

# Adaptive Traffic Signal Control of Bottleneck Subzone based on Grey Qualitative Reinforcement Learning Algorithm

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**Keywords:** Grey Qualitative, Reinforcement Learning, Bottleneck Subzone Control, BP Neural Networks.

**Abstract:** A Grey Qualitative Reinforcement Learning algorithm is present in this paper to realize the adaptive signal control of bottleneck subzone, which is described as a nonlinear optimization problem. In order to handle the uncertainties in the traffic flow system, grey theory model and qualitative method were used to express the sensor data. In order to avoid deducing the function relationship of the traffic flow and the timing plan, grey reinforcement learning algorithm, which is the biggest innovation in this paper, was proposed to seek the solution. In order to enhance the generalization capability of the system and avoid the "curse of dimensionality" and improve the convergence speed, BP neural network was used to approximate the Q-function. We do three simulation experiments (calibrated with real data) using four evaluation indicators for contrast and analyze. Simulation results show that the proposed method can significantly improve the traffic situation of bottleneck subzone, and the algorithm has good robustness and low noise sensitivity.

## 1 INTRODUCTION

Road bottleneck or congestion, which is a special case of imported lanes of road are saturation or near saturation, is the performance of road traffic deterioration. In order to fully enhance the urban road network resource utilization efficiency and avoid bottlenecks or even traced queuing phenomenon of individual sections, the primary objective of regional traffic signal coordination control should be to maximize the number of vehicles leaving the subzone per unit time. Taking into account the conservation of subzone traffic flow, the control target is equivalent to minimize the average carrying vehicles of all bottlenecks in the specified time period.

Adaptive regional coordination control (TSC) has been a goal of intelligence traffic signal control researchers. In traffic signal control system, an signal controller can be seen as an intersection agent, and all signal controllers of controlled subzone can be seen as multi-agent cluster. A controller agent can be viewed as perceiving its environment (traffic flow) through sensors (traffic flow detectors) and acting upon that environment through effectors (traffic signal lights). Based on multi-agent

reinforcement learning (RL) technology, combined with grey system theory and neural network tool, we try to construct an effective traffic signal coordination control model for bottleneck subzone which has some saturated or nearly saturated sections, but also has some relatively smooth sections.

There are many kinds of artificial intelligence methods that have been used to implement adaptive traffic signal control and ease the traffic pressure. The adaptive traffic signal control techniques in Chun-gui (2009), Arel et al (2010) and Prashanth et al (2011) are based on the reinforcement learning and rely on the Q-learning algorithm with function approximation (Baird, 1995), State-Action, Reward-State Action (SARSA) (Loch and Singh, 1998) and the Policy Gradient Actor Critic algorithm (Sutton et al, 2000). Yujie et al (2011) have proposed a traffic signal controller based on three layered neural network to control the traffic lights in urban road traffic conditions. Shen and Kong (2009) used neural network (NN) with back propagation to implement fuzzy logic for devising a technique for traffic coordination control. Choy et al (2006) have also used hybrid system of neural network and fuzzy logic for designing a TSC and carried out simulations for comparing the working of SPSA-

NN, GLIDE and hybrid NN. Teo et al (2010) introduced genetic algorithm in their paper to optimise the traffic flow control. Choy et al (2006) also made use of GA to optimise the parameters of fuzzy controller used in their distributive multi-agent traffic signal controller. Ahmad et al (2014) first proposed an earliest deadline (EDF) based scheduling to reduce urban traffic congestion.

In this paper, the adaptive signal control of bottleneck subzone is described as a nonlinear optimization problem, and solved using a BP Neural Network based Grey Qualitative Reinforcement Learning algorithm (BP-GQRL), which can handle the uncertainty in traffic flow control system and alleviate traffic congestion spread. Grey qualitative theory has been successful in robot navigation applications and qualitative simulation applications (Shujie et al, 2011; Yuanliang et al, 2008; Chunlin et al, 2008; Yuanliang et al, 2004).

The remainder of the paper is organized as follows: Section 2 describes the mathematical model of the problem. Section 3 describes the proposed BP-GQRL algorithm in detail. The simulations are carried out in section 4 to verify the effectiveness and robustness of our method, and section 5 concludes our work.

## 2 PROBLEM DESCRIPTION

Any traffic signal control bottleneck subzone can be defined as a collection of sections of roads. A typical bottleneck subzone topology diagram is shown in Figure 1. Road carrying capacity is determined by the length of road and the effective lengths of the vehicles. When not considering the interaction between the subzones, we can say that the current bottleneck subzone is independent. Some sections with small traffic pressure, whose carrying capacities are considered to be  $+\infty$ , do not require monitoring their carrying capacities, and are called "ordinary sections". Accordingly, other sections are called "bottleneck sections". Our optimization goal is to minimize the average number of carrying vehicles of bottleneck sections and limit the number of carrying vehicles of each bottleneck section within an acceptable range.

Assuming the set of bottleneck sections of subzone is  $R = \{R_i | i = 1, 2, \dots, N\}$ ,  $N$  is the number of bottleneck sections. According to the traveling directions of flow, one bottleneck section  $R_i$  can be divided into two links: input link  $R_{in}^i = \{L_{in,j}^i | j = 1, 2, \dots, N_{in}^i\}$  and output link  $R_{out}^i = \{L_{out,k}^i | k =$

$1, 2, \dots, N_{out}^i\}$  (As shown in Figure 1, there are four input links and four output links).  $L_{in,j}^i$  is the inlet lane  $j$  of  $R_i$  and  $N_{in}^i$  is the number of inlet lanes of  $R_i$ .  $L_{out,k}^i$  is the outlet lane  $k$  of  $R_i$  and  $N_{out}^i$  is the number of outlet lanes of  $R_i$ . The vehicle flows in input link and output link are determined by the traffic signal control schemes of the upstream and downstream signal controllers. The changes of the carrying vehicles in the links are the direct reflection of the control effect.

Supposing that  $f(t, i)$  is the number of carrying vehicles of bottleneck section  $R_i$  at time  $t$ ,  $f_{in}(t, i, j)$  is the instantaneous passing vehicle number of inlet lane  $L_{in,k}^i$ , and  $f_{out}(t, i, j)$  is the instantaneous passing vehicle number of outlet lane  $L_{out,k}^i$ .  $f_{in}(t, i, j)$  and  $f_{out}(t, i, k)$  can be obtained from the traffic flow detectors laying at the inlet and outlet sections, respectively. The detectors may be coil detectors, video detectors, microwave detectors or any other types of detectors. According to road traffic conservation,  $f(t + \Delta t, i)$  can be calculated by the following equation (1):

$$f(t + \Delta t, i) = f(t, i) + \sum_{j=1}^{N_{in}^i} \int_t^{t+\Delta t} f_{in}(t, i, j) dt - \sum_{k=1}^{N_{out}^i} \int_t^{t+\Delta t} f_{out}(t, i, k) dt \quad (1)$$

The objective function of bottleneck control optimization is:

$$\min_{P(t)} \frac{1}{N} \sum_{i=1}^N STA(t, i) \quad (2)$$

$$STA(t, i) = \frac{f(t, i)}{U(i)}$$

$$s. t. FMIN_i \leq f(t, i) \leq FMAX_i, \quad \forall t, i;$$

where  $U(i)$  is the saturated flow of  $R_i$  (which is the maximum carrying capacity of the section, an inherent attribute of the road, and whose value can be selected by experience),  $STA(t, i)$  is the saturation of  $R_i$  (which reflects the traffic state of the section), and  $FMIN_i$  and  $FMAX_i$  are the wanted lower and upper limits of the number of carrying vehicles of  $R_i$ , respectively.  $P(t)$  is the dynamic traffic timing plan, namely the combination of the signal lights state and vehicles release time (including the transition time, such as the green flash time and the yellow light time) of each stage of each intersection in the bottleneck subzone.

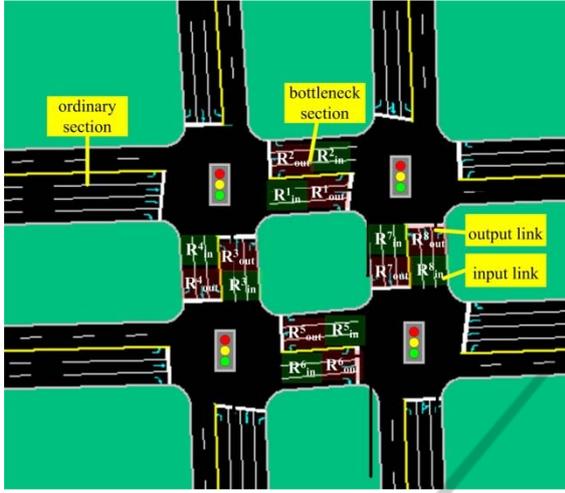


Figure 1: A typical bottleneck subzone topology diagram.

Suppose that the subzone is composed of  $M$  intersections. For any one intersection  $C_m$  ( $m = 1, 2, \dots, M$ ), supposing that the set of traffic signal stage (a combination of signal phases) is  $S_m = \{S_m(g) | g = 1, 2, \dots, B_m\}$ , then, at time  $t$ , the vehicles release time of stage  $S_m(i)$  is  $SG_m(i, t)$  (unit: seconds). The solution of the optimization problem is the optimal dynamic traffic timing plan. Based on the above assumptions, we can get:

$$P(t) = \{SG_m(g, t) | 1 \leq g \leq B_m, 1 \leq m \leq M\} \quad (3)$$

### 3 METHOD

In the optimization problem defined by equations (1) to (3), the function relationship of  $f_c(t, i)$  and  $P(t)$  is not explicitly expressed. Although there are a lot of "model-driven" methods that can deduce the function relationship between  $f_c(t, i)$  and  $P(t)$  approximately, here we do not intend to make this attempt based on the following reasons: (1) Numerous confounding factors, such as pedestrians and non-motorized vehicles etc, make many uncertainties in the traffic flow system, and we can only get the approximate function. (2) Many "data-driven" methods, such as reinforcement learning method, can solve above optimization problem without knowing the exact function relationship. In order to handle the uncertain error of the traffic flow detectors, grey theory model and qualitative method were used to express the sensor data. In order to avoid deducing the function relationship between traffic flow and timing plan, grey reinforcement learning algorithm was adopted to seek the solution. In order to enhance the generalization capability of the

system and avoid the "curse of dimensionality" and improve the convergence speed, BP neural network was used to approximate the Q-function.

### 3.1 Grey Qualitative Representation of Data

#### 3.1.1 Probability Grey Number

The grey system theory, which originated in the 1980s, mainly focuses on modeling a system using "small sample" information (Julong, 1985). Considering the uncertainty, information is usually represented by grey numbers, which are usually intervals. For example, given an observation value  $y$  of a system output, the grey system theory researchers prefer to represent it by a grey number  $\otimes = [y, \bar{y}]$ . It means that the true value of the output is in  $[y, \bar{y}]$ , but we do not know exactly which one it is. To obtain the exact value, a whitening weight function is usually defined on the grey number to indicate the observer's preference of the values in the interval.

In this paper, we use Probability Grey Number to describe the observation value, which is defined as follows:

Definition 1. Probability grey number  $\otimes_{[a,b]}$ , whose whitening weight function is the probability density function of the normal distribution  $N(\frac{a+b}{2}, \sigma^2)$ , is a special interval grey number defined in the real interval  $[a, b]$  ( $a < b$ ), where  $\sigma^2$  satisfies the following formula:

$$\int_{-\infty}^a \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-0.5(a+b))^2}{2\sigma^2}} dx = \frac{\alpha}{2} \quad (4)$$

$1 - \alpha$  is the measure of probability grey number, denoted as follow:

$$\mu(\otimes_{[a,b]}) = 1 - \alpha, \alpha \in [0, 1] \quad (5)$$

From the above definition, the distribution parameters of probability grey number is  $\sigma = \frac{b-a}{2Z_{\alpha/2}}$ ,  $Z_{\alpha/2}$  is the  $\alpha/2$  fractile of standard normal distribution  $N(0, 1)$ . If we know any two of the three elements (i.e. the interval  $[a, b]$ , measure  $1 - \alpha$  and distribution parameter  $\sigma^2$ ) in the probability grey number, and the third will be finalized.

Definition 2. The similarity between two probability grey numbers,  $\otimes_{[a,b]}$  and  $\otimes_{[c,d]}$ , is defined as follows (seeing shadow area in Figure 2 below):

$$\text{Sim}(\otimes_{[a,b]}, \otimes_{[c,d]}) = \mu(\otimes_{[p, +\infty)}) + \mu(\otimes_{(-\infty, p]}) \quad (6)$$

where  $p$  is the intersection of whitening weight functions of  $\otimes_{[a,b]}$  and  $\otimes_{[c,d]}$ .

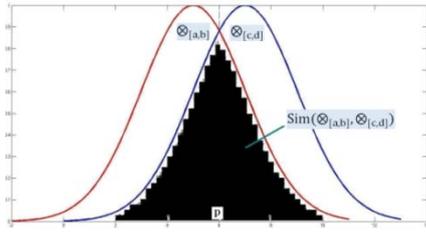


Figure 2: Definition of similarity between grey numbers  $\otimes_{[a,b]}$  and  $\otimes_{[c,d]}$

### 3.1.2 Grey Qualitative Description of the System Observations

As mentioned earlier, saturation  $STA(t, i)$  is the observation value of the traffic system, which reflects the road service level and the control target. Because of the uncertainty of measurement error, supposing that one observation value  $STA(t, i)$  corresponds to a probability grey number  $\otimes(t, i)$ . According to the following six principles based on traffic flow theory, the service level  $SL(t, i)$  of section  $R_i$  at time  $t$  can be qualitatively divided into six levels.

**Principle 1:** If  $STA(t, i) \in (-\infty, 0.35]$ , which means that the traffic is smooth and volume is less than 60% of road capacity, then  $SL(t, i) = 1$  and the corresponding grey number of this level is  $\tilde{\otimes}_1 = \otimes_{(-\infty, 0.35]}$ ,  $\mu(\tilde{\otimes}_1) = \beta_1(t)$ .

**Principle 2:** If  $STA(t, i) \in (0.35, 0.64]$ , which means that the traffic is steady with a slight delay, and volume is nearly 70% of road capacity, then  $SL(t, i) = 2$  and the corresponding grey number of this level is  $\tilde{\otimes}_2 = \otimes_{(0.35, 0.64]}$ ,  $\mu(\tilde{\otimes}_2) = \beta_2(t)$ .

**Principle 3:** If  $STA(t, i) \in (0.64, 0.77]$ , which means that the traffic is steady with some delay, and volume is nearly 80% of road capacity, then  $SL(t, i) = 3$  and the corresponding grey number of this level is  $\tilde{\otimes}_3 = \otimes_{(0.64, 0.77]}$ ,  $\mu(\tilde{\otimes}_3) = \beta_3(t)$ .

**Principle 4:** If  $STA(t, i) \in (0.77, 0.9]$ , which means that the traffic is not steady with tolerable delay, and volume is nearly 90% of road capacity, then  $SL(t, i) = 4$  and the corresponding grey number of this level is  $\tilde{\otimes}_4 = \otimes_{(0.77, 0.9]}$ ,  $\mu(\tilde{\otimes}_4) = \beta_4(t)$ .

**Principle 5:** If  $STA(t, i) \in (0.9, 1.00]$ , which means that the traffic is not steady with intolerable delay, and volume is close to the road capacity, then  $SL(t, i) = 5$  and the corresponding grey number of this level is  $\tilde{\otimes}_5 = \otimes_{(0.9, 1.00]}$ ,  $\mu(\tilde{\otimes}_5) = \beta_5(t)$ .

**Principle 6:** If  $STA(t, i) \in (1.00, +\infty)$ , which

means that the traffic is not steady with tolerable delay, and volume is nearly 90% of road capacity, then  $SL(t, i) = 6$  and the corresponding grey number of this level is  $\tilde{\otimes}_6 = \otimes_{(1.00, +\infty)}$ ,  $\mu(\tilde{\otimes}_6) = \beta_6(t)$ .

$\beta_i(t)$  is the parameter to be optimized, whose initialized value is selected according to the artificial experience or obtained through parameter learning from the historical detector data. With the operation of the system, the grey degree will gradually decrease, the value of  $\beta_i(t)$  will be updated with the change of the data.

## 3.2 System Decision

The output of the traffic signal optimization system described here is the optimized dynamic traffic signal timing plan  $SG_m(g, t)$ . The decision of agent is: at the start time of each stage  $g$  of each intersection  $m$ , increase or reduce the vehicle release time of current stage  $\Delta T$  seconds, or keep it unchanged. In general,  $T$  is set to 4, which reflects the adjustment step length of the green light time, and its value should not be too large. The decisions are executed by the traffic signal controllers in intersections.

## 3.3 BP-GRL Algorithm

The uncertainties of bottleneck subzone signal control system mainly come from the detectors, the environmental status and the feedbacks. For the subzone with  $N$  bottleneck sections and  $M$  intersections, the state set and action set can be expressed as follows:

$$\begin{aligned} GS &= (GS_1, GS_2, \dots, GS_N) \\ GA &= (GA_1, GA_2, \dots, GA_M) \\ GS_i &\in \{\tilde{\otimes}_1, \tilde{\otimes}_2, \tilde{\otimes}_3, \tilde{\otimes}_4, \tilde{\otimes}_5, \tilde{\otimes}_6\} \\ GA_j &\in \{+\Delta T, 0, -\Delta T\} \end{aligned} \quad (7)$$

where  $i = 1, 2, \dots, N$  and  $j = 1, 2, \dots, M$ .

At the start of time step  $t$  of each stage, agent sensing the external environment by the detectors and get the grey state  $GS(t)$ . Then, by using grey RL model, the agent will select an action  $GA(t)$  to execute. At the start of the time step  $t+1$  of next stage, the agent observes subsequent state  $GS(t+1)$ , and gets the the corresponding compensation according to the grey enhancement function  $GR_{(GS(t), GA(t))}$ . Combining equation (7), the grey enhancement function  $GR_{(GS(t), GA(t))}$ , which is used to reward the action which makes the grade of road service level improved and not less than four, is defined as follows:

$$GR_{(GS(t),GA(t))} = \frac{\sum_{i=1}^N SI(t+1) - \sum_{i=1}^N SI(t)}{\max(0.5, \max_{j=1,2,3,4} \text{Sim}(GS_i(t), \mathcal{G}_j))} \quad (8)$$

$$SI(t) = \frac{SL(t,i)}{SL(t,i)}$$

We use GQ values of grey Q learning method to describe the reward discounts and expectations. BP neural networks can be used to learn GQ values, and each bottleneck section corresponds to a neural network. The output of a neural network is the GQ values of each stage of the intersection, and the input of a neural network is the STA(t, i) values.

The traffic bottleneck area signal control algorithm based on grey reinforcement learning and neural networks is described below. For each BP neural network:

**Step0:** Initialize the starting timing plan, the agent state/action set and the neural network.

**Step1:** At the start of time step  $t$  of each stage of each intersection, detect the states  $\{STA(t, i)\}$  of all bottleneck sections and compute the state  $GS(t)$  and the output  $GQ_{(GS(t),GA(t),\omega(t))}$  of network.

**Step2:** According to "greedy exploration strategy" (i.e., choosing the actions which can maximize the GQ value), select the current decision  $GA(t)$  and execute it.

**Step3:** Observe the subsequent state set  $\{STA(t+1, i)\}$ , calculate the grey number  $GS(t+1)$ , and receive the timely feedback value  $r_t = GR_{(GS(t),GA(t))}$  according to the formula (8).

**Step4:** Update  $GQ_{(GS(t),GA(t))}$  based on formula (9) as follows:

$$GQ_{(GS(t),GA(t))} = (1 - \alpha)GQ_{(GS(t),GA(t),\omega(t))} + \alpha[r_t + \gamma \max_{GA'} GQ_{(GS(t+1),GA(t'),\omega(t))}] \quad (9)$$

**Step5:** Update the weight  $\omega(t)$  of neural network as  $\omega(t+1)$  using the error signal  $e(t)$  defined in formula (10), so that the actual output of the neural network can approximate the desired output  $GQ_{(GS(t),GA(t))}$ .

$$e(t) = GQ_{(GS(t),GA(t))} - GQ_{(GS(t),GA(t),\omega(t))} \quad (10)$$

**Step6:** Go to the start time step  $t+1$  of next stage, and repeat **Step1~Step6**.

## 4 SIMULATION AND ANALYSIS

### 4.1 Experimental Method

In order to verify the effectiveness of the proposed method, we use the microscopic traffic simulation commercial software VISSIM to simulate the

control effect of a bottleneck subzone in the city of Lianyungang of China. By collecting the traffic flow and road topology information of the subzone, we calibrate a simulation intersection with real data. The region topology diagram is shown in Figure 3. There are 9 intersections and 10 bottleneck sections in the subzone, and local traffic in the periods of morning and evening peaks is congested.

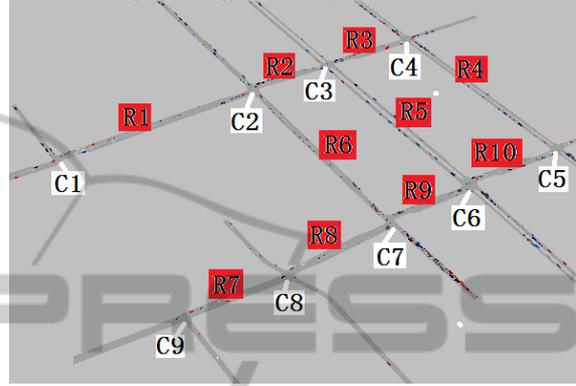


Figure 3: The subzone topology diagram.

Currently, the traffic signal timing plans in the region has been optimized by the professional engineers with more than five years of work experience. The present signal controllers implement the "multi-period timing control" mode, whose plans are fixed without any dynamic adjustment for each period. Commissioned by the traffic management departments, we need to assess the timing plans in this subzone, to determine whether it is necessary to change the timing plan or perform adaptive control scheme.

In order to verify the robustness of BP-GRL model in a situation where the detected data is not accurate or with noise, the Gaussian white noise are added to the output of each VISSIM detector, whose mean is zero and variance is four.

The three experiments that can be used to contrast and analyze include: (a) the experiment with the existing fixed timing plans optimized by professional engineers, which is recorded as FIX method; (2) the experiment with proposed BP-GRL method based on Gaussian white noise data, which is recorded as G-BP-GRL method; (3) the experiment with proposed BP-GRL method without Gaussian white noise, which is recorded as BP-GRL method. The four evaluation indicators that can be used to evaluate the results include: (1) delay, (2) parking time, (3) the number of stops and (4) the number of passing vehicles.

## 4.2 Results and Analysis

We conducted 40,000 seconds simulation three times, corresponding to the FIX experiment, G-BP-GRL experiment and BP-GRL experiment. One simulation second is equal to five simulation steps. The sums of the evaluation indicator values are shown in Figure 4 and the numerical trends are shown in Figure 5 for comparisons.

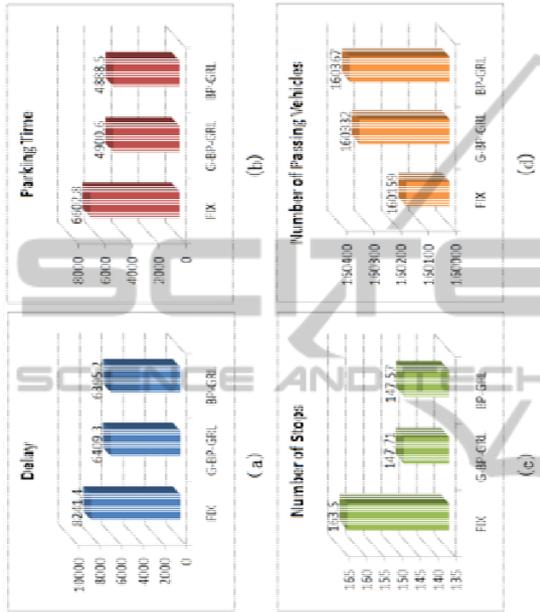


Figure 4: The statistical results of the sums of the evaluation indicator values.

Through data analysis, we find that, compared with FIX, the delays, the parking time, the number of stops and the number of passing vehicles of G-BP-GRL are reduced by 18.05%, 22.1%, 2.07% and -337 pcu (per car unit) respectively, while BP-GRL are reduced 18.57%, 22.41%, 2.03% and -618 pcu respectively. Thus, regardless of whether the data have a certain detector noise or not, the results of BP-GRL algorithm are much better than the FIX method, and the improvement is obvious. Because we use grey qualitative approach to express the observation values, the system has good robustness and low sensitivity to data noise.

By the analysis, we know that the existing fix timing plans have large room for improvement, and recommend that traffic managers should set up the addition of traffic flow detectors and implement the adaptive control strategy to achieve better control effect.

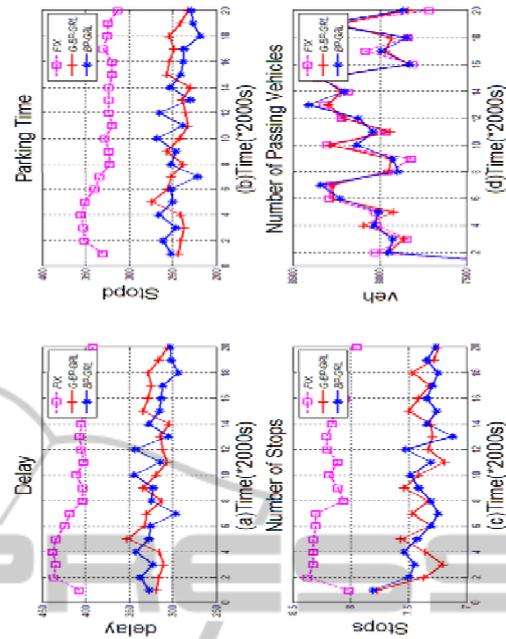


Figure 5: The numerical trends of the valuation indicators.

## 5 CONCLUSIONS

Essentially, the bottleneck is the inevitable result of the growing traffic demand. Excessive traffic demand makes road traffic congestion arising from lack of capacity for queue. A Grey Qualitative Reinforcement Learning algorithm is present in this paper to realize the adaptive signal control of bottleneck subzone, which is described as a nonlinear optimization problem. Firstly, we use grey theory model and qualitative method, which can handle the uncertainties in the traffic flow system, to express the sensor data. Secondly, BP Neural Network based Grey Reinforcement Learning (BP-GRL) algorithm, which is the biggest innovation in this paper, was adopted to seek the solution. So we do not need to deduce the function relationship of traffic flow and timing plan. Finally, we do three experiments using four evaluation indicators for contrast and analyze. Simulation results show that the proposed method can significantly improve the traffic situation of bottleneck subzone, and the method has good robustness and low noise sensitivity. Combining with pedestrian and non-motorized traffic data, the model will be further extended in the future.

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