

# Direct and Inverse Models of Human Arm Dynamics

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**Abstract:** This paper uses experimental data to model the human arm at the elbow joint. Direct models have been published before; this paper addresses inverse models (i.e. relating the force at the hand with the arm angle). Models used were integer, fractional commensurable and fractional non-commensurable order transfer functions, as well as neural networks (used as a term of comparison). Results show the superiority of fractional models, simpler, more exact, and with less parameter uncertainty.

## 1 INTRODUCTION

Dynamic models for the human arm are needed to replicate its behaviour by a robot (Taix et al., 2013; Fu and Cavusoglu, 2012). And controlling a robotic arm so that it will behave as much as possible as a human arm is no idle experience. It seems to be a good option for surgical robots (Park et al., 2006). Such robots can achieve, if properly designed, performances with an accuracy that represent a valuable assistance even to the most seasoned surgeons. But, for this to happen, the surgeon has to be comfortable working with the robot, and that is where the replication of a human arm comes in. A robot that feels more like another person's hand has shown to be a better companion than a robot with some other type of behaviour.

Third-order linear models are a usual assumption for this system; experimental data can be reasonably fitted, and there is furthermore a very reasonable rationale argument in favour of this structure, shown in Figure 1. More accurate results can be obtained with more complicated identification techniques and structures (Adewusi et al., 2012; Nagarsheth et al., 2008; Mobasser and Hashtrudi-Zaad, 2006; Venture et al., 2006); whether this pays off or whether the simpler linear option is better because it is good enough depends, of course, on the intended use.

In a previous paper we presented fractional order linear models for the human arm, obtained from experimental data with the measured force at the hand as model input and the measured arm angle as output, and compared them with the above mentioned

third order models (Tejado et al., 2013). There are reasons to expect this type of models here, because the dynamics of muscles of several animal species (including humans) have been modelled using fractional derivatives (Sommaccal et al., 2008; Sommacal et al., 2007b; Sommacal et al., 2007a; Djordjevic et al., 2003), and because muscles show viscoelastic behaviour, that can also be modelled using fractional derivatives (Mainardi, 2010; Magin, 2004). In their turn, fractional derivatives are expectable here given the fractal nature of muscular tissue.

In this paper, we present inverse (i.e. using now the measured arm angle as model input and the measured force at the hand as output) models for the human arm, using the same data from (Tejado et al., 2013), comparing fractional and third order (integer) linear models with neural networks. Neural networks provide nonlinear models that often obtain excellent performances (Jang et al., 1997; Nørgaard et al., 2003; Haykin, 1999): hence the pertinency of the comparison with fractional models, to see if they can stand the test. Parameter variability is checked and held as an important indicator of model suitability.

The contents of the paper are as follows: section 2 explains how data was obtained, and section 3 which models were used. Then, section 4 presents the results, and section 5 offers comments and conclusions.

## 2 EXPERIMENTAL DATA

As mentioned above, the experimental data is that of (Tejado et al., 2013); further details can be got in that

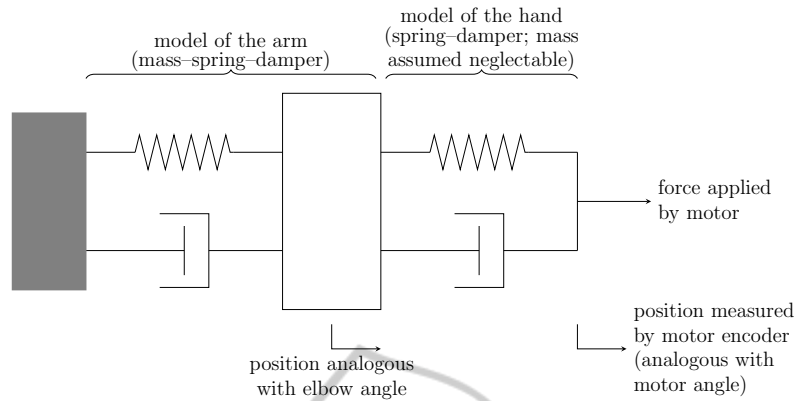


Figure 1: Third order translation analog to elbow dynamics.

paper.

Nine female and nine male volunteers, with ages ranging from 25 to 66, without any known musculo-skeletal injuries of the higher limbs, kneeled or sat holding a horizontal robotic arm and tried to keep it steady, while it moved randomly. The robotic arm was moved by a Kollmorgen direct drive D061M-23-1310 motor, able to produce 5.3 N·m continuous torque and 16.9 N·m peak torque, in current control mode. The rotation range was limited to  $\pm 0.9$  rad for safety reasons. The measured angle was obtained from an encoder with a resolution of 65,535 pulses/revolution. At the end of the aluminium horizontal robotic arm there was a handle for the volunteers to grab, a JR3 12-degree-of-freedom DSP-based force sensor, and a laser pointer which should be kept inside a target. Data was recorded with a 2 kHz sampling frequency.

Experiments (shown in Figure 2), which lasted 40 s to avoid fatigue, could be of three types:

- Type I — Oscillations in both directions around the zero-point;
- Type II — Oscillations only in the positive side of the zero-point (flexion of the elbow);
- Type III — Oscillations only in the negative side of the zero-point (extension of the elbow).

The oscillations were random to avoid anticipatory reflexes, which would interfere with the modelling of the arm dynamics. So the forces generated by the motor were a sum of sinusoids with frequencies in the [0.12, 15] Hz range (chosen because the bandwidth for the human arm is approximated to 10 Hz), limited to not exceed 4 N. Eight volunteers of either sex performed two experiments of type I, one experiment for type II, and one experiment for type III (68 data sets in total). The two other volunteers performed 16 additional experiments: six of type I, seven of type II, and seven of type III. The force measured by the men-

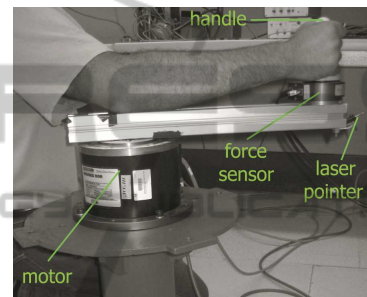


Figure 2: Robotic arm used to obtain experimental data of human arm dynamics (Tejado et al., 2013).

tioned sensor was practically identical to the force input.

### 3 TYPES OF MODELS

This section describes which models were used to find transfer function  $G_{arm, inverse}(s) = \frac{F_{measured}(s)}{\theta(s)}$ , where  $\theta$  is the measured arm angle,  $F_{measured}$  is the measured force at the hand, and  $s$  is the Laplace transform variable.

#### 3.1 Third-order Integer Linear Models

These have already been mentioned in section 1, and the reasoning behind their choice in Figure 1. The identification methods were the same described below for fractional transfer functions, restricting differentiation orders to natural numbers.

#### 3.2 Fractional Linear Models

If initial conditions are zero, fractional derivatives, denoted by  ${}_0D_t^\alpha$ ,  $\alpha \in \mathbb{R}$ , have Laplace transforms given by  $\mathcal{L}[_0D_t^\alpha f(t)] = s^\alpha F(s)$ . Consequently, from a differential equation with fractional derivatives,

fractional transfer functions arise. Those in which all orders share a least common multiple (the commensurability order) are called commensurate. Commensurate transfer functions with a commensurability order of 1 are integer transfer functions. Interested readers can find ample information on fractional derivatives in several books and papers, among which (Valério and Sá da Costa, 2013; Valério and Sá da Costa, 2011; Podlubny, 1999; Miller and Ross, 1993; Samko et al., 1993; Magin, 2004).

To identify a fractional transfer function from the measured data, rather than using a method to do this directly from a time response (Malti et al., 2008; Valério and Sá da Costa, 2013), a frequency response was estimated first (using Welch's method on the filtered output), and then Levy's method was applied, as this leads to less noisy results. Levy's method fits to frequency response  $G(j\omega_p)$ ,  $p = 1, \dots, f$ , a commensurable fractional model with a frequency response given by

$$\hat{G}(j\omega_p) = \frac{\sum_{k=0}^m b_k(j\omega_p)^{k\alpha}}{1 + \sum_{k=1}^n a_k(j\omega_p)^{k\alpha}} = \frac{N(j\omega_p)}{D(j\omega_p)} \quad (1)$$

minimising  $(G(j\omega)D(j\omega) - N(j\omega))^2$  (which is easier than the more obvious alternative of minimising  $\left|G(j\omega) - \frac{N(j\omega)}{D(j\omega)}\right|^2$ , which leads to a nonlinear problem). Commensurable orders of fractional models were found sweeping the  $\alpha \in [0, 2]$  range (outside which no transfer function is stable) with a 0.1 step, and keeping the  $\alpha$  for which results are better, using a heuristic which is better described below in section 4 after performance indexes are introduced. For more details on identification procedures of transfer functions for this plant, see (Tejado et al., 2013). Levy's method for fractional transfer functions is covered in (Valério et al., 2008).

### 3.3 Neural Networks

The (artificial) neural network (NN) models used below have a particular architecture—neural network auto-regressive with exogenous inputs (NNARX) models—represented in Figure 3. Notice how the model dynamics appear through the delay operator  $z^{-1}$  to make use of past values of both the input and the output of the system. Maximum input and output delays, respectively  $m$  and  $n$ , determine the memory that the model has of the input and output signals. Neurons are arranged in layers; as input, each neuron  $i$  in layer  $n$  receives a signal  $y_{n,i}$  that is a linear combination of every output signal of the neurons in the

previous layer  $n - 1$ :

$$y_{n,i} = b_{n,i} + \sum_{i=1}^{N_{n-1}} w_{n-1,i} x_{n-1,i}. \quad (2)$$

Here  $x_{n-1,i}$  is the  $i$ th input of the neuron coming from the previous layer,  $w_{n-1,i}$  is a weight associated with that input,  $N_{n-1}$  is the number of such inputs, and  $b_{n,i}$  is a bias. Obviously, layer number 0 (which does not exist) corresponds to the neural network's inputs themselves. The neuron's output  $x_{n,i}$  is determined by its transfer function  $f_{n,i}$ , usually known as activation function. Activation functions may be transfer functions, but if the neural network already includes a dynamic elsewhere activation functions will probably be static; the hyperbolic tangent sigmoid function  $x_{n,i} = f_{n,i}(y_{n,i}) = \frac{1 - e^{-y_{n,i}}}{1 + e^{-y_{n,i}}}$  is commonly used. Neural networks employed below use the activation function  $y(x) = x$  in the output layer; in other words, their output is a biased linear combination of all the inputs. For more details on neural network architecture, see the references in section 1.

Training a neural network is an optimization process in which weights  $w_{n,i}$  and bias factors  $b_{n,i}$  are iteratively updated in order to minimize the mean square error between the model output signals and the output data. Numbers of delays  $m$  and  $n$  are to be identified along with the weights and bias of the neural network. For the system at stake NNARX, models were trained using the Levenberg-Marquardt backpropagation algorithm, chosen for speed and accuracy. It is, actually, a local minimization method: therefore it is not guaranteed that a global optimal set of networks parameters is achieved. Because of this and the fact that the algorithm initialization has a random basis, the probability of two networks having equal final weights and biases is very low, even when trained with the same data. The data, after being resampled at 500 Hz (the robot's communication frequency), was then actually split into three parts: 60% for training, 20% for validation, and 20% for testing. The best results were consistently obtained for neural networks with 4 input delays, 2 output delays and a single neuron in the hidden layer: this was thus the configuration chosen. Indeed, architectures with more than one neuron in the hidden layer were tested, showing insignificantly better or weaker overall results, depending on the number of input and output delays. As to the number of input and output delays, it was determined as discussed below in section 4. That is why only networks with a single neuron in the hidden layer are considered below. (Mandic and Chambers, 2001; Marquardt, 1963) further elaborate on neural network training.

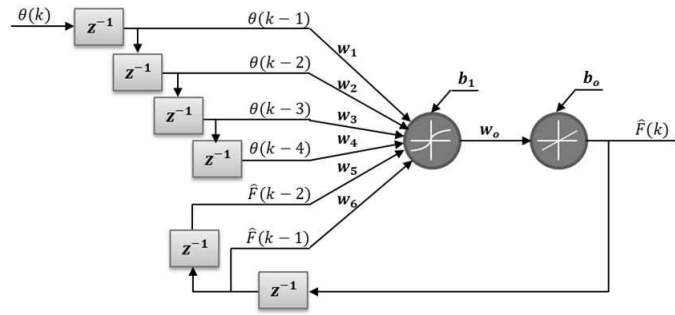


Figure 3: Neural network auto-regressive with exogenous inputs (NNARX) model, with one input and one output, one hidden layer,  $n = 2$  output delays and  $m = 4$  input delays.

#### 4 PERFORMANCE ASSESSMENT

Results were assessed with the following four performance indexes:

$$\text{Mean Squared Error, } MSE = \frac{\sum_{j=1}^N (y_j - \hat{y}_j)^2}{N} \quad (3)$$

$$\text{Mean Absolute Deviation, } MAD = \frac{\sum_{j=1}^N |y_j - \hat{y}_j|}{N} \quad (4)$$

$$\text{Maximum Deviation, } MD = \max_N |y_j - \hat{y}_j| \quad (5)$$

$$\text{Variance Accounted For, } VAF = 1 - \frac{\sigma^2(\mathbf{y} - \hat{\mathbf{y}})}{\sigma^2(\mathbf{y})} \quad (6)$$

The meaning of the variables for frequency and time responses is shown in Table 1. With three series to compare (gain, phase, time response) and four indexes, there are in all 12 values to assess a model's performance.

Recall that in (Tejado et al., 2013) we have shown that fractional models are better than integer order models, inasmuch they achieve a performance which is similar or even slightly better, with one parameter less, and with clearly less parameter uncertainty. Since identified integer direct models have 2 zeros and 3 poles, inverse models with 3 poles and 3 zeros were considered, as they ought to be causal. In the case of fractional order models, the same principle as above was applied: since fractional order direct models have 1 zero and 2 poles, causal inverse models with 2 zeros and 2 poles were considered. But this time there is the commensurability factor. So, in a first stage, the best commensurable order was found sweeping this factor in the  $[0.1, 1.9]$  range with a 0.1 step, keeping the model's dynamical structure (2 zeros and 2 poles). The output of this process is a set of 19 models to compare using the 12 aforementioned performance indexes. The following heuristic, essentially a multi-criteria optimisation algorithm, was used to choose the commensurate order:

1. Initialize  $P = 12$  lists with a length  $L = 1$ ;
2. Select the best  $L$  models according to each performance index;
3. Model by model, check for its presence in any of the  $P$  lists with length  $L$  and compute a histogram that shows the number of presences of every model in all the lists;
4. If one model is found to be present in every single list, that is the best choice and the heuristic stops;
5. Otherwise, increment the value of  $L$  by 1 and repeat from step 2.

It may happen that more than one model comes to appear at all  $P$  lists at the same time. In that case, either may be selected as convenient. With this heuristic a good value for the commensurability order may be got, but a 0.1 step may be a little too rough, so a second stage search was performed. In this stage, the best model was found sweeping the commensurable order, with a 0.01 step, in a range defined by a neighborhood, with a 0.1 radius, centred on the best commensurability value (or values) obtained on the first stage. From this sweeping process, another set of models arise and are, consequently, compared, again, using the same heuristic. Therefore, this second stage is essentially a refined search around the best solution (or solutions) of the first stage.

In the case of NNARX models, training algorithms are rather blind when it comes to the best values of  $m$  and  $n$ . Models with an unrealistically large (and unnecessary) number of input and output delays may still provide good results. So we might assume, initially, an unrealistically large number of input and output delays and analyze the corresponding weights, comparing them to decide if  $m$  and  $n$  should be decremented, until none of the weights is lower than a certain threshold value. But in this case it is possible to use prior knowledge of the system to be identified, assume a maximum value for the number of input

Table 1: Variables in equations (3)–(6).

Variables	Frequency response		Time response
	Gain	Phase	
$y$	Gain curve estimated from measured data for a certain frequency vector	Phase curve estimated from measured data for a certain frequency vector	Time series of measured input data
$\hat{y}$	Gain curve estimated from measured input and identified inverse model for the same frequency vector of $y$	Phase curve estimated from measured input and identified inverse model for the same frequency vector of $y$	Time series of inverse model output
$N$	Length of the frequency vector length, that must be the same for $y$ and $\hat{y}$		Length of the time series

Table 2: Performance comparison between identified inverse models of the human arm (best values in bold).

Model		Frequency response								Time response			
		Magnitude (dB)				Phase (°)				MSE	MAS	MD	VAF
		MSE	MAS	MD	VAF	MSE	MAS	MD	VAF				
All	Int.	<b>15.410</b>	<b>2.903</b>	<b>11.994</b>	<b>92.545</b>	1007.274	28.486	69.760	74.125	0.272	0.371	1.819	41.508
	Frac.	21.632	3.455	14.352	88.903	628.689	18.032	<b>58.400</b>	86.449	<b>0.266</b>	<b>0.354</b>	<b>1.658</b>	47.888
	NN	23.028	3.807	18.378	90.812	<b>559.089</b>	<b>16.279</b>	101.135	<b>87.496</b>	1.616	1.096	2.335	<b>57.827</b>
Type I	Int.	<b>12.294</b>	<b>2.587</b>	<b>11.398</b>	<b>92.703</b>	764.852	24.454	62.375	82.033	0.421	0.490	2.440	51.515
	Frac.	18.769	3.274	12.803	89.150	<b>404.851</b>	<b>14.437</b>	<b>42.690</b>	<b>90.971</b>	0.423	0.477	2.284	55.144
	NN	25.354	4.094	14.948	90.574	467.089	16.026	76.924	88.489	<b>0.286</b>	<b>0.415</b>	<b>2.001</b>	<b>63.803</b>
Type II	Int.	<b>15.315</b>	<b>2.842</b>	<b>12.230</b>	<b>93.656</b>	1146.667	30.209	69.054	68.563	0.143	0.280	1.408	39.239
	Frac.	23.264	3.483	17.098	88.926	<b>624.472</b>	<b>17.813</b>	<b>59.324</b>	<b>85.727</b>	0.136	0.269	1.267	45.956
	NN	22.443	3.908	14.337	93.456	920.851	17.971	157.716	81.771	<b>0.086</b>	<b>0.227</b>	<b>1.086</b>	<b>57.492</b>
Type III	Int.	<b>16.032</b>	<b>3.089</b>	<b>10.971</b>	93.165	1318.925	31.813	80.725	65.715	0.126	0.271	1.363	31.355
	Frac.	21.802	3.507	14.387	89.999	687.664	19.075	<b>67.334</b>	<b>84.818</b>	<b>0.096</b>	<b>0.239</b>	1.143	45.007
	NN	24.498	4.129	12.308	<b>93.450</b>	<b>647.907</b>	<b>17.872</b>	128.764	84.132	0.098	0.250	<b>1.041</b>	<b>54.529</b>

and output delays and try every dynamical structural combination within that maximum number of delays; from this process, results a set of models that should be compared, keeping the best one. It was shown that linear inverse models are of 2nd or 3rd order, so a maximum of 6 input and output delays was considered, to give some margin for possible additional nonlinear dynamics to be identified in measured data. With this maximum value, one gets a set of 36 neural networks to compare and a heuristic similar to the one described above for fractional plants was employed. The only difference here is that, as mentioned previously, very complex networks can have slightly better results, but at the cost of a lower computational efficiency. Therefore, to the 12 values of performance indexes mentioned above, two more were added:  $m$  and  $n$  themselves, thus providing for a neural network which is a compromise between model performance and model complexity.

## 5 COMMENTS AND CONCLUSIONS

Performance results in Table 2 show that integer models often get better results in what the gain of the transfer function is concerned, but not the phase, or above all the time response; NN models, even though nonlinear, do not consistently perform better, and,

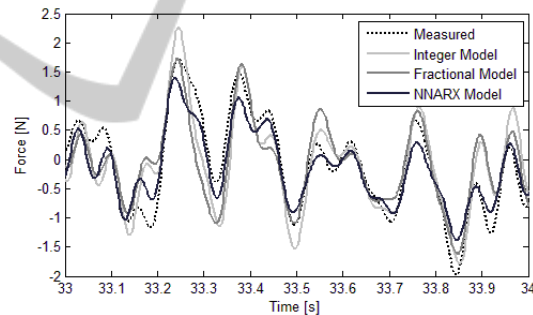


Figure 4: Part of the responses of the several models, compared with experimental data.

when they do, only slightly. Normalising all performance indexes between 0 (worst) and 1 (best), and averaging the results, we obtain the following results: integer models, 0.6224; fractional models, 0.6847; NNARX models, 0.3770. It can be seen that fractional models achieve their performance being linear, continuous (and thus fit for every sampling time) and using one parameter less than integer models. They are thus the simplest possible model.

So again we find a dynamic behaviour which we can conclude is best described by fractional transfer functions, just as was seen in (Tejado et al., 2013) for direct models. Figure 4 shows some seconds of the responses obtained with the different models, compared with experimental data.

Future work consists in using these models to make a KUKA LWR 4+ 7-degree of freedom light

weight robot behave like a human arm, and check this behaviour against the experimental data collected.

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