

# Shape Classification based on Skeleton-branch Distances

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**Abstract:** In recent decades, the need for efficient and effective image search from large databases has increased. In this paper, we present a novel shape matching framework based on structures that are likely to exist in similar shapes. After representing shapes as medial axis graphs, where vertices show skeletons and edges connect nearby skeletons, we determine the branches connecting or representing shape's different parts. Using the shortest path distance from each vertex (skeleton) to each of the branches, we effectively retrieve similar shapes to the given query through a transportation-based distance function. A set of shape retrieval experiments including the comparison with two previous approaches demonstrate the proposed algorithm's effectiveness and perturbation experiments present its robustness.

## 1 INTRODUCTION

In recent decades, the need for efficient and effective image search from large databases has increased. This demand is raised within a number of domains such as content based image retrieval, face recognition, and bioinformatics. A number of powerful approaches have been presented in the literature to address the problem of retrieving database images, which most resembles the given query. While some of these techniques match the query features with those of each database image, some uses indexing algorithms to reduce the number of candidate database images for more efficient retrieval.

In order to retrieve similar database images, an image is segmented into different regions and each region is represented by its distinctive features. Several image features, such as shape, color, texture, central moment, eccentricity, and brightness are used in content based image-retrieval systems (Ardizzone et al., 1996). Among those, shape is an important visual feature to describe the image content. In many applications, the shape of a planar object is described either by its contour or skeleton.

In our framework, we use a skeleton (or, medial axis) based shape descriptor. Our motivation for using skeletons rather than contours is due to the studies showing that skeleton-based shape similarity descriptors perform better than contour-based ones even in the case of partial occlusions (Sebastian et al., 2004; Sebastian and Kimia, 2005). Since skeleton integrates

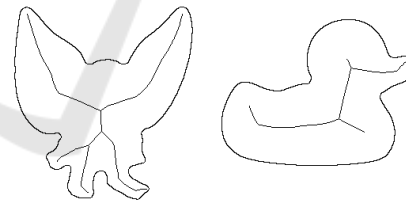


Figure 1: Sample silhouettes superimposed on their shapes.

both topological and geometrical features (Blum and Nagel, 1978), it has been used as a powerful shape descriptor in various applications including character recognition, content-based image retrieval, circuit board inspection, and biomedical imaging. Specifically, the skeleton of a shape forms the centers of the maximal disks inside the shape boundary, and the radii of these maximal disks represent the thickness of an object. Given the radius of such discs associated with each skeletal point, the object can be reconstructed exactly. Figure 1 shows two sample skeletons superimposed on their shapes. Skeleton extraction methods are highly sensitive to the boundary noise, yielding spurious skeleton branches. Therefore, many skeleton pruning approaches have been developed as we review some of them in the next section.

In this paper, we present a novel shape matching framework based on structures that are likely to exist in similar shapes. Specifically, after representing shapes as medial axis graphs, where vertices show skeletons and edges connect nearby skeletons, we determine the branches connecting or representing

shape's different parts. When images of an object are captured from close viewpoints, similar branch points are likely to exist in neighboring shapes. Using the shortest path distance from each vertex (skeleton) to each of the branches, we effectively retrieve similar shapes to the given query through a transportation-based distance function. Since the position of a skeleton changes with respect to shape rotation and scale, various approaches solve for transformation parameters prior to or during the matching. On the other hand, the shortest path distance we compute in this paper is invariant under rotation and takes into account the radius of each skeleton, allowing us to employ a more powerful shape representation.

Our idea for representing a skeleton using its distance from each of the branches, which are likely to exist in similar shapes, is motivated by the work of Vleugels and Veltkamp (Vleugels and Veltkamp, 2002). Assuming that the similarity between a pair of shapes can be computed using a metric distance function, the authors show that their resemblance can be computed by finding their distances from a third shape. Specifically, the distance from each database image to a set of  $n$  predetermined what is called vantage objects is computed. This step represents an image as a point in the  $n$ -dimensional vantage space. The database images that are similar to the vantage objects and similar to each other are determined by their position in the vantage space. Applying the same idea to our framework, we represent similar skeletons as nearby points in the geometric space. However, instead of selecting vantage objects from the database and computing the distance from each database shape to the same vantage objects, we select branches within each shape independently of the other shapes. Thus, while the database has to be given offline to compute the vantage objects for the technique presented in (Vleugels and Veltkamp, 2002), no such restriction is needed for the proposed approach. As an application, we implemented our approach on two datasets for shape retrieval experiments and showed the efficacy of the proposed technique for shape retrieval. Figure 2 demonstrates an overview of the proposed approach.

The rest of the paper is organized as follows. After taking a review of some previous work in Section 2, we describe the proposed approach in Section 3. We present the experimental results in Section 4 and conclude the paper in Section 5.

## 2 RELATED WORK

In this section, we briefly review some existing shape

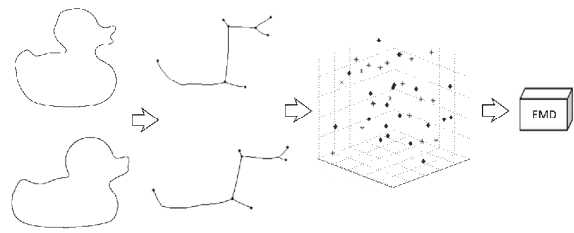


Figure 2: Overview of the proposed approach. The input shapes are first represented as medial axis graphs, whose vertices represent skeletons and whose edges show vertices adjacency. After the branch points are determined and ordered, we find the shortest path distance between each vertex and branch, which, in turn, represents each skeleton in the geometric space. The distance between the points sets are then computed using the Earth Mover's Distance (EMD) algorithm.

matching techniques. Shape context (SC) (Belongie et al., 2002) is a powerful shape descriptor based on counter points. The algorithm starts by generating  $n$  discrete counter points. The set of  $n - 1$  vectors originating from point  $p_i$  to all other points represents the overall shape relative to  $p_i$ . This information is encoded in the spatial histogram called shape context. Although this algorithm has a good descriptive power, it does not perform well for shapes with articulation parts. To deal with this problem, inner distance shape context algorithm (Ling and Jacobs, 2007) has been proposed. Given a set of boundary points on the shape, this algorithm considers the shortest path between each point pair. The approach has been shown to be robust to shape deformation, however, its main shortcoming is its sensitivity to the number of boundary points, e.g., with low number of such points its performance drops dramatically (Guocheng et al., 2010).

Guocheng et al. presents a framework dealing with the problems of describing part structure and articulation for shape recognition (Guocheng et al., 2010). The approach first performs equal space sampling on the shape contours. For each contour  $c$ , the algorithm draws circles centered at  $c$  and computes the ratio of the number of pixels that lie within the shape to the total number of pixels. The distance between two shapes is computed using  $\chi^2$  test statistics without using the point coordinates. Recently, Sirin and Demirci (Sirin and Demirci, 2014) extended this technique by relaxing the requirement of the same number of points for each input shape, which are represented as skeletons and taking into consideration the point coordinates during the matching stage.

A number of methods have been presented for skeleton-based shape matching. In (Liu and Geiger, 1999) shape axis tree defined by the locus of mid-points for optimally corresponding boundary points

are matched. Using graph topology changing operations may result in matches that do not preserve the coherence of the shape. Sharvit et al. proposes a shock graph-based shape recognition framework (Sharvit et al., 1998). Although this method has shown promising results, the errors of fundamental flows can break the hierarchical relations among parts of the shape. In (Siddiqi et al., 2002), shape recognition is achieved by solving the problem of subgraph isomorphism. Shock graphs are converted into rooted trees, which in turn are matched using a tree matching algorithm. In (Sebastian et al., 2004), the distance between two shapes is computed using the least action path deforming one shape to another. Yang et al. presents a shape classification framework based on statistics of dissimilarities between shortest skeleton paths (Yang et al., 2007). After finding end node in a medial axis graph, the algorithm computes the shortest path distances between all pairs of end nodes. Based on the sequences of radii of the maximal disks at corresponding skeleton points on these paths, the algorithm obtains similarity scores between input skeletons.

Given two similar shapes, it is well known that minor boundary deformation and part articulation may lead to significantly different skeletal representations. This, in turn, will have a negative impact on the shape matching algorithms, as it induces a large distance. To address this problem, skeleton pruning algorithms have been developed. Before the computing the skeletons, some of these algorithms perform boundary smoothing process (Dimitrov et al., 2000; Siddiqi et al., 2002), which may shift the skeleton positions due to the change in boundary locations. In some other methods (Shaked and Bruckstein, 1998; Ogniewicz and Kübler, 1995), an importance of each skeleton is computed and skeletons whose importance values are less than a predefined threshold is removed. In our framework, we used a recently developed skeleton pruning technique, where the decision regarding whether a skeletal branch should be pruned or not is based on the context of the boundary segment (Shen et al., 2011).

### 3 SHAPE REPRESENTATION THROUGH BRANCH NODES

In this section, we first describe the shape descriptor for the proposed approach. We then present the matching algorithm used in our framework.

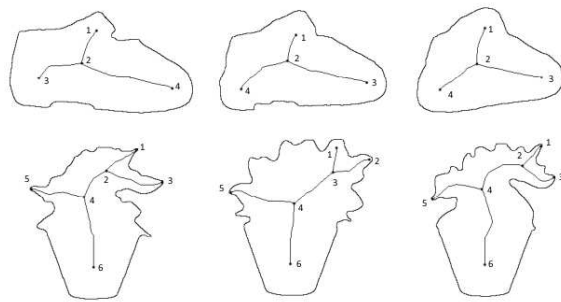


Figure 3: Shapes are represented as a medial axis graphs, whose vertices represent the regions skeletons and whose edges represent vertices adjacency. A branch in the graph (shown with a black dot) represents a point where either three or more region's parts meet or an end point of a region. The number associated with each branch indicates its order based on the local histogram.

#### 3.1 Shape Descriptor

The shape of a region can be represented as a medial axis graph, whose vertices represent the regions skeletons and whose edges represent vertices adjacency. The medial axis (Blum, 1967) captures the symmetries of a region, and its branches can be thought of as the regions parts (Demirci et al., 2009). A junction in the medial axis graph, thus, represents a point where three or more region's parts meet. To find branches in the medial axis graph, we use vertices of degree 1 or  $\geq 3$ . Assuming that shapes of similar objects have similar stable skeletons, the branches of degree 1 or degree  $\geq 3$  are likely to represent or connect similar parts. Figure 3 presents an example, where each branch is shown with a dot. Although the shapes are captured from different viewpoints, their branches are similar, representing and connecting similar region's parts.

After computing the branches, we find the shortest path distance from each vertex (skeleton) to each branch in the medial axis graph. Let the vertex set  $V$  and the branch set  $B$  in the graph are denoted by  $V = \{v_1, v_2, \dots, v_n\}$  and  $B = \{b_1, b_2, \dots, b_k\}$ , respectively. Finding the distance from each vertex to each branch in the medial axis graph represents vertex  $v_i$  in a  $k$ -dimensional vector space such that coordinate  $c_j$  corresponds to the weight of the shortest path between  $v_i$  and  $b_j$ , where  $v_i \in V$ ,  $b_j \in B$ ,  $1 \leq i \leq n$ , and  $1 \leq j \leq k$ . Based on the work of Eberly who finds the distance between two points in the scale-space (Eberly, 1994), we define the distance  $d(s_i, s_j)$  between skeletons  $s_i$  and  $s_j$ , located respectively at  $(x_i, y_i)$  and  $(x_j, y_j)$  as follows:

$$d(s_i, s_j) = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (r_i - r_j)^2}, \quad (1)$$

where  $r_i$  ( $r_j$ ) corresponds to the radii of the maximal

disk associated with  $s_i(s_j)$ . Let  $p = \{v_1, v_2, \dots, v_m\}$  denote the shortest path between a vertex and a branch point. The weight of this path is then computed as:

$$w(p) = \sum_{i=1}^{m-1} d(v_i, v_{i+1}). \quad (2)$$

Using the above procedure, skeletons are likely to be represented into geometric spaces of different dimensions as the dimensionality of the geometric space is defined by the number of branches. Therefore, a registration step whose objective is to represent the skeletons in the same space must be performed. To do this, we bring up lower dimensional skeletons to higher dimensions by padding them with zeros. Let  $b_m$  denote the maximum number of branches in a database shape. Suppose that shape  $S_1$  has  $b_1$  branches and  $b_1 \leq b_m$ . By adding  $b_m - b_1$  0-valued coordinates, we make the dimensions of the skeletons equal in the geometric space. In case the number of branches in the the query is greater than  $b_m$ , we reduce dimension of its skeletons using a dimensionality reduction technique, e.g., Principal Component Analysis.

One may notice that the position of a skeleton in the vector space is effected by the order of the branches, i.e., with different branch ordering the same skeleton is represented in different coordinates. In order to obtain consistency in our procedure, we order the branches by their relative position with respect to their neighbors. Specifically, we compute the local histogram around each branch using the radii of its neighbors. If we map the radii of branch neighbors containing  $n$  bins, then the histogram  $H$  becomes a vector  $(h_1, h_2, \dots, h_n)$ , where each element  $h_i$  represents the number of skeletons whose radii lies in the range associated with bin  $i$ . Stricker and Orengo showed that a slight shift in the histogram may result in a large dissimilarity between histograms of similar images (Stricker and Orengo, 1995). To make our branch ordering process robust to small shifts, we use cumulative histograms as suggested by this work. Given a cumulative histogram  $H = (h_1, h_2, \dots, h_n)$  associated with each branch, we first compute  $\alpha_1 \times h_1 + \alpha_2 \times h_2 + \dots + \alpha_n \times h_n$ , where each  $\alpha$  is a constant between 0 and 1,  $\alpha_1 \leq \alpha_2 \leq \dots \leq \alpha_n$  and  $\sum_{i=1}^n \alpha_i = 1$ , and order the branches by this value. In Figure 3, each branch is shown with its order number obtained using this procedure.

### 3.2 Matching in the Geometric Space

The final step of our algorithm is to match the skeleton representations in high dimensional geometric spaces. To compute the matching, we use the Earth

Movers' Distance (EMD) algorithm (Rubner et al., 2000), which has been successfully used in several applications, e.g., (Wang and Guibas, 2012; Xu et al., 2012; Li et al., 2013; Shokoufandeh et al., 2012). The EMD finds the optimum match between two sets by computing the minimum amount of work required to transform the first point set into the other.

Formally, let  $P_1$  and  $P_2$  be the first and second point sets with  $n$  and  $m$  points, respectively. Let  $D = [d_{ij}]$  be the ground distance matrix, where  $d_{ij}$  is the ground distance between points  $s_i \in P_1$  and  $s_j \in P_2$ . We define  $d_{ij}$  based on Equation 1. The objective of EMD is to compute a flow matrix  $F = [f_{ij}]$ , with  $f_{ij}$  being the flow between  $p_i$  and  $q_j$ , minimizing the overall distance:

$$\text{Work}(P_1, P_2, F) = \sum_{i=1}^m \sum_{j=1}^n f_{ij} d_{ij} \quad (3)$$

subject to:

$$\begin{aligned} f_{ij} &\geq 0, \quad 1 \leq i \leq m, \quad 1 \leq j \leq n \\ \sum_{j=1}^n f_{ij} &\leq w_{s_i}, \quad 1 \leq i \leq m \\ \sum_{i=1}^m f_{ij} &\leq w_{s_j}, \quad 1 \leq j \leq n \\ \sum_{i=1}^m \sum_{j=1}^n f_{ij} &= \min \left( \sum_{i=1}^m w_{p_i}, \sum_{j=1}^n w_{q_j} \right), \end{aligned}$$

where  $w_{s_i}(w_{s_j})$  represents the radius of the maximal disk associated with skeleton  $s_i(s_j)$ .

## 4 EXPERIMENTS

The proposed approach is evaluated in the context of a shape retrieval experiment using two datasets. The first dataset is a subset of ALOI (Amsterdam Library Object Images) (Geusebroek et al., 2005) database and consists of 1440 silhouettes of 20 classes, with 72 rotated views for each. The top four rows of Figure 4 present sample silhouettes, while the bottom row shows sample views for the same object.

We used leave-one-out procedure to the dataset in our experimental setup. Precisely, the first shape from the database is removed and used as a query for the remaining database shapes. After the query shape is put back in the database, and the procedure is repeated with the next shape from the database. Ideally, given a view of an object class, the shape retrieval algorithm should return an other view of the same class as its nearest neighbor. Based on the overall matching statistics, we observe that the proposed method obtains 96.2% correct nearest neighbor retrieval rate. In Figure 5, we present 2 correct experimental results for each class and the images of the last row are selected from the wrong classifications, which are shown in



Figure 4: The top four rows present sample views for each class from the subset of the ALOI dataset, while the bottom row shows different views for the same class.

red. Upon taking a closer look at the results, we note that the misclassifications are mostly due to the similar skeletons of different objects from specific views. Thus, taking a closer look at the last row we notice that the similarity between the skeletons of the cup and water sprayer, toy keys and mouse, shell7 and white shoe, plant in pot and cauliflower have a negative effect on the retrieval results.

To compare our results to two alternative shape classification algorithms, we use the Aslan and Tari dataset consisting of 14 classes with 4 shapes in each class (Aslan and Tari, 2005). The silhouettes are shown in Figure 6. While the first approach uses contour points (Sun and Super, 2005), the second approach is based on the statistics of dissimilarities of the shortest paths between a pair of skeletons (Yang et al., 2007). According to the results, out of 56 queries while the proposed framework yields 2 wrong results, these methods result in 4 and 1 wrong classifications, respectively. The correct classification accuracies are, thus, recorded as 96.4% for our approach, 92.8% for the first, and 98.2% for the second techniques. Although our performance is slightly worse than that of (Yang et al., 2007), the results still indicate an important retrieval potential of our framework. In addition, these two alternative approaches employ the Bayesian classifier to perform the retrieval task. We expect that arming our framework with a classifier will improve its results.

Finally, to evaluate the fitness of our approach for dealing with noise, we perturbed each query by randomly deleting its skeletons whose size was chosen randomly to fall between 10% and 35% of the total number of skeletons. The same experimental setup is

Query	Result	Query	Result	Query	Result	Query	Result
	halter toy		halter toy		wooden massage		wooden massage
	smiling duck		smiling duck		four vikings		four vikings
	white shoe		white shoe		yellow princess		yellow princess
	white cup		white cup		big pink eared animal		big pink eared animal
	toy keys		toy keys		water sprayer		water sprayer
	horse		horse		plant in pot		plant in pot
	eland		eland		cauliflower		cauliflower
	mouse		mouse		death bonsai tree		death bonsai tree
	shell7		shell7		feather ring		feather ring
	blue/red car		blue/red car		head		head
	water sprayer		mouse		white shoe		cauliflower

Figure 5: Part of the classification results. The text with black and red colors indicate correct and wrong classifications, respectively. The misclassifications are mostly due to the similar skeletons of different objects from some specific views.



Figure 6: Aslan and Tari dataset consists of 14 classes with 4 shapes in each class. Each row shows images of two different classes.

then used with the perturbed queries for both datasets. According to the results, the nearest neighbor retrieval score for our approach was dropped around 5% in both datasets, reflecting the algorithm’s robustness to missing data. The robustness of the proposed method can be attributed to (i) the stable skeleton extraction, (ii) the branch ordering, (iii) the effectiveness of the

EMD for partial matching. Although the true occlusion experiment would require replacing some part of the shape with an occluder, this experiment still presents promising results for handling the occlusion.

## 5 CONCLUSIONS

In this paper, we have proposed a novel method to classify a given shape using its skeletal representation. The algorithm starts by representing a shape as a medial axis graph, whose vertices represent the skeletons and whose edges represent vertices adjacency. After obtaining the branches in the graph, we compute the shortest path distance from each vertex to each branch, representing the corresponding skeleton in a geometric space. The distance between skeletons in the geometric space is computed based on the Earth Movers' Distance (EMD) algorithm. A set of shape retrieval experiments including the comparison with two previous approaches demonstrate the proposed algorithm's effectiveness and perturbation experiments present its robustness.

Although we applied our method to skeletal shape representations in this paper, we will test the framework to color object representations in the future. Our future work will also include employing a classifier into the framework and performing a more comprehensive comparison of our approach to more leading shape retrieval algorithms using larger datasets, including a test regarding the time efficiency of each system. In addition, designing an indexing system based on the similar idea is an interesting research direction on which we will work in the future.

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