Medial Width of Polygonal and Circular Figures
Approach via Line Segment Voronoi Diagram

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Abstract: The paper proposes the concept of building the so-called medial width function - integral shape descriptor of figures used in image recognition tasks. Medial width function is determined based on the skeleton of the shape and the radial function. An algorithm to compute the medial width function for polygonal figures based on the line segment Voronoi diagram is also presented here. Generalized solution to the circular figures obtained by rounding corners in a polygonal figure is presented. Computational experiment demonstrates the efficiency and effectiveness of the approach to the problem of palm shapes comparing for personal identification.

1 INTRODUCTION

Features generation for classification of objects of variable shape, such as a human figure or an animal is to build shape descriptors which remain invariant during object deformation.

A useful tool for object shape classification is a skeleton or medial axis of the figure. Skeleton of figure is defined as the set of points-centers of the circles inscribed in the figure. Skeleton looks like flat geometric graph and analysis of this graph gives the ability to generate multiple topological and metric features of the object's shape.

Another source for shape features generating is the width of the object with respect to medial axis. Width of the object is described by the radial function, which establishes a correspondence between the points of the skeleton and the radii of the inscribed circles with centers at these points. Medial axis and radial function together form a medial representation of figures (Siddiqi, 2008).

Radial function gives a local description of the width of the figures at the points of the skeleton. This width is tied to the skeleton and allows us to compare objects that have isomorphic skeletons. To use these widths for the classification of objects having different skeletal structure, we need to construct an integral descriptor of the object width. As an example of such descriptor pattern spectrum (Maragos, 1989) with the disk structuring element can be used. Calculation of the pattern spectrum is based on the operations of mathematical morphology that are associated with the transformation of discrete raster images. The limitation of these methods is the high computational complexity. For example, in the problem of biometric identification by hand geometry pattern spectrum shows good results (Ramirez-cortes, 2008). But because of the slow work allows you to work only with images of small size with low resolution. The time consumed to compute the pattern spectrum precludes its use in processing of video sequences and the analysis of complex high-resolution images.

We propose an alternative approach that can significantly reduce the time required for the calculation of the integral shape descriptors based on the width of objects. A new descriptor, called the *medial width*, the calculation of which is performed by means of efficient algorithms of computational geometry is proposed. The approach is based on the following principles.

1. Introduce the concept of the medial width of figure at a point on the basis of the medial representation.
2. In the figure, define the region of given width as the set of points at which the medial width does not exceed a predetermined value.
3. Define the medial width function of figure describing the area ofthe region of given width as a function of the width parameter.

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The paper proposes a method for direct calculation of the medial width function of polygonal figure (polygon with polygonal holes). The method is based on Voronoi diagram of line segments formed by the boundary of a polygonal figure. On the one hand, efficient algorithms for constructing such Voronoi diagrams are known (Held, 2011, Karavelas, 2004 , Mestetskiy, 2013). On the other hand, figures that have non-linear boundary, as well as figures from the bitmaps can be easily approximated by polygonal figures.

For a more adequate approximation of such shapes, we introduced the circular figures. Circular figure is obtained by the process of skeleton pruning, leading to "rounding" corners of the polygonal figure by circular arcs. The proposed method of direct calculation of the medial width function for polygonal figures allows us to calculate the same ones for circular figures as well.

Implementation and experimental evaluation of the proposed approach is made with respect to the problem of personal identification through the hand geometry. We compute the medial width function for circular figures approximating the shape of palm in bitmap. Later we construct a measure of difference of palm shapes based on the comparison of these functions.

2 MEDIAL REPRESENTATION AND MEDIAL WIDTH OF FIGURES

We consider medial representation of bound closed regions in Euclidean plane and name them figures. The skeleton of figure is a locus of centers of maximum empty circles in this region. The circle is considered to be *empty* if all its internal points are internal points of the region and the *maximum empty circle* which is not contained in any other empty circle is called *inscribed circle*. Radial function is defined in a point of skeleton and is equal to the radius of inscribed circle centered at this point.

Definition 1. A *spoke* is a line segment from the skeleton point to any nearest boundary point.

Spokes have important properties (Mestetskiy, 2014):

1) Each point in figure belongs to at least one spoke, hence spokes cover the entire figure.
2) If the point of figure does not belong to the skeleton, then it is incident on one spoke only.

Definition 2. *Medial width* of figure in an internal point is equal to the length of its incidence spoke.

All spokes of a point of skeleton have the same length. Therefore medial width for points of skeleton is equal to the radial function. The incidence spoke of the non-skeletal internal point is unique. Hence medial width in this point is well defined too.

Boundary points of the figure may have several incident spokes of different length. But the total area of the boundary is 0. Consequently, these points do not contribute to the area calculation of the region of given width. Therefore, medial width at the boundary points can be set arbitrarily, for example, put it equal to zero.

We will use the following notation:

R^2 – Euclidean plane,

G – figure, bound closed region $G \subset R^2$,

∂G – boundary of figure G,

G' – internal open region of figure G, $G' = G \setminus \partial G$,

$C(P)$ – inscribed circle centered in the point $P \in G$,

S – skeleton of figure G.

We denote $\tau(g)$, $g \in G'$ – medial width of the figure at the points g, $G'_z = \{g \in G', \tau(g) \le z\}$ – the region of given width $z \ge 0$.

Definition 3. *Medial width function* $\mathcal{F}(z)$ of figure G is an area of given width $z \ge 0$ $\mathcal{F}(z) = \mu(G'_z)$.

3 POLYGONAL AND CIRCULAR FIGURES AND THEIR MEDIAL WIDTH

Polygonal figure is closed bounded region with boundary consisting of polygons. Polygonal figures can be used as convenient continuous models for approximating binary bitmap objects.

The boundary of the polygonal figure can be represented as the set of *point-sites* (vertices of a figure) and *segment-sites* (sides of boundary polygons). Voronoi diagram (VD) of line segments is defined for these set of sites. The part of this VD, lying inside the figure is termed as VD of polygonal figure, which is a geometric graph whose edges are of straight line segments and quadratic parabola segments.

Let G is a polygonal figure, $Vor(G) = \langle V, E \rangle$ is VD of figure G. Here V – the set of vertices, E – set of VD edges. Each edge of E is associated with a pair of sites to which this edge is the bisector – the common boundary of their Voronoi cells. Consider the VD subgraph $\langle V', E' \rangle$, formed from $Vor(G)$ by cutting of some terminal vertices and edges incident to these vertices. If cut vertices and edges of $Vor(G)$

which incident to concave vertices of polygonal figure, then the union of the edges of the VD subgraph $\langle V', E' \rangle$ is the skeleton of figure, i.e. $S = \langle V', E' \rangle$, $V' \subseteq V$, $E' \subseteq E$. Therefore, the skeleton of a polygonal figure can be considered as a subgraph of VD $S = \langle V', E' \rangle, V' \subseteq V, E' \subseteq E$.

Let S be a polygonal skeleton of G. Pruning, a process of sequential cutting of some terminal vertices and their incident edges helps in the construction of subgraphs $S_1, S_2, \dots, S_n$ such that $S_i = \langle V_i, E_i \rangle$, $S_{i+1} = \langle V_{i+1}, E_{i+1} \rangle$, $V_{i+1} = V_i \setminus \{v_i\}$, $E_{i+1} = E_i \setminus \{e_i\}$, $v_i \in V_i$, $e_i \in E_i$, and the vertex v_i is terminal in the subgraph S_i, and e_i is its incident edge.

Definition 4. Subgraphs of VD resulting from pruning process are called *skeletal subgraphs*.

Definition 5. Union $G' = \bigcup_{P \in S'} C(P)$ of inscribed circles centered on skeletal subgraph $S \subseteq S'$, is called a *circular figure*.

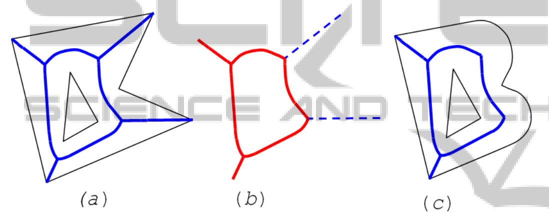

Figure 1: (a) the polygonal figure and the skeleton, (b) the skeleton subgraph resulting from pruning, (c) the circular figure corresponding skeleton subgraph.

Polygonal figure G can be represented as the union of all the inscribed circles which are centered at the skeleton points $G = \bigcup_{P \in S} C(P)$, i.e., it is a particular case of circular figure. The example in Fig.1 presents a polygonal figure, its skeleton, skeletal subgraph, and circular figure formed by circles of this subgraph.

4 BICIRCLES IN THE POLYGONAL AND CIRCULAR FIGURES

An edge $e \in E$ of the skeletal subgraph is a segment of a straight line or a parabola. This segment has two endpoints at the graph vertices. The remaining points of the edge will be named as internal.

Definition 6. A *bicircle* of the edge $e \in E$ is the union of all inscribed circles centered on e. The edge is called axes of bicircle.

Definition 7. *Proper region* of bicircle of edge e is the closure of the union of all the spokes incident to an interior point of e.

Proper region of bicircle is included to bicircle. Boundary of proper region includes two spokes. Two circles with centers at the edge endpoints are *end circles* of bicircle. Couple spokes divides end circle into two sectors – external and internal. External sector includes a part of the border of bicircle whereas the internal sector comprises the remainder of the end circle (Fig.2).

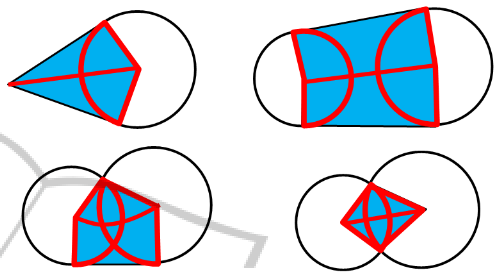

Figure 2: Bicircles, proper regions, internal and external sectors of end circles.

Let B^e – a proper region of the bicircle B^e of edge e.

Definition 8. The subset $B_z^e \subseteq B^e$ of bicircle $B_z^e = \{g \in B^e,\ \tau(g) \leq z\}$, in which the medial width does not exceed $z \geq 0$, be called the *region of width* z.

Denote $\mu(B_z^e)$ – area of B_z^e.

Definition 9. *Medial width function* of bicircle B^e is $\mathcal{F}^e(z) = \mu(B_z^e)$.

Proper regions of two bicircles may have intersection over the boundary spokes only. The area of this intersection is zero. On the other hand proper regions of bicircles cover the entire polygonal figure completely. Therefore, the medial width function of the polygonal figure is equal to the sum of the medial width functions of bicircles

$$\mathcal{F}(z) = \sum_{e \in E} \mathcal{F}^e(z) \tag{1}$$

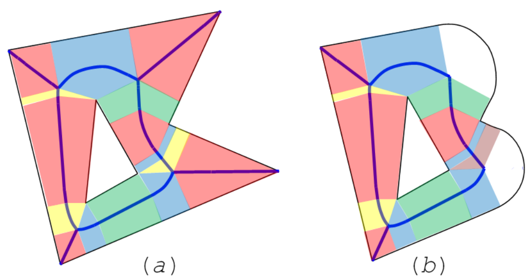

Figure 3: (a) Coverage of the polygonal figure by proper regions of bicircles, (b) coverage of the circular figure by proper regions and border sectors.

The circular figure is the union of all bicircles of its skeletal graph. But proper regions of these bicircles do not cover the entire circular figure. Therefore, the remaining portion of the circular figure is covered by the external sectors of bicircles (Fig. 3).

Definition 10. In the circular figure the part of the inscribed circle $C(v)$ centered in the skeleton vertex $v \in V$, which is not covered by proper regions of bicircles, is called the *border sector*.

The inscribed circle $C(v)$ exists for every skeleton vertex $v \in V$. Let $\theta(v)$ is the area of the border sector of $C(v)$. Denote $V_z \subseteq V$ the set of vertices of the skeleton, which radii of the inscribed circles $r(v) \leq z$.

Then the medial width function of the circular figure is

$$\mathcal{F}(z) = \sum_{e \in E} \mathcal{F}^e(z) + \sum_{v \in V_z} \theta(v). \qquad (2)$$

The first term is the area of proper regions of bicircles, and the second term is the area of the external sectors of vertices.

5 MEDIAL WIDTH OF BICIRCLES

Each edge of the skeleton has two site generators. Couple sites "point-segment" forms a parabolic edge and the corresponding bicircle is said to be *parabolic*. Couples "point-point" and "segment-segment" form linear edges. In these cases graphs of the dependence of the inscribed circle radius with the position of the circle center on the edge are a straight line (for a pair "segment-segment") or hyperbola (for a pair "point-point"). For convenience, corresponding bicircle are said to be *linear* and *hyperbolic*.

We wish to obtain an explicit formula for the calculation of the medial width functions: $\mathcal{F}_{lin}(z)$ – for linear bicircle, $\mathcal{F}_{par}(z)$ – for parabolic bicircle, $\mathcal{F}_{hyp}(z)$ – for hyperbolic bicircle as a function of the width parameter z.

The formulas for calculating these functions are provided below. Detailed formation of these formulas performed on the basis of the geometric analysis and was described in (Mestetskiy, 2014).

Denote

r, R – radii of bicircle's end circles, $r \leq R$,

l – distance between end circle centers,

$t = \sqrt{l^2 - (R-r)^2}$ – length of the bicircle axis projection on the segment-site in linear and parabolic bicircles.

5.1 Medial Width of a Linear Bicircle

Medial width function of linear bicircle can be computed as

$$\mathcal{F}_{lin}(z) = \begin{cases} 0 & \text{if } z < r \\ az^2 + b & \text{if } r \leq z \leq R \\ t(R+r) & if\ z > R \end{cases}$$

where

$$a = \begin{cases} 0 & \text{if } r = R \\ \dfrac{t}{R-r} & \text{if } r < R \end{cases}$$

$$b = \begin{cases} 2lr & \text{if } r = R \\ -\frac{tr^2}{R-r} & \text{if } r < R \end{cases}.$$

5.2 Medial Width of a Parabolic Bicircle

Parabolic bicircle axis is a segment of a parabola. To calculate the medial width of the bicircle, it is necessary to determine the position of the vertex of the parabola with respect to the axis of the bicircle. Position of the vertex of the parabola defined by the parameters of the parabolic bicycle.

Let where $t^* = 2\sqrt{r(R-r)}$.

The variants of the parabola vertices are:

(a) if $t = t^*$ then the vertex of the parabola is the endpoint of axis,

(b) if $t > t^*$ then the vertex of the parabola is an interior point of the axis,

(c) if $t < t^*$ then the vertex of the parabola lies outside the axis.

Definition 11. Parabolic bicircle having vertex of the parabola coinciding with the endpoint of axis is called as *root parabolic bicircle*.

Position of the parabola vertex is defined by the relation: at $t = t^*$ option (a), $t > t^*$ option (b), $t < t^*$ option (c), where $t^* = 2\sqrt{r(R-r)}$.

Parabola parameter for parabolic bicircle is

$$p = \frac{t^2}{2l^2}\left(R + r + \sqrt{(R+r)^2 - l^2}\right).$$

Area of proper region of root parabolic bicircle with parameter p and the end circle radius z is

$$\varphi(z) = (z+p)\sqrt{\frac{p}{2}\left(z - \frac{p}{2}\right)}.$$

Medial width function of root parabolic bicircle with parameter p and end circle radius R is

$$\Phi(z,p,R) = \begin{cases} 0 & \text{if } z \le \frac{p}{2} \\ \varphi(z) & \text{if } \frac{p}{2} < z \le R \cdot \\ \varphi(R) & \text{if } z > R \end{cases}$$

Now the medial width function of the parabolic bicircle can be calculated through areas of 2 root bicircles:

if the vertex of the parabola lies on the axis, then

$$\mathcal{F}_{par}(z) = \Phi(z,p,r) + \Phi(z,p,R)$$

if the vertex of the parabola lies outside the axis, then

$$\mathcal{F}_{par}(z) = \Phi(z,p,R) - \Phi(z,p,r)$$

5.3 Medial Width of a Hyperbolic Bicircle

Definition 12. Midpoint of the segment connecting the point-sites of hyperbolic bicircle, called the *center* of hyperbolic bicircle.

The position of the center relative to the axis of the hyperbolic bicircle is also important for calculating of the medial width function. Depending on the values of r, R, l the center lies on the axis of the bicircle, or outside the axis.

(a) if $l^2 + r^2 = R^2$ then the center coincides with the endpoint of axis,

(b) if $l^2 + r^2 > R^2$ then the center is an interior point of the axis,

(c) if $l^2 + r^2 < R^2$ then the center lies outside the axis.

Let q is the distance between point-sites of hyperbolic bicircle. We name it as the parameter of hyperbolic bicircle.

The parameter is calculated by the formula

$$q = \frac{1}{l}\sqrt{[(l+r)^2 - R^2]\cdot[R^2 - (l-r)^2]}\ .$$

Definition 13. Hyperbolic bicircle is called as the *root bicircle*, if the center of the bicircle coincides with the endpoint of axis.

Area of proper region of root hyperbolic bicircle with parameter q and the end circle radius z is

$$\psi(z) = \frac{q}{2}\sqrt{z^2 - \frac{q^2}{4}}\ .$$

Medial width function of root hyperbolic bicircle with parameter q and end circle radius R is

$$\Psi(z,p,R) = \begin{cases} 0 & \text{if } z \le \frac{q}{2} \\ \psi(z) & \text{if } \frac{q}{2} < z \le R \cdot \\ \psi(R) & \text{if } z > R \end{cases}$$

Medial width function of the hyperbolic bicircle can now be calculated through areas of 2 root hyperbolic bicircles:

if the center of the bicircle lies on the axis

$$\mathcal{F}_{hyp}(z) = \Psi(z,q,r) + \Psi(z,q,R)$$

if the center of the bicircle lies outside the axis

$$\mathcal{F}_{hyp}(z) = \Psi(z,q,R) - \Psi(z,q,r)$$

5.4 Medial Width of End Sectors

To evaluate the medial width function of the circular figure, we must calculate the areas of border sectors of vertices, which are not covered by bicircle proper regions.

Suppose that a skeleton vertex $v \in V$, has incident edges $e_1, e_2, \ldots, e_k$, $k \ge 1$ and bicircles of these edges have a common end circle centered at v.

The border sector is the intersection of external sectors of all incident bicircle, whereas the internal sectors in these bicircles do not overlap. Therefore, if the angular size of the internal sectors are $\alpha_1, \alpha_2, \ldots, \alpha_k$, then their sum does not exceed 2π, i.e., $\alpha_1 + \alpha_2 + \cdots + \alpha_k \le 2\pi$.

If a vertex v preserved all incident edges after pruning, then $\alpha_1 + \alpha_2 + \cdots + \alpha_k = 2\pi$. But, if some edges have been removed during the pruning, then $\alpha_1 + \alpha_2 + \cdots + \alpha_k < 2\pi$. Thus, the angular size of the border sector of the vertex v is

$$\xi(v) = 2\pi - (\alpha_1 + \alpha_2 + \cdots + \alpha_k).$$

If r_v is the radius of the inscribed circle $C(v)$ centered at vertex v, then the area of the border sector is

$$\theta(v) = \frac{1}{2}\xi(v)\cdot(r_v)^2.$$

Thus, to calculate the area of border sectors $\xi(v)$ for all vertices $v \in V$, there is a need to find the angular size of all internal sectors of bicircles. These sizes are calculated depending on the type of bicircle (linear, parabolic, or hyperbolic).

In linear bicircle the size of internal arc of small end circle is $\beta = \pi + 2\cdot arcsin\frac{R-r}{l}$, and of large end circle is $= \pi - 2\cdot arcsin\frac{R-r}{l}$.

Internal arc of a large end circle of the parabolic bicircle with the parameter p is

$$\alpha = arccos\left(1 - \frac{p}{R}\right).$$

An internal arc of a small circle is $\beta = arccos\left(1 - \frac{p}{r}\right)$ in the case where the parabola vertex lies on the axis of the bicircle, and $\beta = 2\pi - arccos\left(1 - \frac{p}{r}\right)$, if it lies outside axis.

Internal arc of a large end circle of the hyperbolic bicircle with the parameter q has a size $\alpha = arcsin\left(\frac{q}{2R}\right)$. If the center lies on the axis of the bicircle then the internal arc of the small circle has the size $\beta = arcsin\left(\frac{q}{2r}\right)$, and if it lies outside the axis, $\beta = 2\pi - arcsin\left(\frac{q}{2r}\right)$.

Formulas (1), (2) allow us to calculate the value of the medial width function $\mathcal{F}(x)$ for a fixed value of the argument x. As can be seen from the formulas obtained, the calculation of the medial width function of one bicycle $\mathcal{F}^e(x)$ is $O(1)$. Hence, the computational complexity for the sum $\sum_{e \in E} \mathcal{F}^e(x)$ is $O(|E|)$, where $|E|$ - is the number of edges in skeletal graph of figure. Calculation of the areas of border sectors $\theta(v)$ for all vertices $v \in V$ is carried in a single pass over the edges of the skeletal graph, i.e. has the complexity $O(|E|)$. Calculating the sum $\sum_{v \in V_x} \theta(v)$ adds to this $O(|V|)$. Thus, the calculation of $\mathcal{F}(x)$ has complexity $O(|E| + |V|)$. Since skeletal graph is planar, single complexity of computing $\mathcal{F}(x)$ can be written as $O(m)$, where m - is the number of vertices in the skeletal graph.

For feature generation it is necessary to calculate the medial width function for the argument $x = r_0, r_1, \dots, r_N$, where N - the dimension of the feature vector. Thus, the total computational complexity of constructing the feature vector based on the medial width function will be $O(mN)$ in the worst case.

6 APLICATION TO PALM SHAPE COMPARING

Our example is intended to demonstrate the utility of the medial width function and effectiveness of the method of its calculation. We consider an application for biometric identification by hand geometry. The task is to construct a measure of distinction palm shapes, presented in the form of binary images. We use our method of circular approximation and constructing a continuous skeleton of a binary bitmap image (Mestetskiy, 2008). It contains the following steps.

1. We model binary bitmap as an integer lattice points in the plane. First, we construct a polygonal figure approximating a binary raster image. The boundary of figure consists of separating polygons of the minimum perimeter.

2. Construct the VD of line segments formed from approximating polygonal figure boundaries. Extract the internal part of the VD of the figure.

3. To obtain an approximating circular figure, pruning of internal Voronoi diagram is performed.

4. Calculate the medial width function of circular figure using the algorithm discussed in this article.

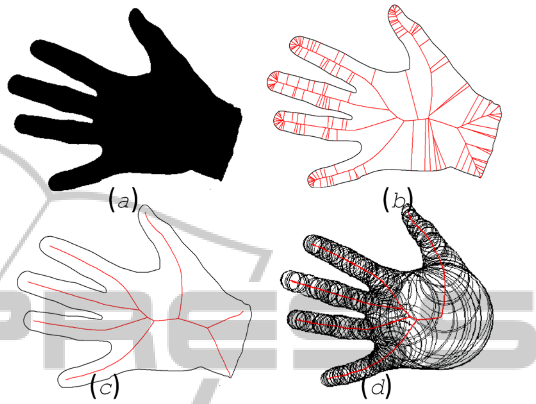

Figure 4: (a) binary raster image, (b) approximating polygonal figure and continuous skeleton, (c) skeleton after pruning, (d) medial representation of image.

Fig.4 illustrates an example for the described scheme. Original binary image is the 640×480 bitmap (Fig.4*a*). The resulting polygonal figure is the simple polygon with 346 vertices. Skeletal graph (VD) has 689 edges (Fig. 4*b*). Simple pruning (regularization by parameter 1) leaves 435 edges in the skeletal graph (Fig. 4*c*).

The resulting sub-graph generates circular figure that approximates the original bitmap image with accuracy ε in the Hausdorff metric. In our example, ε = 1. Further semantic segmentation leaves in the skeletal graph only significant part which describes a hand (removes wrist).

The result is a graph with the edges 382. This graph gives a circular shape consisting of 382 bicircles (Fig. 4*d*), among them are 182 linear, 152 parabolic and 48 hyperbolic bicircles.

We consider three measures of palm differences based on different features: line of hand geometric points, the curvature of the fingers and the medial width.

Line of hand geometric points is a polyline whose vertices are the singular points on the boundary contour of the palm: 5 tips and 4 valleys points (Fig.5). The method of allocation of these points in the image is described in (Mestetskiy, 2011).

Let $T_1, T_2, \cdots, T_9$ – sequential vertices of the polyline, and $\gamma_i = |T_i T_{i+1}|, i = 1, \dots, 8$ – the length of

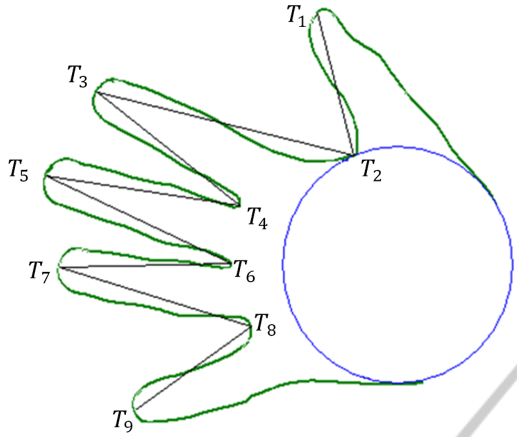

Figure 5: Line of hand geometric points.

segments, $\gamma=\sum_{i=1}^{8}\gamma_i$ – total length. Feature vector is defined as $\Gamma=\left(\frac{\gamma_1}{\gamma},\frac{\gamma_2}{\gamma},\ldots,\frac{\gamma_8}{\gamma}\right)$. Measure of the difference of two palms $\Gamma^{(1)}$ and $\Gamma^{(2)}$ is calculated as the Euclidean distance $\mu_1(\Gamma^{(1)},\Gamma^{(2)})$ between these vectors.

Measure the curvature of the fingers is constructed as follows (Fig. 6). For each finger, $i=1,\ldots,5$ in the continuous skeleton find centers of the inscribed circles: the tip B_i and a base A_i. The method of obtaining these points is described in (Mestetskiy, 2011). Then, on a skeleton branch A_iB_i find most distant points from straight line A_iB_i to the right (point R_i) and to the left (point L_i). Let δ_i – distance from R_i to A_iB_i, ε_i – distance from L_i to A_iB_i, and $\eta_i=|A_iB_i|$ – segment length. Feature vector $\Delta=\left(\frac{\delta_1}{\eta_1},\frac{\varepsilon_1}{\eta_1},\frac{\delta_2}{\eta_2},\frac{\varepsilon_2}{\eta_2},\ldots,\frac{\delta_5}{\eta_5},\frac{\varepsilon_5}{\eta_5}\right)$ is a vector of curvature of fingers. Measure differences of palms $\Delta^{(1)}$ and $\Delta^{(2)}$ is calculated as the Euclidean distance $\mu_2(\Delta^{(1)},\Delta^{(2)})$.

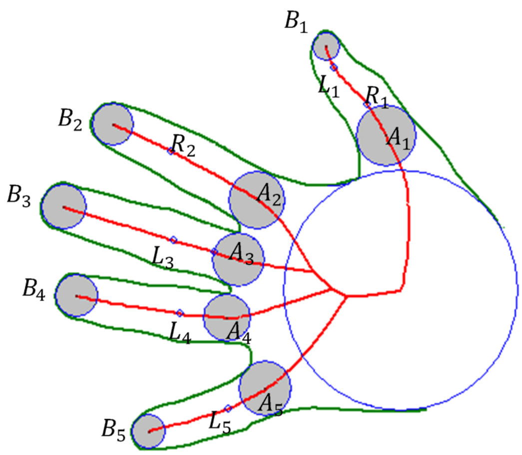

Figure 6: Curvature of the fingers.

Feature vector of palm width is calculated using the normalized of the medial width function of the circular figure. Normalization is necessary for compare images of different sizes, obtained under various shooting conditions. Let the radius of the maximum inscribed circle of palm image is R_{max}. Scale the virtual circular figure so that the radius of its maximum inscribed circle became R_{norm}. For this set of scaling coefficient $\lambda=R_{norm}/R_{max}$. Then we obtain the normalized function of the medial width$\mathcal{F}_{norm}(z)=\frac{\mathcal{F}(\lambda z)}{\lambda^2}$. In our experiments we used $R_{norm}=100$. Feature vector describing the medial width of the palm has the form $\Omega=(\omega_0,\omega_2,\ldots,\omega_m)$, $m=R_{norm}$, $\omega_k=\mathcal{F}_{norm}(k)$, $k=0,1,\ldots,m$. The difference of palms $\Omega^{(1)}$ and $\Omega^{(2)}$ on the medial width is calculated as the Euclidean distance $\mu_3(\Omega^{(1)},\Omega^{(2)})$.

General measure of differences for pairs of images of palms $I^{(1)}=\left(\Gamma^{(1)},\Delta^{(1)},\Omega^{(1)}\right)$ and $I^{(2)}=\left(\Gamma^{(2)},\Delta^{(2)},\Omega^{(2)}\right)$, combining all three measures, is

$$\mu(I^{(1)},I^{(2)})=C_1\mu_1(\Gamma^{(1)},\Gamma^{(2)})+$$
$$+C_2\mu_2(\Delta^{(1)},\Delta^{(2)})+C_3\mu_3(\Omega^{(1)},\Omega^{(2)}).$$

To prove the efficacy of proposed approach, we conducted the experiments including 160 binary 640 × 480 images of palms of 35 people, 4-5 samples for each person. Based on a comparison of the distances between samples with a threshold occurs classification of a pair as their "own" or "alien". The threshold is set so that rates FAR and FRR are equal. The value obtained Equal Error Rate (EER) is considered by us as a quality criterion for the construction of the metric. The values of the coefficients C_1, C_2, C_3 are obtained by minimizing the EER.

The table 1 shows the EER values for different formations of $\mu(I^{(1)},I^{(2)})$ by combining measures Hand Geometric Points (HGP), Finger Curvature (FC), Palm Width (PW).

Table 1: Efficiency of the medial width for measuring the of palm shape similarities.

Measure		**EER**
FC	$(C_1=C_3=0)$	15.9%
PW	$(C_1=C_2=0)$	11.8%
HGP	$(C_2=C_3=0)$	8.5%
HGP & FC	$(C_3=0)$	7.7%
FC & **PW**	$(C_1=0)$	6.7%
HGP & **PW**	$(C_2=0)$	5.1%
HGP & FC & **PW**		4.0%

The experiment shows that the medial width (PW) substantially improves the classification level in comparison with features based on the use of only the boundary (HGP) and skeleton (FC).

The table 2 shows the computation time (in millisecond) for the processor Intel® Core™ i5-3210M CPU @ 2.50GHz. Operation "Calculation of the medial width function" includes the construction of approximating polygonal figure, the calculation of the medial representation, regularization of the skeleton, as well as a direct computation of three measures based on medial representation.

Table 2: Expenses of time for the palm medial width calculating.

Operation	Amount	Time spent	Time per step
Medial width function	160 images	2325 ms	14.53 ms
Comparisons	12720	3200 ms	0.25 ms

High computational efficiency of our approach enables the use of the medial width for image recognition in real-time computer vision systems.

7 CONCLUSION

The proposed method opens up new possibilities for the application of high-performance computational geometry algorithms in the analysis and recognition of discrete raster images. Known approaches to the calculation of descriptors for the width of the figures on the basis of pattern spectrum is not suitable for use in real-time computer vision systems, as they have high computational complexity. The proposed transition to a continuous model based on polygonal and circular figures, as well as a highly effective method of calculating the medial width function for these figures allow us to overcome this short coming.

Medial width is a universal feature, it does not include structural analysis of shapes, therefore, its use requires a combination with other features, such as the image skeleton. Future work should build such combined classification methods.

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