

# Discrete Optimal View Path Planning

Sebastian Haner and Anders Heyden

Centre for Mathematical Sciences, Lund University, Lund, Sweden

**Keywords:** Path Planning, Next Best View Planning, Active Vision, Discrete Optimization, Semidefinite Programming, Genetic Algorithm.

**Abstract:** This paper presents a discrete model of a sensor path planning problem, with a long-term planning horizon. The goal is to minimize the covariance of the reconstructed structures while meeting constraints on the length of the traversed path of the sensor. The sensor is restricted to move on a graph representing a discrete set of configurations, and additional constraints can be incorporated by altering the graph connectivity. This combinatorial problem is formulated as an integer semi-definite program, the relaxation of which provides both a lower bound on the objective cost and input to a proposed genetic algorithm for solving the original problem. An evaluation on synthetic data indicates good performance.

## 1 INTRODUCTION

As an experimental design problem, *camera network design* has been studied extensively in the photogrammetry literature. The goal is to obtain the most accurate reconstruction of a scene or object given a limited number of observations, and the task is to find the optimal set of camera poses or sensor configurations. In robotics, the *next best view* problem is similarly concerned with finding the next sensor position to most improve a sequential reconstruction of the environment. Both problems are hard due to the non-convex, multi-modal costs arising (cf. (Fraser, 1984)), but also to the sometimes high computational burden of evaluating the cost function. Recent research has mostly focused on the latter problem of accurately and efficiently evaluating the expected information gain of a potential sensor configuration (Low and Lastra, 2006; Vasquez-Gomez et al., 2013; Foix et al., 2012) and on achieving coverage of the scene (Blaer and Allen, 2007), while other works tackle minimizing the resulting cost functions to find one or a series of optimal sensor configurations. As the name implies, next best view planning usually employs a myopic planning horizon of only one step ahead, mainly due to these difficulties. Camera network optimization, on the other hand, can be seen as a long horizon planning problem, but without constraints on the order in which observations are made.

Problems of this type have mainly been addressed using stochastic optimization algorithms or by solving a relaxed, easier version. For example, in (Dunn

et al., 2006) the camera network problem is solved using a genetic optimization algorithm searching the high-dimensional parameter space of camera placements. In (Wenhardt et al., 2006) the authors reconstruct objects using a camera mounted on a robotic arm. The object geometry is estimated using a Kalman filter, and the next imaging location is determined by searching a discrete parameter space and evaluating the expected information gain in the filter at each position. A different approach is taken in (Trummer et al., 2010) where the next imaging location is decided based only on the single currently least well-determined feature, allowing a simple closed form solution. In (Dunn et al., 2009) the path of a robot moving in the plane is planned based on the expected reconstruction accuracy of an observed object. An approximation of the geometry is given and the expected information gain from observing the object from a particular vantage point is determined on a discrete grid of camera locations. Each grid cell is assigned a cost proportional to the inverse of the information gain, and a minimum cost path is found between the starting point and the global minimum grid cell. This does not take into account the fact that an observation may be more or less valuable depending on what other observations are available, and thus does not optimize the desired objective. In (Haner and Heyden, 2011) a continuous optimization approach is used to solve the similar problem of finding the best reconstruction while also reaching a predefined destination as quickly as possible, and while taking all future observations into account, this method can only

guarantee locally optimal solutions using gradient descent. In this paper, we formulate discrete analogs of the problem formulation of (Haner and Heyden, 2011) and cast them as integer semi-definite programs (SDPs). The relaxations to continuous SDPs may be used in a branch-and-bound scheme to find optimal solutions, or as input to a stochastic optimization algorithm proposed below in Section 4.

Related discrete approaches include (Englot and Hover, 2010) where a shortest path linear program formulation similar to this work is used, but only considers view coverage and not uncertainty. (Hollinger et al., 2012) uses a two-stage approach where good views are selected based on uncertainty, and then connected by solving a traveling salesman problem.

In (Singh et al., 2009; Golovin and Krause, 2010) approximation algorithms for the constrained path problem using greedy strategies are shown to provide solutions within a constant factor of the optimum, given that the underlying cost function is submodular. Unfortunately, the maximum eigenvalue metric used in this paper is not submodular and such guarantees cannot be given; however, an optimality gap can always be computed.

## 2 PROBLEM DESCRIPTION

Assume that the goal of a moving sensor is to reach a predefined target destination, while simultaneously reconstructing its surroundings as accurately as possible, based on observations taken along the path to the destination. There is a trade-off between reaching the destination quickly, and reducing the reconstruction uncertainty; for a bearing-only sensor such as a camera, a longer path can accommodate more observations with greater parallax, thus improving triangulation accuracy. Given a trade-off preference, or a bound on the path length, an optimal path can be found by solving a discrete optimization problem. The space between and around the start and destination positions is discretized into a finite number of possible sensor positions, and these positions constitute the nodes of a graph. The edges of the graph encode a neighborhood connectivity, i.e. the possible motions between the fixed positions. Thus a path in the graph corresponds to a physical path. With each node is also associated an information matrix encoding how much information about the environment we can expect to gain, if performing a measurement at that node's location.

The problem formulations in this paper are agnostic to the graph geometry and topology, and to how the information matrices are generated. Thus there

are no restrictions such as continuity or smoothness on the function used to evaluate the information content of a proposed sensor configuration, but which are typically required by continuous optimization approaches. Furthermore, the information of each view can be computed in parallel to leverage modern multi-core processors and GPU:s.

## 3 PROBLEM FORMULATION

Define a directed graph  $G = (V, E)$  and a set of positive semi-definite information matrices  $\{I_i \in S_+^n \mid i = 0, \dots, |V|\}$ . For a given trade-off parameter  $\lambda$ , define the optimization problem

$$\min_{p \in P} \text{length}(p) + \frac{1}{\lambda} F \left( \left( I_0 + \sum_{i \in p} I_i \right)^{-1} \right) \quad (\text{P1})$$

where  $P$  is the set of all simple paths in  $G$  from the source node to the destination node.  $I_0$  is the initially available information matrix of the environment structure, and  $I_i$  the expected information to be gained at node  $i$ . The inverse of the information matrix is the covariance matrix of the reconstructed structure, so the second term measures the variance using the scalarizing function  $F$ . This function is typically the trace or maximum eigenvalue, corresponding to  $A$ - and  $E$ -optimality in the experiment design literature (Montgomery, 2000). For these choices of  $F$ , we note that the second term is always decreasing as a function of the number of nodes on the path<sup>1</sup>. We now make two observations: if  $\lambda$  is large enough, the problem is equivalent to finding the shortest path through the graph, and may be solved efficiently using standard algorithms. If  $\lambda$  is sufficiently small, it is optimal to include all (non-zero) information matrices in the sum, while still minimizing the distance traveled, thus the problem is equivalent to the traveling salesman problem (TSP). Since TSP is known to be  $NP$ -hard, an efficient exact algorithm for the general case is out of reach. Also, the recognition version of TSP ("Is there a tour of length less than  $L$ ?") is  $NP$ -complete, so we should not expect even to be able to verify if a given solution is optimal. This is true even for graphs with nodes of degree at most four, e.g. planar grids (Papadimitriou and Steiglitz, 1998).

In the above formulation (P1) the parameter  $\lambda$  is used to control the trade-off between a short path and a more accurate reconstruction of the surroundings. It is however not obvious how to select this parameter, or even its suitable range, without some trial-and-error. In fact, another problem formulation may be

<sup>1</sup>This follows from the Courant-Fischer theorem.

more natural: given a time or distance budget, what is the best reconstruction obtainable? In other words, given an upper bound on the length of the path traveled, minimize the reconstruction error, i.e.

$$\begin{aligned} \min. & F\left(\left(I_0 + \sum_{i \in p} I_i\right)^{-1}\right) \\ \text{s.t.} & \text{length}(p) \leq L. \end{aligned} \quad (\text{P2})$$

Note that with this formulation, as the allowed path length grows we no longer approach TSP. Instead, for  $L$  large enough, any Hamiltonian path on the graph will be optimal, and for the types of graphs considered here, these are usually easily generated. Unfortunately, the problem still appears difficult for length limits of practical interest. There are several other variations on the problem formulation, for example one could minimize the path length under the constraint that the covariance is reduced by a certain amount. However, all of them appear equally hard to solve.

Below, we derive convex relaxations of (P1) and (P2) and show how these can be used to solve the original problems in a branch-and-bound scheme, or more practically as guides for more local optimization methods. The convex relaxation and optimization methods presented are easily adapted to alternative problem formulations.

### 3.1 Shortest Path as a Linear Program

The problem of finding the shortest path between two nodes in a graph with positive edge weights is often solved using Dijkstra's algorithm. It can be shown that this algorithm is equivalent to applying a primal-dual solver to the following linear program (Papadimitriou and Steiglitz, 1998):

$$\begin{aligned} \min. & \sum_{(i,j) \in E} c_{ij} x_{ij} \\ \text{s.t.} & \sum_{j: (i,j) \in E} x_{ij} = \sum_{j: (i,j) \in E} x_{ji}, \quad i \in \{1, \dots, |V| \setminus \{s, t\}\} \\ & \sum_{j: (s,j) \in E} x_{sj} = 1, \quad \sum_{j: (j,t) \in E} x_{jt} = 1 \\ & 0 \leq x_{ij} \leq 1. \end{aligned} \quad (\text{LP})$$

Here  $x_{ij}$  is a variable indicating if the edge between node  $i$  and  $j$  is part of the path or not, and  $c_{ij}$  the associated non-negative edge weight. The constraints express flow conservation, so that the number of edges incident on a node equal the number exiting, except for the source ( $s$ ) and terminal ( $t$ ) nodes which have one outgoing and one incident edge respectively. These constraints can be summarized into  $A_G x = b$  where  $A_G$  is the  $|V|$ -by- $|E|$  edge incidence matrix of

$$G \text{ with entries } a_{ij} = \begin{cases} -1 & \text{if edge } j \text{ leaves node } i \\ +1 & \text{if edge } j \text{ enters node } i \\ 0 & \text{otherwise} \end{cases}, \quad (1)$$

and  $x$  are the edge indicator variables suitably stacked. It is easily shown that (LP) must have an integer optimal solution. Note that this formulation does not explicitly forbid solutions consisting of a path between the source and terminal, plus any number of closed loops; these are only eliminated by virtue of not being optimal.

### 3.2 View Planning as a Semidefinite Program

We adapt the shortest path problem formulation above to the planning problem (P1). For convenience, introduce binary variables  $\alpha_i$  for each node of the graph, indicating whether that node is on the path or not. We form the relaxed optimization problem

$$\begin{aligned} \min. & \sum_{(i,j) \in E} c_{ij} x_{ij} + \frac{1}{\lambda} F\left(\left(I_0 + \sum_{i=1}^{|V|} \alpha_i I_i\right)^{-1}\right) \\ \text{s.t.} & A_G x = b \\ & \alpha_i = \sum_{j: (j,i) \in E} x_{ji}, \quad i \neq s \\ & \alpha_s = 1, \quad 0 \leq \alpha_i \leq 1, \end{aligned} \quad (\text{P3})$$

where  $\alpha$  and  $x$  are not required to be binary. The cost functions used in next best view planning are generally non-smooth and multi-modal, and difficult to optimize. However, due to the discretization, the argument to the second term of the objective above is affine in  $\alpha$ . Both the trace-of-inverse and maximum eigenvalue-of-inverse functions are convex, and using the epigraph trick the second term may be formulated as a convex semidefinite constraint (see e.g. (Boyd and Vandenberghe, 2004)). As one would expect, this semidefinite program no longer has all the desirable properties of the linear program; integrality of  $x$  or  $\alpha$  is no longer guaranteed, and a solution with disjoint loops may in fact be optimal.

The corresponding convex relaxation of (P2) is the same as (P3), except that the first term of the objective is transformed into a linear inequality:

$$\begin{aligned} \min. & F\left(\left(I_0 + \sum_{i=1}^{|V|} \alpha_i I_i\right)^{-1}\right) \\ \text{s.t.} & A_G x = b \\ & \sum_{(i,j) \in E} c_{ij} x_{ij} \leq L \\ & \alpha_i = \sum_{j: (j,i) \in E} x_{ji}, \quad i \neq s \\ & \alpha_s = 1, \quad 0 \leq \alpha_i \leq 1. \end{aligned} \quad (\text{P4})$$

Selecting  $L$  is more intuitive than choosing  $\lambda$ ; one must only be careful not to produce an infeasible problem by selecting  $L$  too low, but the lower limit is readily obtained using a standard shortest path algorithm.

It is possible to find an optimal integer solution to (P3) or (P4) using a standard branch-and-bound search, but this is also known to be *NP*-hard and may take a large number of iterations, each involving solving a potentially quite large SDP. If a solution is found, it may also contain unwanted disjoint loops. While it is easy to include linear constraints forbidding any *particular* loop in the SDP, since there are exponentially many possible loops in the graph, adding constraints against them all at the outset is infeasible. But, they can be added on an as-needed basis; if loops are present in the solution, add constraints against them and solve again until no loops remain. As it turns out, 2-cycles are quite common in the solutions, and as their number is typically linear in the number of nodes, it is feasible to remove them at the outset which may lead to faster convergence to a loop-free solution.

Obviously, the above procedure may be very time consuming or completely intractable for all but the smallest problem instances. However, we also noted above that depending on the trade-off parameter  $\lambda$ , the original problem (P1) should vary in difficulty between simple shortest path (typically  $O(|E| \log |V|)$  for Dijkstra's) up to exponential complexity. Empirically, it turns out that many instances are in fact "easy", in that very few steps of branch-and-bound are required and few or no loops are included in the solution. Yet, many other instances are indeed difficult and not amenable to this approach.

## 4 APPROXIMATE SOLUTION

Despite the problems of tractability in finding optimal solutions described above, it can be noted that the relaxed SDP formulations (P3) and (P4) provide lower bounds on the optimal objective values of (P1) and (P2). This may be used to verify the performance of approximation algorithms. Also, if the problem instance at hand is "easy enough", the relaxed solution  $x^*$  may be quite close to a valid integer, loop-free solution. In these cases it is possible to construct a valid solution to (P1) using a simple shortest path search on the graph  $G$  with edge weights  $c_{ij} = 1 - x_{ij}^*$ . This solution may be good enough, or can serve as initialization for local or stochastic optimization algorithms. For the formulation (P2) a slightly different approach is needed, which will not be explored here.

### 4.1 A Simplified Formulation

Given the hardness of (P1) and (P2), it is natural to seek a simplified problem formulation which might admit faster solution algorithms. Part of the difficulty is the nonlinearity of the interaction between the information matrices when taking the inverse to obtain the covariance; the value of any particular contribution to the information depends on all the others. Forgoing this interaction, instead of minimizing the covariance, one can maximize the trace of the information matrix, yielding the problem

$$\min_{p \in P} \text{length}(p) - \frac{1}{\lambda} \text{trace} \left( I_0 + \sum_{i \in p} I_i \right). \quad (\text{P5})$$

Since the trace is linear, this results in a shortest path problem on  $G$  with modified weights (subtract  $\text{trace}(I_i)/\lambda$  from each edge incident on node  $i$ ). As long as this does not result in any negative cycles, this may be efficiently solved using e.g. the Bellman-Ford algorithm, or even Dijkstra's if all weights are non-negative. If negative cycles are present, the problem again becomes much more difficult. In the extreme where all edge weights are negative, the problem is equivalent to the longest simple path problem on  $-G$ , which is known to be *NP*-hard (Schrijver, 2004). Unfortunately, for many scenarios and reasonable choices of  $\lambda$  negative cycles will be present, and in these cases (P5) can be formulated as (LP) but with binary constraints on  $x$ . While this ILP may be significantly faster to solve than (P3), the complexity is the same and no-loop constraints must also be introduced incrementally. However, in some scenarios it may be reasonable to restrict the graph  $G$  to be acyclic, and then the shortest path problem can always be solved in linear time. A general graph may be reduced to a directed acyclic graph by ordering the nodes by decreasing distance to the target node, and only keeping edges reducing the distance, thus forcing the sensor to move monotonically towards the destination. This will of course not work for purely exploratory scenarios where the start and end points may be near.

Even with this significantly simplified formulation sacrificing the interdependence of measurements, the problem is still not easy in general. We therefore introduce a stochastic genetic algorithm applicable to all problem formulations.

### 4.2 A Genetic Algorithm

Genetic algorithms (GA) are a class of evolutionary optimization algorithms which emulate the process of natural selection. A population of candidate solutions is maintained, and in every iteration of the algorithm,

a new population is generated by mutation and crossings of individuals of the previous generation. The chance of an individual producing offspring in the next generation is proportional to that individual's fitness, calculated from the corresponding value of the function being minimized. Genetic algorithms have been found to be quite efficient in providing good solutions to many combinatorial optimization problems, including TSP (Choi et al., 2003; Schmitt and Amini, 1998) and path planning (Davoodi et al., 2013), which motivates the use here.

To use a genetic algorithm, one must choose a representation for a candidate solution, and define unary mutation and binary crossover operators. In this work, each individual is described simply as a sequence of vertices constituting the path. The GA framework used is completely standard; for brevity we describe only the important steps below.

**Initialization** The first step is to generate candidate solutions, in this case paths in  $G$  from the source to the terminal node. Unless we have some a priori information on the characteristics of the optimal solution, these should be spread out uniformly across the space of all feasible paths. Unfortunately, truly uniform sampling of simple paths on a general graph appears to be a difficult problem. Reasonably random paths, however, may be obtained using random order depth-first search (DFS), *loop-erased random walk* (Lawler, 2012), or for undirected graphs by computing the minimum cost spanning tree with randomized edge weights, and extracting the unique path in the tree. If candidate solutions have been obtained using any of the heuristic methods based on the SDP relaxation, these can be included in the initial population and will then be refined.

**Mutation Operators** A mutation operator should introduce “noise” or randomness into an existing path, while preserving its main features. In practice, several mutation operators are often employed, exploiting problem-specific heuristics. To modify a path, just randomly replacing vertices is not possible, since not every sequence of vertices is a valid path in the graph  $G$ . Instead, our first operator selects two random cut points along the path, and replaces the path in between with a random one generated using either randomized DFS or loop-erased random walk. A second operator instead replaces the section with the shortest path between the cut points. This is motivated by the fact that optimal paths are often quite regular, so it makes sense to smooth out kinks. Both these operators are comparatively slow, so we also use a much faster but more local operator which simply selects a

random vertex on the path, and replaces it with one picked from the intersection of nodes reachable from the preceding node with those with outgoing edges incident on the next node on the path.

**Crossover Operator** The crossover operator takes two existing paths as input and produces a mixed path, containing parts of both, assuming they cross at some point. This is accomplished by selecting a random simple path on the graph obtained from  $G$  consisting only of the edges on the two paths.

**Local refinement** To speed up convergence to a locally optimal solution, paths may be optimized by systematically applying the fast local mutation operator described above in a deterministic fashion. Each node on the path, visited in random order, is replaced with the neighbor which minimizes the objective function.

With the formulation (P2), we run the risk of generating infeasible paths in the course of the genetic algorithm. A simple solution is to reject any infeasible path obtained and repeat the mutation or crossover operation until a feasible realization is produced. It is easy to verify that if the inputs are feasible, the operators defined above will eventually produce feasible output. However, depending on how close  $L$  is to the lower bound of feasibility, this may take an unreasonable amount of time. The very simplest solution is to transform the length constraint to a hard penalty term in the objective, e.g.

$$\min. F \left( (I_0 + \sum_{i \in p} I_i)^{-1} \right) + B_{\{p: \text{length}(p) \leq L\}} \quad (2)$$

where  $B_S$  is the barrier function for the set  $S$  s.t.  $B(x) = 0$  if  $x \in S$  and  $\infty$  otherwise.

## 5 STRATIFIED SOLUTION STRATEGY

The genetic algorithm will quickly find good solutions if the search space is not too large. For large grids with many hundreds of nodes and large neighborhood connectivities, the algorithm risks getting stuck in local optima, often producing implausible-looking paths. We therefore propose to reduce the search space by substituting a smaller graph, based on the solution of the SDP relaxation of the problem, to obtain a good initialization which can then be refined on the full graph.

## 5.1 Reducing the Graph

The idea is to keep only a subset of the  $N$  most important nodes, as indicated by the fractional SDP solution  $\alpha^*$ . Interpreting these values as probabilities, we draw  $N$  nodes without replacement, selecting nodes in proportion to their  $\alpha^*$ -value. The reduction in the covariance achieved using only these nodes is computed and maximized through repeated random sampling of the subset.

Once a subset has been chosen, a new fully connected graph is formed, where edges between the nodes represent the shortest path between them in the original graph. This allows the mapping of paths on the reduced graph to the full graph where they can be evaluated. The genetic algorithm can now be run without modification on the reduced graph, where the parameter  $N$  can be chosen to trade fidelity for convergence speed.

## 6 EXPERIMENTS

Due to the general formulation of the basic problem, many different scenarios can be accommodated by adapting the graph  $G$  and edge weights  $c_{ij}$ , which do not need to fulfill geometric constraints such as the triangle inequality. For example, each node can represent a camera position and an orientation, and the connectivity between poses can be defined so as to constrain angular velocity on the path. Purely exploratory behavior can be achieved by selecting start and/or destination nodes as “super-nodes” connected to every other node with zero weight, thus effectively permitting arbitrary start and destination points. Typically, we know less about the scene further away from the starting point, so the predictions of what will be seen, or what obstacles lay ahead, may be incorrect. Therefore one should plan with caution; by weighting the information matrices based on distance to the starting node, such behavior can be incorporated.

In the synthetic experiments below, each node (except super-nodes) has an associated camera pose defining position and orientation. The environment structure is represented by 3D points, each considered independently estimated such that the initial information matrix  $I_0$  is block diagonal (in fact we let it be the identity matrix so that the uncertainty is on the order of the scene scale). To compute the information gained from acquiring an image at a certain pose, the standard pinhole camera model is used, so that the relation  $\hat{x} = f(P, X)$  between a world point  $X$  and its projection  $\hat{x}$  is given by

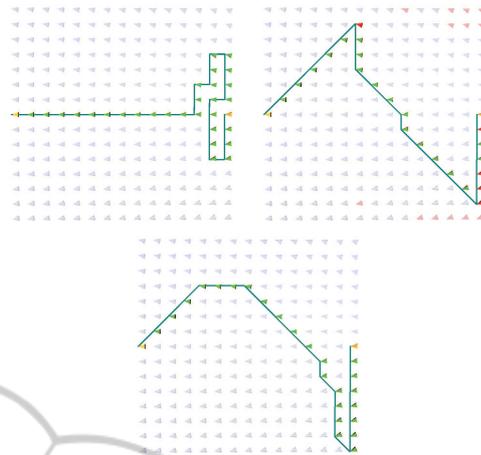


Figure 1: Cameras looking at a point cloud are placed on an 8-connected unit grid. Top left, the greenness and thickness of edges is proportional to  $\alpha_{ij}^*$ , the corresponding variable of the solution to the relaxed SDP (P3), using the maximum eigenvalue scalarizing function and  $\lambda = 2$ . Due to the symmetry of the problem, both a left and right path seem to be given equal consideration. The lower bound obtained is 61.1. Top right, the same problem with the path (green) found by the genetic algorithm among possible positions (blue), with a cost of 67.3. The third plot shows the same problem but with  $\lambda = 40$  and final cost 32.6, as compared to the lower bound of 23.0. Depending on problem characteristics, bounds may be more or less tight.

$$f(P, X) = \left( \frac{P^1 X + P_{13}}{P^3 X + P_{33}}, \frac{P^2 X + P_{23}}{P^3 X + P_{33}} \right)^\top \quad (3)$$

where  $P^i$  is the  $i$ :th row of the camera matrix. Given a point  $\bar{X}$ , the corresponding block of  $I_i$  is computed as  $J^\top \Sigma^{-1} J$ , where  $J = \frac{df}{dX} \Big|_{(P_i, \bar{X})}$  is the projection Jacobian and  $\Sigma$  the assumed measurement error covariance on the image plane. However, if  $\bar{X}$  is out of the camera’s field of view or too far away, the block is set to zero. In Fig. 1–5 different experiments are shown<sup>2</sup>; the details of the setup and results are in the figure captions. Note that in these experiments, the online nature of the problem is not demonstrated; in real use, as the sensor moves and new information becomes available, the structure and uncertainties need to be updated and the path re-planned. By seeding the optimizer at each stage with the previous solution, rapid convergence is possible.

### 6.1 Practical Considerations

The choice of scalarizing function  $F$  can have a large impact on the solution time of the SDP, depending on

<sup>2</sup>Many of the plots in the PDF version of this paper are embedded 3D models which may be viewed on-screen in recent versions of Adobe Reader.

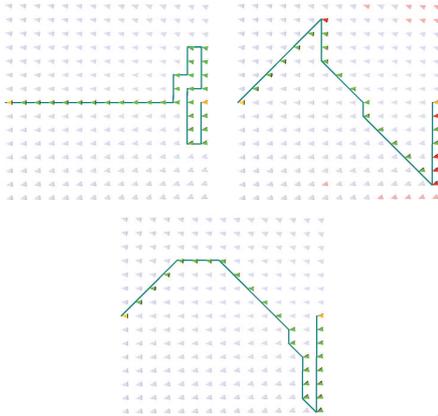


Figure 2: The same situation as in Fig. 1, now solving problem (P2) with  $L = 30$ . Top left, the solution obtained by solving the analog of the simpler problem (P5), giving a cost of 92.3 compared to the SDP lower bound of 67.7. Top right, the red nodes indicate the reduced graph obtained by sampling the SDP solution with  $N = 20$ , and applying the GA gives a path with cost 88.7. In the bottom plot, the GA run on the full graph with cost 86.36. The simplified formulation is qualitatively different from the others and focuses on getting as close to the structure as possible while neglecting the parallax effects. It is therefore not a suitable approximation in many situations.

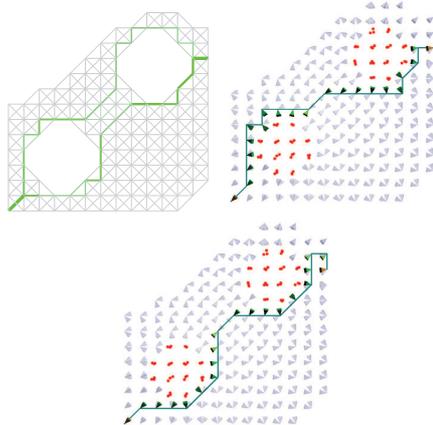


Figure 3: In this example, two nodes each are placed in every point of an 8-connected unit grid. The two nodes represent a camera looking at either of two objects/obstacles. Top left, the solution to the relaxed SDP (P4), using the trace scalarizing function and  $L = 22$ . Nodes of the original square grid whose combined shortest distance to the start and destination nodes is greater than  $L$  have been removed, since they cannot be part of a feasible solution. As in Fig. 1, there appears to be two competing paths, with edge values  $x_{ij}^* \approx 1/2$ . Top right, the path obtained from the simplified formulation (P5), which works reasonably for this problem instance. In the bottom plot, the solution obtained using the proposed genetic algorithm. The corresponding objective values are 192.1, 271.8 and 262.1. The gap between the final objective and the lower bound given by the SDP is relatively large, but the path obtained directly from the SDP solution is still quite reasonable.

the dimension of the information matrices. To minimize the trace of the covariance matrix, one variable per eigenvalue is required, while the maximum eigenvalue cost only needs one. On the other hand, evaluating the trace cost function is typically faster. Furthermore, the maximum eigenvalue is vulnerable to outliers e.g. features which are not seen in any or too few views. If such features are not removed in a pre-processing step, the cost function can never decrease below the initial uncertainty.

The algorithms were implemented in Matlab with core functions in MEX C++. SDPs were set up using YALMIP (Löfberg, 2004) and solved using the MOSEK interior-point optimizer (Dahl, 2012). For the experiment in Fig. 4, solving the SDP with the trace cost took 8.7 s as opposed to 4.2 s for the maximum eigenvalue, while the genetic algorithm (on the full problem) runs at about 10 iterations per second on the same Core 2 Duo 3.0 GHz computer, with a population of 60 individuals. With parallel processing of individuals, speed can likely be increased manyfold.

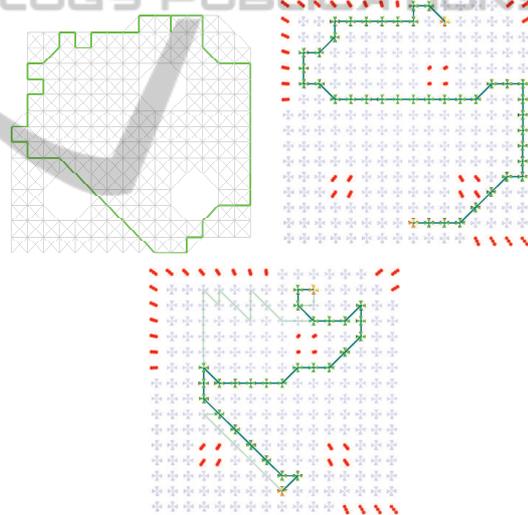


Figure 4: In this experiment, we simulate an omnidirectional camera by adding the information matrices of four cameras at each location. Each cluster thus corresponds to only one node of the graph. Top left, the SDP (P4) with  $L = 43$  gives a lower bound at 4.11. Top right, the solution using the stratified approach, with cost 5.47. In the last plot, the graph has been reduced to a DAG such that the observer must move towards the target at every step. This limits the search space making branch-and-bound tractable, the optimal solution with cost 17.98 is shown next to the solution of the simplified problem (P5) (pale green) with cost 30.81.

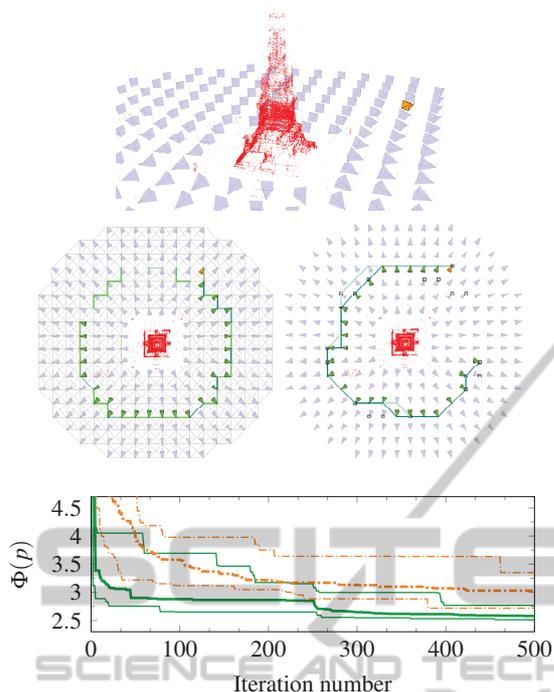


Figure 5: Exploratory scenario with fixed start and free end point. The point cloud of the monument has been reduced to a few hundred representative points (by random sampling) to constrain the information matrix dimension. In the middle left plot, the SDP solution giving lower bound 1.76 along with the “filled in” path found as the shortest path on the graph with weights  $1 - x^*$  (see Section 4), having cost 3.18. Middle right, the GA solution on the reduced graph obtained by sampling the SDP solution, with nodes marked with black squares, shown in light green with cost 2.78. The dark green path is the result of the GA on the full graph, seeded with the reduced solution, with cost 2.57. Achieving similar cost using random initialization takes significantly longer; the bottom graph shows the progression of the objective value ( $\Phi$ ) over iterations of the proposed genetic algorithms. The orange dashed curves show the maximum, minimum and median over 20 runs of the GA on the full graph with random initialization. The green curves show the same for the stratified approach, first running 250 iterations on the reduced graph, then switching to refinement on the full graph. It is clear that the stratified scheme converges much faster.

## 7 CONCLUSIONS

While the general problems considered in this paper are demonstrably hard, satisfactory solutions can be found sometimes directly from the SDP relaxation, and often by the proposed genetic algorithm. In many scenarios, the SDP solution gives hints as to what a good path might look like, while in others it consists of seemingly random, disconnected edges only. In those cases the lower bound obtained is usually not

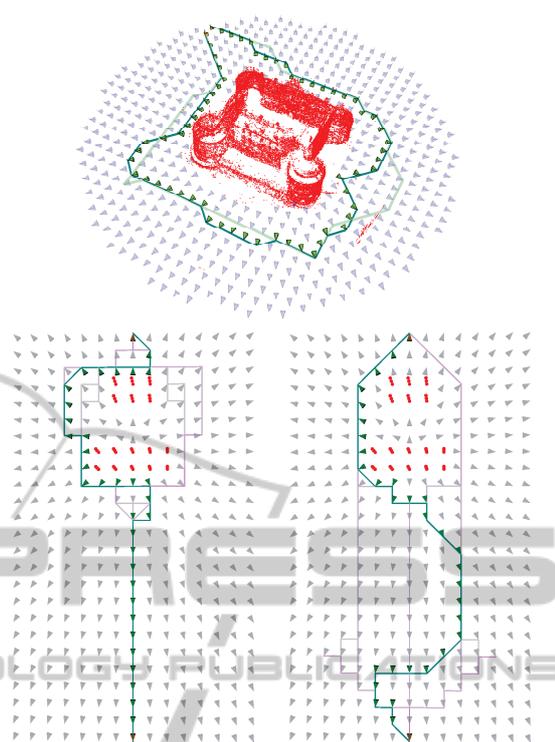


Figure 6: Top: stratified algorithm run on castle dataset, reduced to one hundred representative points using random sampling. Bottom right: same scenario as on the left, but here the information matrix at each node has been down-weighted by the distance from the start node, to signify reduced confidence in future measurements. The pale purple lines indicate the SDP solution, the green path the GA solution.

very tight and it is difficult to draw any conclusions about the optimality of any path. This is of course to be expected given the hardness of the problem. Nevertheless, the SDP solution can always be used to seed the GA optimizer in the proposed three-stage stratified algorithm.

The linearized approximation (P5) sometimes gives reasonable solutions, as in Fig. 3, but most often does not show any proper long-term planning behavior, as in Fig. 2. On a directed acyclic graph (Fig. 4 right) it does have the advantage of being extremely fast compared to the other methods.

For computational tractability, structure points must be considered independent, and real point cloud data need to be subsampled. How to best subsample while preserving data characteristics has not yet been considered. As the graph size and connectivity increases, computational complexity also rises and the quality of solutions attainable in reasonable time drops. This limits the resolution of the discretization, particularly in the orientation space, which means

local, continuous refinement may be a desirable second step. This we leave to future work.

Full source code to replicate the experiments in this paper are available at [http://github.com/sebhaner/nbv\\_discrete](http://github.com/sebhaner/nbv_discrete).

## REFERENCES

- Blaer, P. and Allen, P. (2007). Data acquisition and view planning for 3-d modeling tasks. In *Intelligent Robots and Systems, 2007. IROS 2007. IEEE/RSJ International Conference on*, pages 417–422.
- Boyd, S. and Vandenberghe, L. (2004). *Convex Optimization*. Cambridge University Press, New York, NY, USA.
- Choi, I.-C., Kim, S.-I., and Kim, H.-S. (2003). A genetic algorithm with a mixed region search for the asymmetric traveling salesman problem. *Computers & Operations Research*, 30(5):773 – 786.
- Dahl, J. (2012). Semidefinite optimization using MOSEK. In *21st International Symposium on Mathematical Programming*.
- Davoodi, M., Panahi, F., Mohades, A., and Hashemi, S. N. (2013). Multi-objective path planning in discrete space. *Applied Soft Computing*, 13(1):709 – 720.
- Dunn, E., Olague, G., and Lutton, E. (2006). Parisian camera placement for vision metrology. *Pattern Recognition Letters*, 27(11):1209–1219.
- Dunn, E., van den Berg, J., and Frahm, J.-M. (2009). Developing visual sensing strategies through next best view planning. In *Intelligent Robots and Systems, 2009. IROS 2009. IEEE/RSJ International Conference on*, pages 4001–4008.
- Englot, B. and Hover, F. (2010). Inspection planning for sensor coverage of 3d marine structures. In *IROS*, pages 4412–4417. IEEE.
- Foix, S., Kriegel, S., Fuchs, S., Aleny, G., and Torras, C. (2012). Information-gain view planning for free-form object reconstruction with a 3d tof camera. In *Advanced Concepts for Intelligent Vision Systems*, volume 7517 of *Lecture Notes in Computer Science*, pages 36–47. Springer Berlin Heidelberg.
- Fraser, C. S. (1984). Network Design Considerations for Non-Topographic Photogrammetry. *Photo Eng. and Remote Sensing*, 50(8):1115–1126.
- Golovin, D. and Krause, A. (2010). Adaptive submodularity: A new approach to active learning and stochastic optimization. In *23rd Annual Conference on Learning Theory*, pages 333–345.
- Haner, S. and Heyden, A. (2011). Optimal view path planning for visual SLAM. In Heyden, A. and Kahl, F., editors, *Image Analysis*, volume 6688 of *Lecture Notes in Computer Science*, pages 370–380. Springer Berlin / Heidelberg.
- Hollinger, G. A., Englot, B., Hover, F., Mitra, U., and Sukhatme, G. S. (2012). Uncertainty-driven view planning for underwater inspection. In *ICRA*, pages 4884–4891. IEEE.
- Lawler, G. (2012). *Intersections of Random Walks*. Modern Birkhäuser Classics. Springer-Verlag New York.
- Löfberg, J. (2004). Yalmip : A toolbox for modeling and optimization in MATLAB. In *Proceedings of the CACSD Conference*, Taipei, Taiwan.
- Low, K.-L. and Lastra, A. (2006). An adaptive hierarchical next-best-view algorithm for 3d reconstruction of indoor scenes. *14th Pacific Conference on Computer Graphics and Applications, Taipei, Taiwan*.
- Montgomery, D. C. (2000). *Design and Analysis of Experiments*. John Wiley & Sons Canada, Ltd., 5th edition.
- Papadimitriou, C. H. and Steiglitz, K. (1998). *Combinatorial Optimization: Algorithms and Complexity*. Dover Publications.
- Schmitt, L. J. and Amini, M. M. (1998). Performance characteristics of alternative genetic algorithmic approaches to the traveling salesman problem using path representation: An empirical study. *European Journal of Operational Research*, 108(3):551 – 570.
- Schrijver, A. (2004). *Combinatorial Optimization : Polyhedra and Efficiency (Algorithms and Combinatorics)*. Springer.
- Singh, A., Krause, A., Guestrin, C., and Kaiser, W. J. (2009). Efficient informative sensing using multiple robots. *J. Artif. Intell. Res. (JAIR)*, 34:707–755.
- Trummer, M., Munkelt, C., and Denzler, J. (2010). Online Next-Best-View Planning for Accuracy Optimization Using an Extended E-Criterion. In *Proc. International Conference on Pattern Recognition (ICPR'10)*, volume 0, pages 1642–1645. IEEE Computer Society.
- Vasquez-Gomez, J. I., Sucar, L. E., and Murrieta-Cid, R. (2013). Hierarchical ray tracing for fast volumetric next-best-view planning. *2013 International Conference on Computer and Robot Vision*, 0:181–187.
- Wenhardt, S., Deutsch, B., Hornegger, J., Niemann, H., and Denzler, J. (2006). An Information Theoretic Approach for Next Best View Planning in 3-D Reconstruction. In *Proc. International Conference on Pattern Recognition (ICPR'06)*, volume 1, pages 103–106. IEEE Computer Society.