

# Deep Learning for Facial Keypoints Detection

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**Abstract:** A new area of machine learning research called deep learning has moved machine learning closer to one of its original goals: artificial intelligence and feature learning. Originally the key idea of training deep networks was to pretrain models in completely unsupervised way and then fine-tune the parameters for the task at hand using supervised learning. In this study, deep learning is applied to a facial keypoints detection. The task is to predict the positions of 15 keypoints on grayscale face images. Each predicted keypoint is specified by a real-valued pair in the space of pixel coordinates. In the experiments, we pretrained a Deep Belief Network (DBN) and finally performed discriminative fine-tuning. We varied the depth and size of the network. We tested both deterministic and sampled hidden activations, and the effect of additional unlabeled data on pretraining. The experimental results show that our model provides better results than the publicly available benchmarks for the dataset.

## 1 INTRODUCTION

A feature learning algorithm could be described as a way of automatically learn features from data at multiple levels of abstraction. The learned features might be different depending on the task, even for the same data. For example, a classification task requires distinctive features between classes whereas more generative features are needed in an image inpainting task. In the case of images, the first abstraction level could contain simple filters like straight lines with different orientations. The level above the first one forms more complex shapes, such as polygons, based on the abstractions from the layer below.

There are at least three reasons why a feature learning is of interest to researchers. First, higher-level abstractions often become hard for humans to specify in terms of raw data. Second, learned features are adapted for the task at hand. To obtain features for a specific task normally requires a huge amount of domain knowledge, for example features used in image processing are quite different from features in speech recognition. Third, a feature learning scales and adapts to the data compared to the traditional feature extractor methods. The last one is a fascinating property: More data will provide better features. For smaller problems, the lack of sufficient data might also be a crucial limitation.

The depth of the architecture refers to the num-

ber of levels of composition of non-linear operations in the learned function. Most current supervised learning algorithms correspond to shallow architectures (1, 2 or 3 levels), such as neural networks with one hidden layer, support vector machines and random forests. In addition to the limitations caused by the shallow architectures (Bengio, 2009), these algorithms are trained using only labeled data and they are unable to benefit from unlabeled data. Even though researchers have put huge effort on labeling data sets, this underlying problem still remains in every machine learning application using supervised learning.

On the contrary, for example, object recognition in the visual cortex, uses many layers of nonlinear processing and requires very little labeled input (Lee et al., 1998). Object recognition, which is a common example of artificial intelligence (AI) related task, is still a difficult task in computer vision research. This is because the image of an object vary in different view points, scale and sizes or when an object is translated or rotated into a different pose. Objects can also be partially occluded from the view, which is extremely common in video sequences and real world images.

The goal of this study is to find out how well a deep learning model called Deep Belief Net (DBN) can learn features from small grayscale images to detect facial keypoints. The facial keypoints dataset available through the Kaggle competition, was used

in the experiments.

Section 2 gives a brief overview of deep learning and its success on different application areas. Section 3 gives a detailed description of methods used in this study. The experiments that were done are explained in Section 4. The results are discussed in Section 5 and a short conclusion of this study is given in Section 6.

## 2 DEEP LEARNING

Increased computing power and massive data sets have made the implementation of larger and more complex machine learning systems feasible. These factors have had a huge impact on the development of artificial neural networks research, which is now commonly referred as deep learning.

In 2006, Hinton et al. (Hinton et al., 2006) introduced unsupervised learning algorithm for deep generative models called Deep Belief Net. The building block for DBN is an energy-based graphical model called restricted Boltzmann machine (RBM) that can learn a probability distribution over its set of inputs.

Recently in 2012, Hinton et al. (Hinton et al., 2012) developed an interesting biology-inspired method to prevent overfitting by randomly omitting half of the feature detectors on each training case. This prevents complex co-adaptations in which a feature detector is only helpful in the context of several other specific feature detectors. This method, referred as “dropout”, can be seen as an efficient way of performing model averaging.

Experiments in speech recognition show that pre-training the model with DBNs can significantly outperform the conventional context-dependent Gaussian mixture model (GMM) (Dahl et al., 2012)(Jaitly et al., 2012). In the field of object recognition, large, deep Convolutional Neural Network (CNN) was considerably better than the previous state-of-the-art methods in classification and localization tasks in ImageNet Large Scale Visual Recognition Challenge 2012 (Krizhevsky et al., 2012). The network contained 60 million parameters and 650,000 neurons, and was trained on raw RGB pixel intensities.

High-dimensional data can be converted to low-dimensional codes by training a multilayer neural network with a small central layer to reconstruct high-dimensional input vectors. This deep autoencoder network significantly outperforms common methods, such as Principal Component Analysis (PCA) and Latent Semantics Analysis (LSA), and autoencoder networks can be used for data visualization purposes (Hinton and Salakhutdinov, 2006).

Also, a technique called “semantic hashing”, provides very fast method for document and image retrieval (Salakhutdinov and Hinton, 2009).

An important notion is that deep learning methods do not necessarily require domain specific knowledge of the particular task at hand. Deep learning moves machine learning closer to one of its original goals: artificial intelligence and automatic feature learning. Deep learning models have been applied successfully to a wide variety of machine learning competitions in Kaggle (Kaggle, 2010), for example, molecular activity prediction (Dahl, 2012) and job salary prediction (Mnih, 2013).

Hierarchical face parsing (Luo et al., 2012) is an interesting method on how to combine prior knowledge on facial parts and components with deep learning strategy for several face-related tasks. Another interesting work in the domain is the convolutional network cascade model (Sun et al., 2013), which fuses CNNs with multi-level regression. While the results of both studies are very good, one drawback is that the architectures used are carefully designed requiring a lot of tedious manual engineering.

## 3 MODELING OF DEEP BELIEF NETS

### 3.1 Binary RBM

A Restricted Boltzmann machine is a particular type of Markov random field that has a two-layer architecture (Smolensky, 1986). In the machine the visible, binary stochastic units  $v \in \{0, 1\}^V$  are connected to hidden binary stochastic units  $h \in \{0, 1\}^H$ . The energy of the model with  $V$  visible units and  $H$  hidden units is defined as follows:

$$E(v, h) = - \sum_{i=1}^V \sum_{j=1}^H v_i h_j W_{ij} - \sum_{i=1}^V v_i b_i - \sum_{j=1}^H h_j a_j, \quad (1)$$

where  $W^{V \times H}$  represents the symmetric weights and  $b^V$  and  $a^H$  are bias terms. The conditional distribution over the hidden units is as follows:

$$P(h_j = 1|v) = g \left( \sum_{i=1}^V v_i W_{ij} + a_j \right), \quad (2)$$

where  $g(x) = 1/(1 + \exp(-x))$  is the logistic function. Since  $v$  and  $h$  play a symmetric role in the energy function,

$$P(v_i = 1|h) = g \left( \sum_{j=1}^H h_j W_{ij} + b_i \right). \quad (3)$$

### 3.1.1 Contrastive Divergence

Because the maximum likelihood learning for energy based models is often infeasible (Lecun et al., 2005), an approximation is needed. Contrastive Divergence (CD) (Hinton, 2002) is an efficient learning algorithm for such models and in the case of binary RBMs, this equals to the following learning rule:

$$\Delta W = \alpha (E_{P_{data}} [v^T h] - E_{P_T} [v^T h]) \quad (4)$$

$$\Delta a = \alpha (E_{P_{data}} [h] - E_{P_T} [h]) \quad (5)$$

$$\Delta b = \alpha (E_{P_{data}} [v] - E_{P_T} [v]), \quad (6)$$

where  $\alpha$  is the learning rate,  $P_{data}$  is the training set empirical distribution and  $P_T$  represents a distribution defined by running a Gibbs sampling, initialized at the data, for  $T$  full steps. Setting  $T = \infty$  recovers maximum likelihood learning. However, the CD learning with  $T = 1$  has been shown to work well (Hinton, 2002).

### 3.2 Gaussian RBM

With the usual parameterization of the energy function for Gaussian RBM, it is difficult to learn variances from natural images using Contrastive Divergence. Therefore, variances are often fixed to unity (Cho et al., 2011) (Wang et al., 2012). To overcome this limitation, we introduce a modified energy function for Gaussian RBM that can be trained with Denoising Score Matching (DSM) algorithm.

#### 3.2.1 Denoising Score Matching

Score matching (SM) is another approach to estimate parameters of probability models whose partition functions are intractable. SM minimizes the expected squared distance between the gradient of the log-density given by the model, referred as score  $\psi$ , and the gradient of the log-density of the observed data (Hyvärinen, 2005).

Pascal Vincent showed that there is an equivalence between Denoising Autoencoder (DAE), a successful alternative to RBM that reconstructs the input from a corrupted version of it (Vincent et al., 2008), and Gaussian RBM trained using denoising score matching (Vincent, 2011):

$$J_{DSM} = E_{q_{\sigma}(\tilde{v}, v)} \left[ \frac{1}{2} \left\| \psi(\tilde{v}) - \frac{\partial \log q_{\sigma}(\tilde{v}|v)}{\partial \tilde{v}} \right\|^2 \right]. \quad (7)$$

$E_{q_{\sigma}(\tilde{v}, v)} [\cdot]$  is the expectation with respect to a noise model and using additive Gaussian noise model, the latter term in the squared distance corresponds to

$$\frac{\partial \log q_{\sigma}(\tilde{v}|v)}{\partial \tilde{v}} = \frac{1}{\sigma^2} (v - \tilde{v}). \quad (8)$$

By introducing the following energy function of Gaussian RBM, we achieve the equivalence:<sup>1</sup>

$$E(v, h) = -\frac{1}{\sigma^2} (v^T b + v^T W h + h^T a - \frac{1}{2} v^T v) \quad (9)$$

$$P(h|v) = g \left( \frac{Wv + a}{\sigma^2} \right) \quad (10)$$

$$P(v|h) = \mathcal{N}(Wh + b, \sigma^2) \quad (11)$$

$$\psi(v) = -\frac{1}{\sigma^2} (v - b - g \left( \frac{Wv + a}{\sigma^2} \right) W^T), \quad (12)$$

where  $\{W, b, a, \sigma\}$  are the model parameters similar to binary RBM with an added variance term  $\sigma^V$  for visible units. Substituting Eq. 8 and Eq. 12 in Eq. 7, leads to the final objective function, which can then be minimized by using gradient descent.

### 3.3 Deep Belief Nets

To extract higher-level features, a Deep Belief Network combines a set of RBMs that are learned sequentially (Hinton et al., 2006). The greedy layer-wise algorithm trains one RBM at a time, using the previous layer's activations as inputs.

Finally, a normal feed-forward neural network can be initialized using the weights learned by the pre-trained RBMs. For output, a softmax regression layer is usually chosen for a classification task. For continuous values, a linear Gaussian layer can be used to model conditionally Gaussian data. The full neural network can be then fine-tuned with Stochastic Gradient Descent (SGD) algorithm. The gradients for the parameter updates are obtained using the standard backpropagation algorithm (Rumelhart et al., 1988).

### 3.4 Momentum

Nesterov's Accelerated Gradient (NAG) is a momentum-based acceleration method that is capable of accelerating directions of low-curvature, which is particularly useful in training deep networks. It explores new regions of the parameter space that are local minima of better quality (Sutskever et al., 2013). The NAG update rule for parameters  $\theta$  is given by:

$$v_{t+1} = \mu v_t - \alpha \nabla f(\theta_t + \mu v_t) \quad (13)$$

$$\theta_{t+1} = \theta_t + v_{t+1}, \quad (14)$$

<sup>1</sup>This is not the same energy function as presented in Eq. 4.3. in (Vincent, 2011) because in that energy function there is no hidden variables and therefore is not an RBM. Parameterization in Eq. 9 is proposed by Ian Goodfellow in Pylearn2 (Goodfellow et al., 2013) implementation of Gaussian RBM.

where  $\alpha > 0$  is the learning rate,  $\mu \in [0, 1]$  is the momentum coefficient, and  $\nabla f(x)$  is the gradient at  $x$ . Intuitively, this tells us first to take a step in the direction of the velocity  $v_t$ , which is the direction of the previous parameter update, and then calculate the gradient update. By setting  $\mu = 0$ , this becomes the normal gradient descent update rule.

### 3.5 Regularization

While layer-wise generative pretraining itself is a powerful regularizer, we introduce three regularization methods that improve generalization of large models even further.

#### 3.5.1 L1 Weight-decay

Sparse features effectively lower the dimensions and mitigates the curse of dimensionality. An easy way to increase sparsity in a model is to use the L1 weight-decay (Bach et al., 2012). By penalizing weights with the L1-norm, the weight-decay induces sparsity to the model in the sense that it forces many of the weights to become exactly zero while allowing a few of the weights to grow large.

#### 3.5.2 Dropout

Dropout is a technique that prevents overfitting and provides an efficient way of doing model averaging. The first phase corresponds to dropping out units during training: Each unit in a layer, with all its incoming and outgoing connections, is dropped out with a fixed probability  $p$  independent of other units. With  $p = 0.5$ , a neural net with  $n$  units can be seen as a collection of  $2^n$  possible thinned networks that all share weights. At test time, a very simple approximate model averaging method is used: Using network without dropout and multiplying weights by  $p$ . This ensures that the expected output of any hidden unit is the same as the actual output at test time (Srivastava et al., 2014).

#### 3.5.3 Data Augmentation

A very fundamental approach to improve generalization is getting more training data. If a training set is scarce and has some transformation-invariance properties, generating additional data may be useful. In this particular case, we assume that the facial images are extracted using standard face extractor. Thus, we assume frontal faces with some rotational variance and for example images in much smaller scale are errors of the extractor and they are considered as outliers. This analysis is also supported by the distortion

intervals that produced the best generalization error on the validation set.

We randomly added small variations to the training set, corresponding to rotation and scale. The amount of variation for each distortion were uniformly sampled from the interval, which were determined using the validation set. In supervised fine-tuning, the keypoints were also transformed accordingly. Small distortions provided additional training data that is consistent with the original data distribution. For images, even small random rotation could be enough to prevent deep network to memoize its input.

## 4 EXPERIMENTS

In this section, we experimentally evaluated different DBN architectures to find out a suitable model for facial keypoints detection. We varied the depth and the size of an architecture. In unsupervised pretraining, we tested sampled and expected activations and the effect of additional unlabeled data.

### 4.1 Facial Keypoints Dataset

The facial keypoints dataset available through Kaggle competition (Kaggle, 2013) was used in experiments. The task was to predict keypoint positions on  $96 \times 96$  pixel grayscale facial images. The problem is challenging because facial features vary from one individual to another. Environmental conditions such as illumination, viewing angle and pose express a large amount of variation as well. Features learned to predict the keypoints can also be suitable in several other applications, such as tracking faces, analyzing facial expressions, detecting dysmorphic facial signs for medical diagnosis and face recognition.

The dataset was divided into a training set and a public test set. Each predicted keypoint is specified by an (x,y) real-valued pair in the space of pixel coordinates. There are 15 keypoints<sup>2</sup>, which represent locations of eyes, eyebrows, nose and mouth. 1783 images in the public test set do not have keypoint values annotated and the set is used to test performance against other competitors through the Kaggle platform. There are also two benchmarks in the public leaderboard: Averages benchmark and patch search

<sup>2</sup>Keypoint ids: left\_eye\_center, right\_eye\_center, left\_eye\_inner\_corner, left\_eye\_outer\_corner, right\_eye\_inner\_corner, right\_eye\_outer\_corner, left\_eyebrow\_inner\_end, left\_eyebrow\_outer\_end, right\_eyebrow\_inner\_end, right\_eyebrow\_outer\_end, nose\_tip, mouth\_left\_corner, mouth\_right\_corner, mouth\_center\_top\_lip and mouth\_center\_bottom\_lip.

benchmark. Even though the training set size is 7049 images, only 2140 samples have all the 15 keypoints and the rest have only 4 keypoints labeled. To make use of all the training data, algorithms need to somehow handle this missing label information.

## 4.2 Setup for Experiments

We used Pylearn2 (Goodfellow et al., 2013) machine learning research library to implement the models. It is built on top of Theano (Bergstra et al., 2010) which can compile code for both CPU and GPU backends. Many deep learning algorithms, like Contrastive Divergence for RBM, can be parallelized for GPU which significantly speeds up the learning.

In data preprocessing, we first downsampled the images by a factor of 3 which decreased the input dimensions to  $32 \times 32 = 1024$ . This common practise in image processing is usually applied to achieve reasonable memory usage and faster training times. Then we applied global contrast normalization so that the images became to have zero mean and unit variance.

The set of 2140 fully labeled images was split into a training, validation and a test set. The test and the validation sets consisted of 300 fully labeled examples. Images with imperfect label information gave a convenient way to test the effect of additional unlabeled data. The additional data were used in unsupervised pretraining. Training epochs were fixed and the best model was selected based on results on the validation set.

## 4.3 Architectures

We varied the size of the network, more specifically the number of hidden units in each layer. These three hidden layer configurations were tested: 1000–1000–4000 (small), 1500–1500–6000 (medium) and 2000–2000–8000 (large). The linear Gaussian output layer has 30 units which corresponds to  $(x,y)$  pixel coordinates of 15 keypoints. The medium size architecture used in experiments is outlined in Figure 1.

The surrounding feed-forward neural network with a linear Gaussian output layer was the same for each pretrained DBN. To test the effect of increasing the depth, we trained a new RBM above the previous DBN. We fine-tuned each DBN separately, which led to the total of three DBNs per architecture. We transformed the data from the lower layer by calculating the expectations of the hidden units. These deterministic activations can be considered as features that the layer can extract. Because the hidden units are binary stochastic units, it is natural to interpret the activations to include a sampling from the Bernoulli

distribution. We also ran tests by using sampled activations from deterministic expectations each time a training example is shown to the model.

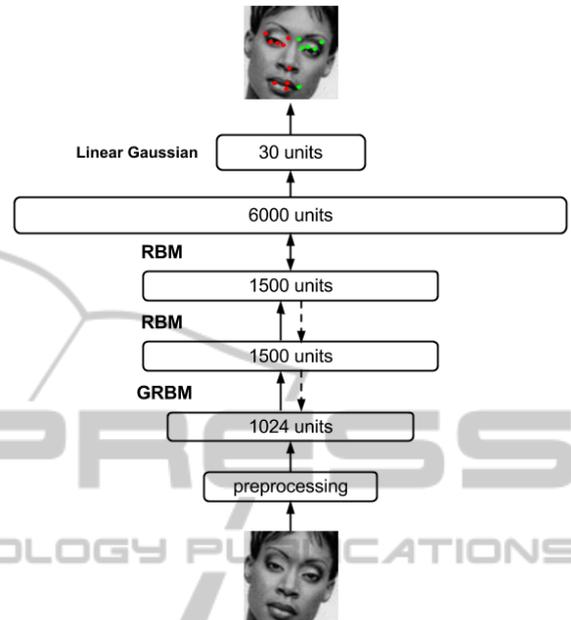


Figure 1: The architecture of medium size Deep Belief Net used in experiments.

## 4.4 Pretraining

We trained the first layer Gaussian RBM with 1024 visible units ( $32 \times 32$  pixels) using Denoising Score Matching with 0.4 Gaussian corruptor and 0.01 learning rate. The second and third layers are normal binary RBMs which we trained using CD-1 algorithm with 0.001 learning rate. The coefficient for L1 weight-decay was set to 0.0001. Momentum constant for NAG was initialized at 0.5 and after 300 epochs it reached its maximum value of 0.7. The data distortion intervals of scale  $[1.1, 0.9]$  and rotation  $[-8^\circ, 8^\circ]$  in data augmentation were used. The values for hyperparameters were determined using the validation set. We trained the models for 300 epochs. We used the objective function on validation set to select the best model with Denoising Score Matching and reconstruction error with Contrastive Divergence.

## 4.5 Fine-Tuning

The same values for the momentum constant, data distortion and L1 weight-decay coefficient were used as in pretraining. We used the dropout probability of 0.8 for the input layer and 0.5 for the other layers. 0.001 was used as the learning rate. We trained the

models for 3000 epochs. In supervised fine-tuning, the best model was selected on Mean Squared Error of predictions on validation set.

## 4.6 Results

We followed the convention from the Kaggle platform and used Root Mean Squared Error (RMSE) between the ground truth and the predicted  $(x,y)$  points as a performance metric.

The experiments showed that there were a little difference on performance whether deterministic or sampled activations were used on pretraining. Also, the results did not provide decisive evidence that additional unlabeled data on pretraining improves the performance. However, the largest model was able to benefit from the additional unlabeled data. There is also a trend that increasing the size and depth of the network improved the performance.

Our best model was the large size DBN-3, trained using extended training set and deterministic activations between layers. The best DBN-3 in each size category overcame the reference benchmarks in Kaggle competition (Table 1). Test results are summarized in Table 2.

Table 1: Comparison to Kaggle benchmarks. “(e)” denotes the usage of extended training set and “(s)” denotes sampled hidden activations between layers on pretraining.

Model	RMSE
Averages Benchmark	3.96244
Patch Search Benchmark	3.80685
DBN-3 (Small) (s)	3.25079
DBN-3 (Medium) (s)	3.23646
DBN-3 (Large) (e)	3.21211

Table 2: Prediction metrics (in RMSE) for various DBNs. “(e)” denotes the usage of extended training set and “(s)” denotes sampled hidden activations between layers on pretraining.

Model	Small	Medium	Large
DBN-1	1.71733	1.62732	1.60542
DBN-2	1.58002	1.54632	1.51221
DBN-3	1.53438	1.50519	1.50385
DBN-2 (s)	1.56710	1.56018	1.49564
DBN-3 (s)	1.53192	1.49432	1.49945
DBN-1 (e)	1.67990	1.63921	1.63162
DBN-2 (e)	1.59267	1.52828	1.51318
DBN-3 (e)	1.53839	1.50037	1.47267

## 4.7 Visualizations

Figure 2 shows some of the filters learned by the first-level Gaussian RBM. Example keypoint predictions

by the best model are shown in Figure 3. Keypoints from faces, which match the dominant scale in the dataset, were predicted well. The mouth keypoints were causing some error on these images when the face was not aligned. When an image was in notably smaller scale, predictions were clearly wrong and somewhat corresponded to an average guess. The model tolerated quite well occlusions like glasses, hair and beard.



Figure 2: Some of the weights learned by Gaussian RBM in the first layer.

## 5 DISCUSSION

The filters in Figure 2 show that modeling the real-valued input data using Gaussian RBM and Denoising Score Matching gave meaningful results. The first layer plays an important role in the whole network because the upper layers are trained based on its hidden activations. Filters clearly corresponds to recognizable face images. This is due the fact that images in the dataset have roughly the same scale and alignment. These weights are clearly better than random values to initialize a feed-forward network. We want to emphasize that these useful filters were learned in completely unsupervised way. Experiments showed that using the extended training set improved the results on the largest model and we are intrigued by the possibility to exploit large databases such as Google image search. There were no significant difference between the deterministic and sampled activations, when transforming the data from the lower layer during pretraining.

Analysis of keypoint predictions tells us that our



Figure 3: Keypoint predictions. Green color indicates point positions from the left side and red color from the right side of the face relative to subject.

model can predict meaningful keypoint positions on facial images which have the same scale. Faces with small rotations got some error typically on one side, but the prediction is probably still good enough for tracking, for example. The largest errors were on images that are at a completely different scale. Predictions for those cases are useless and can be interpreted that the model just made an educated guess. We found that the model tolerated well occlusions that were caused by glasses, hair and beard. That is a useful feature to have because real-world use cases most certainly contain noisy data.

There is a relatively large bias between the test results and Kaggle benchmarks. Some of that is probably caused by the fact that for the 4-keypoint images, the nose tip is systematically labeled below the nose. These partially labeled images were only used in the pretraining when testing the effect of additional data and 4-keypoint images were not used in the more powerful supervised training stage. The public test set contained more examples with a smaller scale. Similar scale variance were present in particular in the 4-keypoints dataset. This imbalance did not cause error on our local tests because we only used the fully labeled examples in the supervised learning and tests.

The fully connected neural network architectures used in our experiments does not scale linearly with respect to the number of parameters. For example, the small architecture had about 6 million parameters

while the large architecture had over 22 million parameters. If we increased the size further, for example to 3000–3000–12000 architecture, it would correspond to 48 million parameters. This suggests that a different network architecture would have to be exploited to train even larger models.

## 6 CONCLUSIONS

The goal of this study was to find out how well Deep Belief Nets can learn features from small grayscale images to detect facial keypoints. We introduced the main building block of deep learning, an energy-based generative model called Restricted Boltzmann Machine and an unsupervised learning algorithm called Contrastive Divergence. A novel approach, Denoising Score Matching, was introduced to train Gaussian RBM to model real-valued data. We described how Deep Belief Nets can be constructed by using the greedy layer-wise training.

The facial keypoints dataset available through the Kaggle competition, was used in the experiments. The analysis of predicted keypoints showed that the proposed deep model was able to predict meaningful keypoints on facial images which have a similar scale. It also overcame the reference benchmarks in the Kaggle competition. The model tolerated occlusions that were caused by glasses, hair and beard, well. On pretraining, the largest model was able to benefit from the extended unlabeled training set.

The results of this work imply that a Gaussian RBM trained with DSM can successfully model real-valued pixel intensities, and increasing the depth and the size of the network improves the performance. To further improve the model, a small amount of translation invariance could be added to the architecture by using a combination of convolutional and max-pooling layers. The weight sharing principle of the convolutional layer will significantly decrease the number of parameters, which makes it feasible to train larger networks using images with realistic sizes. The scale invariance is harder to achieve and it would require larger changes in the presented network architecture.

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