\right|^8]}{(E[\left|x\right|^8])^2 - E[\left|x\right|^6] E[\left|x\right|^{10}]} \end{aligned} \tag{16}$$

Two input signals were considered: 32QAM source and 128QAM source, both complex and dependent signals. Three different channels were considered.

Channel 1 (initial ISI = 0.44): The channel parameters were determined according to Shalvi and Weinstein (Shalvi, Weinstein, 1990):
$h_n$ ={0 for $n < 0$; -0.4 for $n = 0$; $0.84 \cdot 0.4^{n-1}$ }

Channel 2 (initial ISI = 0.88): The channel parameters were determined according to Pinchas (Pinchas, 2010b): $h_n = (0.4851, -0.72765, -0.4851)$.

Channel 3 (initial ISI = 0.5): The channel parameters were determined according to Fiori (Fiori, 2001): $h_n = (-0.0144, 0.0006, 0.0427, 0.0090, -0.4842, -0.0376, 0.8163, 0.0247, 0.2976, 0.0122, 0.0764, 0.0111, 0.0162, 0.0063)$ (a sampled telephonic channel).

The ISI defined in (3) was calculated every iteration, and compared to the new closed-form expression that was presented at (6).

Figures 2-7 present the performance comparison between the calculated and simulated achievable residual ISI according to the ad hoc fifth order equalizer (noted as $P_{new}$) and Godard's algorithm for 32QAM input signal, as a function of iteration number for various step-size parameters (noted as $\mu_{new}$ and $\mu_G$ for $P_{new}$ and Godard respectively), channel characteristics, equalizer's tap length and SNR. High correlation was found between the calculated and simulated achievable residual ISI.

Figures 8-9 present the performance comparison between the calculated and simulated achievable residual ISI according to the ad hoc fifth order equalizer (noted as $P_{new}$) and Godard's algorithm respectively, for 128QAM input signal at SNR of 30dB. Figure 8 (fifth order polynomial equalizer) shows fine correlation between the calculated and simulated achievable residual ISI according (3dB difference) while Figure 9 (Godard) shows very high correlation.

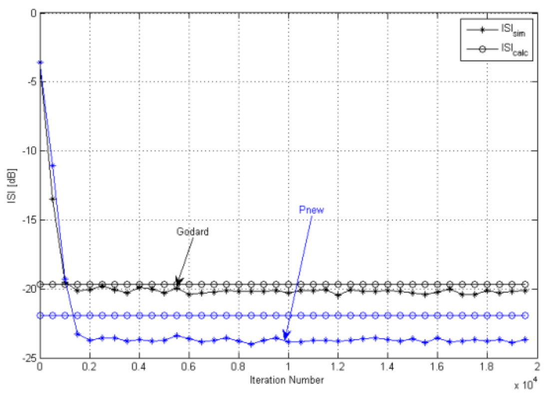


Figure 2: A comparison between the calculated and simulated (with Pnew and Godard's algorithm) residual ISI for 32QAM input signal going through channel 1. The averaged results were obtained in 100 Monte Carlo trials for 25dB. The equalizer tap length was set to 13, the step-size parameters μG and μnew were set to 1.5e-5 and -5e-5 respectively.

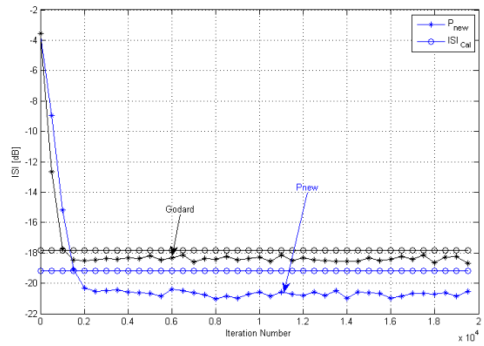


Figure 3: A comparison between the calculated and simulated (with Pnew and Godard's algorithm) residual ISI for 32QAM input signal going through channel 1. The averaged results were obtained in 100 Monte Carlo trials for 15dB. The equalizer tap length was set to 13, the step-size parameters μG and μnew were set to 1.5e-5 and -3e-5 respectively.

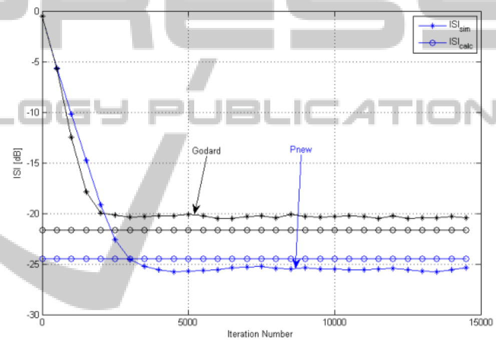


Figure 4: A comparison between the calculated and simulated (with Pnew and Godard's algorithm) residual ISI for 32QAM input signal going through channel 2. The averaged results were obtained in 100 Monte Carlo trials for 25dB. The equalizer tap length was set to 15, the step-size parameters μG and μnew were set to 8e-6 and -2.5e-5 respectively.

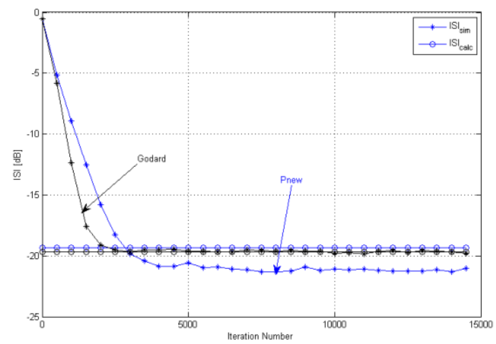


Figure 5: A comparison between the calculated and simulated (with Pnew and Godard's algorithm) residual ISI for 32QAM input signal going through channel 2. The averaged results were obtained in 100 Monte Carlo trials for 15dB. The equalizer tap length was set to 15, the step-size parameters μG and μnew were set to 8e-6 and -3e-5 respectively.

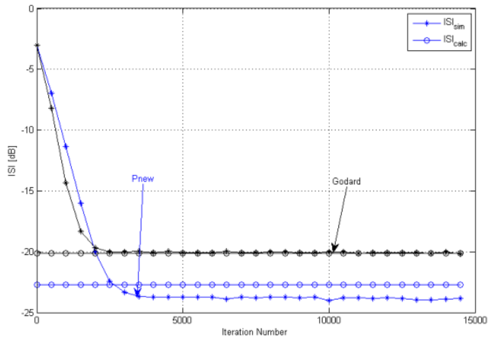


Figure 6: A comparison between the calculated and simulated (with Pnew and Godard's algorithm) residual ISI for 32QAM input signal going through channel 3. The averaged results were obtained in 100 Monte Carlo trials for 25dB. The equalizer tap length was set to 21, the step-size parameters µG and µnew were set to 8e-6 and -2.5e-5 respectively.

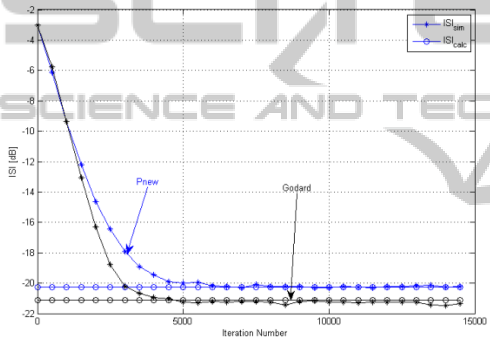


Figure 7: A comparison between the calculated and simulated (with Pnew and Godard's algorithm) residual ISI for 32QAM input signal going through channel 3. The averaged results were obtained in 100 Monte Carlo trials for 15dB. The equalizer tap length was set to 21, the step-size parameters µG and µnew were set to 4.5e-6 and -1.5e-5 respectively.

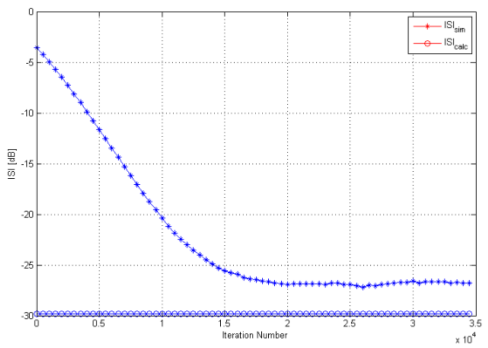


Figure 8: A comparison between the calculated and simulated (with Pnew) residual ISI for 128QAM input signal going through channel 1. The averaged results were obtained in 100 Monte Carlo trials for 30dB. The equalizer tap length was set to 13, the step-size parameter $\mu_{new}$ was set to -1.2e-7.

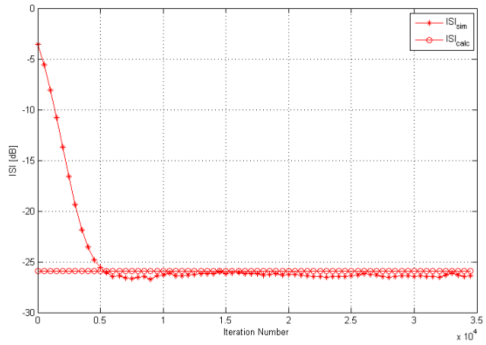


Figure 9: A comparison between the calculated and simulated (with Godard's algorithm) residual ISI for 128QAM input signal going through channel 1. The averaged results were obtained in 100 Monte Carlo trials for 30dB. The equalizer tap length was set to 13, the step-size parameter µG was set to 2e-7.



## 5 CONCLUSIONS

In this paper, a new closed-form approximated expression was developed for the achievable residual ISI valid for SNR values down to 15 dB for the complex and dependent quadrature carrier case applicable for type of blind equalizers where the error that is fed into the adaptive mechanism which updates the equalizer's taps can be expressed as a polynomial function of order five of the equalized output. The developed expression for the achievable residual ISI depends on the channel power (which is measurable or can be calculated if the channel coefficients are given), on the step-size parameter, equalizer's tap length, input signal statistics and SNR. The knowledge of these parameters enables the system designer to use the described evaluation instead of carrying out multiple simulations with various step-size parameters and SNR in order to get the optimal step-size parameter for a required residual ISI.

## ACKNOWLEDGEMENTS

We would like to thank the anonymous reviewers for their helpful comments.

## REFERENCES

Shafayat Abrar, Azzedine Zerguine and Asoke Kumar Nandi (2012). *Adaptive Blind Channel Equalization, Digital Communication*, Prof. C Palanisamy (Ed.), ISBN: 978-953-51-0215-1, InTech, Available from:

http://www.intechopen.com/books/digital-communication/adaptive-blind-channel-equalization.

Zhi Ding, *Digital Signal Processing Fundamentals*, CRC Press 2009, Chapter 24. Adaptive Filters for Blind Equalization.

C.L. Nikias, A.P. Petropulu, *Higher-Order Spectra Analysis A Nonlinear Signal Processing Framework*, Prentice-Hall, Englewood Cliffs, NJ, 1993, pp. 419–425 (Chapter 9).

Pinchas M. A Closed Approximated formed Expression for the Achievable Residual Intersymbol Interference obtained by Blind Equalizers, *Signal Processing. (Eurasip)*, Jun 2010; Volume 90, Issue 6, pp. 1940-1962.

Pinchas M. A New Closed Approximated Formed Expression for the Achievable Residual Intersymbol Interference obtained by Blind Equalizers for the Noisy Case, *IEEE International Conference, Beijing, China Jun 2010*; pp. 26-30.

Kupchan S., Pinchas M. A Closed-Form Approximated Expression for the Residual ISI Obtained by Blind Adaptive Equalizers with Gain Equal or Less than One, *Radioengineering*, Sep. 2014, Volume 23, Issue 3, pp. 954.

Pinchas M. *The whole story behind blind adaptive equalizers/blind deconvolution*, Bentham Science Publishers, 2012, (chapter 6).

Godard DN. Self recovering equalization and carrier tracking in two-dimenional data communication system. *IEEE Trans. Comm*. Nov 1980; Volume 28, Issue 11, pp. 1867-1875.

Shalvi O., Weinstein E. New criteria for blind deconvolution of nonminimum phase systems (channels). *IEEE Trans. Information Theory* Mar 1990; Volume 36, Issue 2, pp. 312-321.

Fiori S. A contribution to (neuromorphic) blind deconvolution by flexible approximated Bayesian estimation, *Signal Processing (Eurasip)* 2001; 81: 2131-2153.