

A MapReduce-based Approach for Finding Inexact Patterns in Large Graphs*

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Abstract: Finding patterns in graphs is a fundamental problem in graph theory, and also a practical challenge during the analysis of big data sets. Inexact patterns may correspond to a set of possible exact graphs. Their use is important in many fields where pattern matching is applied (e.g. mining in social networks or criminal investigations). Based on previous work, this paper introduces a pattern specification language with special language features to express inexact patterns. We also show a MapReduce approach-based algorithm that is able to find matches. Our methods make it possible to define inexact patterns and to find the exact matches in large graphs efficiently.

1 INTRODUCTION

Finding patterns efficiently in large graphs is a core challenge in the analysis of big data sets. Pattern matching is based on subgraph isomorphism that deals with the problem of deciding if there is an injective morphism from one graph to another. It is already proven that finding such a morphism is NP-complete in general (Plump, 1998). To perform the pattern matching efficiently, especially in the processing of big data sets, distributed algorithms such as MapReduce gain focus. To find a match, we need a pattern specification language and a matcher algorithm that performs the actual searching. The language may contain special features that make it possible to define not just a single graph but inexact patterns as well. An inexact pattern covers a set of possible exact graphs by using constraints. Being able to detect inexact patterns might be critical in some cases, for example, when the analyzer needs to match a general pattern without knowing all of the details, the host graph is not completely known, or some aspect of the searched pattern is incorrect (Coffman et al., 2004). Moreover, inexact patterns lead to more concise specifications that is an important aspect in graph rewriting-based model transformations (Ehrig et al., 2006).

As an example, consider the graph depicted in

Figure 1. This graph illustrates the retail banks belonging to the central bank. The graph also represents the different type of loans the retail banks offered to their customers. Moreover, the dotted lines between the customers indicate personal relationships. Although this example contains only a several dozens of elements, it is easy to see, that a real life example from this field might consist of millions of elements, therefore its analysis with MapReduce can be not only reasonable but inevitable.

Assume that the central bank wants to know the households, where at least two members of the family have mortgages. Since it does not matter to the central bank whether the relatives have their mortgages at the same bank or not, there are two different patterns. In the first case (depicted in Figure 2(a)), the customers have their mortgages at different banks, therefore the pattern should match two bank elements. However, the family members might get their mortgages from the same bank, and in this case there should be only one bank element matched (depicted in Figure 2(b)). The answer is formulated from the union of the two cases.

By the usage of traditional pattern matching languages and solutions, the central bank needs to match two different patterns. However, with the help of inexact patterns, a simple pattern can be formulated to answer the question of the central bank. The inexact pattern to solve this problem is presented in Figure 3 (the purpose of the labels is to identify the elements later). The optional blocks (depicted as dot-

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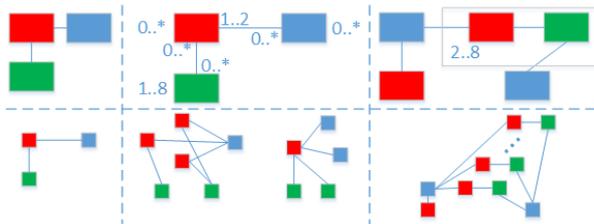


Figure 4: Language features of pattern matchers.

The most trivial specification language would simply allow us to specify exact patterns, i.e. each element that will be present in the match. In other words, we provide a pattern where we specify the types of the nodes and edges along with optional additional constraints, and the actual matches will be isomorph occurrences of this pattern. In many cases, we may need more advanced language features (such as multiplicities, negative application conditions etc.) that lead to inexact patterns. Multiplicities (or cardinalities) added to the nodes and to the ends of the edges mean that the given element of the pattern may occur several times in the actual match. With this feature, the patterns of the specification language can be turned into small metamodels whose all possible instances that are present in the model are valid matches. Two sample patterns (with and without multiplicities) along with possible matches are depicted in the first and second columns of Figure 4. Colors denote the types of the elements. Negative application conditions (NACs) consists of additional elements that will not be matched, on the contrary, they must not be able to be found.

The language features detailed above obviously increase the expressiveness of the language, but also make the match finder algorithms more complex and harder to be implemented. In the following, we introduce our pattern specification language. Our goal is to find a good balance between these two goals. Therefore, this language contains several language features, but has a few limitations compared to the previously mentioned fully metamodel-based approach.

In theory, patterns of our language consists of nodes and edges. We define their types and additional constraints on their attributes in the usual way. We can define groups in a pattern. A group is a set of connected nodes. Each node can be a member of at most one group, so the groups are disjoint. Moreover, it is required that if two nodes are members of two different groups, they cannot have edges between them in the pattern. We can assign multiplicity constraints to the groups. When searching for the possible matches of the pattern, each group can be present multiple times according to their multiplicity constraints. The nodes that are not members of any groups represent

single nodes in the matches just as in exact patterns. When an edge of the pattern connects a group member and a traditional node, this means that the instance of the exact node will be connected to every instances of the group. The third column of Figure 4 shows a sample pattern where there is one group and its multiplicity is 2..8. This means that at least 2, but at most 8 instances of the group must be present in a possible match. The figure also presents an instance of the pattern. It can be seen that the node in the top left corner of the pattern is connected to all instances of the group.

Because of the space limits of this paper, we do not go into deeper details about other features of the language, but we mention that application conditions (negative and positive) and logical statements on optional blocks can be specified as well.

We have implemented a textual language. According to the previous concepts, we can define nodes, edges, and blocks with multiplicities. Nodes and edges may have labels. Additionally, logical statements using block identifiers can be specified. The code belonging to the inexact pattern presented in Figure 3 is as follows:

```
rule MultipleMortgages {
  node p1 [Person] node l1 [Loan] node b1 [Bank]
  node p2 [Person] node l2 [Loan] node b2 [Bank]
  node p3 [Person] node l3 [Loan]
  edge from p1 to l1 [Mortgage]
  edge from b1 to p1 [Customer]
  edge from p1 to p2 [Relationship]
  edge from p2 to l2 [Mortgage]
  edge from b2 to p2 [Customer]
  edge from p1 to p3 [Relationship]
  edge from b1 to p3 [Customer]
  edge from p3 to l3 [Mortgage]
  block A (b2, p2, l2)?
  block B (b1, p3, l3)?
  block C (l1)1..*
  with A or B
}
```

We believe that our concept is efficient enough to be applied in real-world scenarios. Moreover, in the next sections, we will show an efficient distributed algorithm that is able to find the actual matches specified by the patterns of the language. This demonstrates the practical applicability of our approach. Moreover, our experiences gained from practical solutions show that the presented language is expressive enough to be used in model transformations as well, which is an important application field of pattern specification languages.

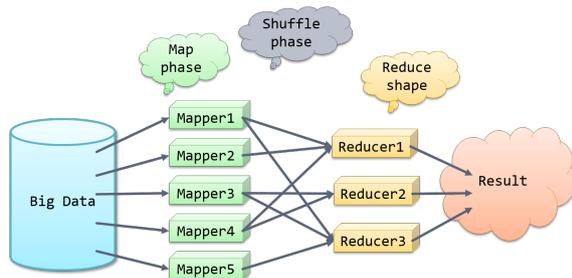


Figure 5: The structure of a MapReduce algorithm.

3 THE MAPREDUCE TECHNIQUE

The MapReduce framework is a parallel computing paradigm (Dean and Ghemawat, 2008). The framework was originally developed at Google, and today has become the *de facto* approach for processing large-scale data. One of its open source implementations is the Apache Hadoop (Apache Hadoop, 2011). The MapReduce framework was designed for utilizing distributed computing resources and for handling large data sets. Data is always assumed to be distributed, and the data balancing is managed by the Hadoop Distributed File System (HDFS).

One of the biggest differences between the MapReduce and other parallel computing frameworks is that MapReduce implements both parallel and sequential computation: the framework calls the map and reduce functions sequentially, but in each phase the functions are executed in parallel.

The structure of a MapReduce algorithm is illustrated in Figure 5. Basically, the MapReduce algorithm consists of three phases. First, the map function is executed. The framework assigns the same mapper task μ to several processes, but these processes take different inputs as the lone argument of μ . The mapper function is stateless, because it does not have any information on the previously processed data, nor can it communicate with other mapper tasks. In this manner, it only requires the input value to compute its output value. This way the mapper function can be run against the values in parallel, which provides great efficiency. Generally, the mapper μ takes a single string input and produces any number of $\langle key, value \rangle$ pairs as output.

The second phase is referred to as shuffle or as group. At this stage, the framework seamlessly sorts the outputs of the mappers by their keys. As a result, the values are grouped by their key values.

The shuffle step makes it possible to assign every value with the same key to the same reducer in the

final phase. A reducer ρ takes a single key and all the values associated with it as input. After processing, the reducer produces arbitrary number of $\langle key, value \rangle$ pairs, as output. Like the mapper tasks, the reducers are stateless as well. An important attribute of the MapReduce framework is that all mapper tasks must finish before the reducer tasks can begin. This aspect of the framework supports the sequential processing. However, by assigning different keys to different processes, the reducer task also supports parallelization.

The map, shuffle, and reduce phases constitute a MapReduce job. The MapReduce algorithm can be implemented as a series of MapReduce jobs. In this case, the output of the λ_1 job (i.e. the $\langle key, value \rangle$ pairs produced by the ρ_1 reduces) is the input of the λ_2 job (i.e. the string input of the μ_2 mapper) (Fehér et al., 2013).

Based on (Karloff et al., 2010), the formal definition of the MapReduce programming paradigm is the following.

Definition 1. A mapper is a function that takes a binary string (which might be a $\langle key, value \rangle$ pair) as input. As output, the mapper produces a finite set of $\langle key, value \rangle$ pairs.

Definition 2. A reducer is a function whose input is a binary string, a key (k), and a sequence of values (v_1, v_2, \dots) that are also binary strings. As output, the reducer produces a finite multiset of pairs of binary strings $\langle k, v_{k,1} \rangle, \langle k, v_{k,2} \rangle, \langle k, v_{k,3} \rangle, \dots$. The key in the output pair is identical to the key in the input.

As it was mentioned before, the main strength of the MapReduce approach is the ease of parallelization. Since one mapper task processes one element from the input set at a time, the framework can have multiple instances from the same map function assigned to different computing nodes in parallel. Similarly, one reducer task operates on one key value and on the values associated with it. Therefore, if the number of different keys is denoted with n , then at most n different reducer instances can be executed simultaneously.

4 THE PATTERN MATCHING ALGORITHM

The MapReduce MetaMatcher algorithm (*MRMM*) is a novel approach to detect arbitrary patterns in graphs with millions of edges and vertices. The algorithm is constructed to extend the MapReduce Subgraph Isomorphism - version S algorithm (*MRSIS*), therefore

the pattern matching utilizes the advantageous properties of the MapReduce framework. In this manner, the algorithms make it possible to move the pattern matching into the cloud, which offers the following benefits: (i) The graph can be stored in the cloud (typically in a BLOB), (ii) the size of the cluster can be dynamically changed depending on the graph and the pattern to match. There are a number of different cloud vendors offering these possibilities, for example (Amazon Web Services, 2013) and (Windows Azure, 2013).

This section briefly presents the basics of the $MRSI_S$ algorithm, then introduces the $MRMM$ algorithm, which enables the inexact pattern matching.

4.1 The $MRSI_S$ Algorithm

The $MRSI_S$ is a MapReduce algorithm to detect subgraph isomorphism. At each iteration, the algorithm adds a new pair of vertices to each partial mapping. All partial mappings are subgraphs isomorphic to the target graph. In case the partial mapping contains the same number of vertices as the target graph, then the two sets are isomorphic to each other. In this manner, the algorithm terminates after at most n iterations, where n denotes the number of vertices contained by the target graph. In case a partial mapping cannot be extended with a new pair of vertices in a way to remain subgraph isomorphic to the target graph, the algorithm does not emit any information related to this partial mapping, therefore, it does not participate in the next iteration. The algorithm terminates either after the n^{th} iteration, or when there are no partial mappings that can be extended with a new pair of vertices.

In order to minimize the required I/O operations, the algorithm uses a special data structure, which contains only the necessary information about a source graph node. Since the processing of a textual file in a MapReduce framework is performed line-by-line, each line represents a source graph vertex. The necessary information about a given node is the following:

- The identifier of the node,
- In case of labeled graph, the labels of the vertex (if there is any),
- The sources of the incoming edges and their labels (if there is any),
- The targets of the outgoing edges and their labels (if there is any).

Since the node may have multiple vertex labels and incoming/outgoing edges with multiple edge labels, the information must be carefully separated. The data structure in Extended Backus–Naur Form (EBNF) is depicted in the following listing, where the rule of thumb is the following:

1. The identifier of the node is the most important information, therefore it is the first, and is followed by a tabulator that is typically used as the separator between the key and the value emitted by either the mapper or the reducer.
2. The remaining sets, i.e. the labels, incoming edges and outgoing edges are separated by a semi-colon character.
3. Each item in an array (e.g. the different labels of the node) are separated by a comma character.
4. The end points of the edges are separated from their labels by a dash character.
5. Each item in a subarray (e.g. the different labels of the edges) are separated by a colon character.

```
row = ID, "\t", { NodeLabels }, ";",
      { IncomingEdges }, ";",
      { OutgoingEdges }, ";", HelperBit;
ID = alphabetic character | digit,
    { alphabetic character | digit };
NodeLabels = Label, { ",", NodeLabels };
IncomingEdges = ID, { "-", EdgeLabels },
               { ",", IncomingEdges };
OutgoingEdges = ID, { "-", EdgeLabels },
               { ",", OutgoingEdges };
EdgeLabels = Label, { ":", EdgeLabels };
HelperBit = "0" | "1";
```

The $MRSI_S$ algorithm consists of a chain of two MapReduce jobs. The first job is responsible for adding a new node pair to the partial mappings. Each mapper function in this job takes a partial mapping and a candidate node from the source graph. The function then calculates the next target graph node to match and determines whether this node can be mapped to the candidate node regarding to the partial mapping. In case of a successful match, the function extends the partial mapping with this node pair. In order to be able to produce all possible candidate nodes for this new partial mapping, the mapper function emits a new $\langle key, value \rangle$ pair for each source graph node contained by the mapping, where the *key* is the source graph node and the value is the new partial mapping. The function then moves on to the next partial mapping related to this candidate node. The reducer functions group the emitted $\langle key, value \rangle$ pairs by their *keys*. In this manner, after the first job, the partial mappings are extended with another node pair, and each emitted line consists of all information of a source node, and all partial mappings the given source node is part of.

The second job is responsible for generating the candidate pairs for each partial mappings. This means, that the job transforms the data in such a way that the partial mappings appear in the line of every candidate source graph node. Therefore, each mapper function obtains a source graph node and the

mappings the node is part of. Then, for each partial mapping, the function iterates through the neighbors of the source graph node, and if the neighbor is not contained by the given partial mapping, the function emits it as the *key* of a $\langle key, value \rangle$ pair. The related *value* is the partial mapping. The reducer functions group the emitted lines by their *key* values and drop the recurrences.

4.2 The MRMM Algorithm

Recall that, the MRMM algorithm is based on the MRSI_S algorithm. This section presents the main differences between the two algorithms.

The MRMM algorithm is capable of detecting patterns with optional parts. These parts are organized into blocks. The blocks have the following three properties:

- The identifier of the block, which must be unique.
- The identifiers of the nodes contained by the block. None of the nodes can be contained by more than one block.
- The multiplicity of the block.

Since the algorithm supports optional blocks, that is, the final match is valid with and without the nodes contained by the block as well, the identifier of the already examined blocks must be maintained throughout the MapReduce iterations. This is particularly important in those cases, when a block cannot be part of a valid partial mapping, because otherwise the algorithm would repeatedly check the nodes of this block, causing an infinite loop.

The obligation of maintaining the block identifiers means that the data structure must be extended. The information about the examined blocks is related to the partial mapping, therefore, each partial mapping has an additional value in the outputs, which contains the identifier of the last unsuccessfully matched block. This information is enough, because of the following:

- The identifier unambiguously determines the block.
- The identifiers of the successfully matched blocks can be deduced from the partial mapping.
- The algorithm orders the nodes of the target graph, therefore, if there are more than one unsuccessfully matched blocks, the last one determines the previous ones as well.

Since the MRSI_S algorithm detects isomorphic subgraph patterns, the number of nodes contained by the final match equals to the number of nodes contained by the target graph. However, in case of the pattern matching this is not true anymore, because of the multiplicity of the blocks, the number

of nodes contained by the valid matches might differ from the number of nodes contained by the target graph. Therefore, the MRMM uses an indicator that signals when at least one partial mapping is extended with a new node pair. In this case, another iteration of MapReduce jobs is justified. Otherwise, the algorithm terminates.

The main difference between the two algorithms is the selection of the next candidate node. As it was mentioned, the algorithm maintains a sorted list based on the nodes of the target graph. The MRSI_S algorithm simply chooses the first from this list that is not already contained by the partial mapping. However, in case of optional blocks this cannot be a solution. Therefore, a new node selection algorithm was needed, which is shown in Algorithm 1.

Algorithm 1: The algorithm of selecting the next candidate node for a partial mapping.

Require: the given partial mapping as *actualMatch* and the identifier of the last unsuccessfully matched block as *lastBlockId*
Ensure: the identifier of the next candidate node

```

1: if actualMatch.isEmpty then
2:   return GETFIRSTNODE()
3: Nodes[] SortedNodes ← GETSORTEDNODES()
4: Nodes[] possibleNodes ← SortedNodes
5: if not lastBlockId.isEmpty then
6:   Nodes[] blockNodes ← GETUNUSEDBLOCKNODES()
7:   possibleNodes.Remove(blockNodes)
8: Node lastNode ← GETNODEBYID(actualMatch.Last.TargetNode)
9: string lastNodeBlockId ← lastNode.BlockId
10: if not lastNodeBlockId.isEmpty ∧ lastNodeBlockId ≠ lastBlockId then
11:   Nodes[] blockNodes ← GETNODESFORBLOCK(lastNodeBlockId)
12:   if blockNodes.count > 0 ∧ lastNode ≠ SortedNodes.LastInBlock(blockNodes) then
13:     return SortedNodes.GetNextFromBlock(lastNode).Id
14:   if blockNodes.count > 0 ∧ lastNode ≠ SortedNodes.LastInBlock(blockNodes) ∧ ISBLOCKSTILLANOPTION(lastNodeBlockId, actualMatch) then
15:     return SortedNodes.GetFirstInBlock(lastNodeBlockId).Id
16: for all m|m ∈ actualMatch ∧ GETNODEBYID(m.TargetNode) ∈ possibleNodes do
17:   possibleNodes.Remove(m)
18: Node nextNode ← possibleNodes.First()
19: return nextNode.Id

```

The algorithm consists of four parts. First, if the partial mapping is empty, then the identifier of the first node in the sorted list is returned. This node cannot be part of any optional block, because then the algorithm could not find the mappings without this optional node.

The second part (lines 4–8) filters the possible nodes. The GETUNUSEDBLOCKNODES() method returns all nodes that are contained by any block that appears before the last unsuccessfully matched block

in the sorted list. These nodes cannot be part of the candidate nodes because they are either already matched or determined as an unsuccessful match. Therefore, these nodes are removed from the possible nodes.

The third part (lines 10–17) is responsible for returning the next candidate node when the lastly matched node was part of a block. In case there are still nodes contained by this block left in the sorted list, the algorithm returns with the identifier of the first one of these. In case the lastly matched node was the last one contained by the block, but the multiplicity of the block indicates another matching, the algorithm returns the first node of the sorted list contained by the block.

Finally, the fourth part (lines 19–22) returns the identifier of the next possible node. This way the algorithm handles all possibilities regarding to the defined language.

The first mapper function of the *MRMM*, which is responsible for adding new pairs to the partial mappings, now might take an optional node as the candidate. In this case the candidate node either extends an optional block currently contained by the partial mapping or not. If it is an extension then the algorithm runs as usual. However, if it is not an extension, that is, this is a new block, then the algorithm emits the original partial mapping as well, and sets its last unsuccessfully matched block to the identifier of this block. This way the partial mapping with and without the optional block constitutes two different branches.

The second mapper function, which is responsible for moving the partial mappings to the row of candidate source graph nodes, must also check, whether the partial mapping contains all non-optional nodes from the target graph. If so, and there is no candidate source node related to this mapping, then the mapping should be marked as a complete match.

With the help of these modifications, the *MRMM* algorithm is able to detect different patterns with optional parts. The algorithm also possesses the advantageous properties of the *MRSIS* algorithm.

5 RELATED WORK

In related work (Berry et al., 2007), Berry et al. developed a parallel implementation for supporting massive multithreading. The implementation is based on the large shared memory of the machines in the cluster. The algorithms are designed for finding connected components and st-connectivity. Experimental results were measured on a synthetic power law graph with 234 million edges. Berry also developed heuris-

tics for inexact subgraph isomorphism (Berry, 2010). Our approach is capable of handling large problems as well with no need for large shared memory.

In (Plantenga, 2012), Plantenga presents a new algorithm, which finds inexact subgraph isomorphisms. The main contribution of his work is a scalable MapReduce algorithm for finding all type-isomorphic matching subgraphs and also introduces the concept of walk-level constraints. The suggested algorithm adds new edges to the already matched ones at each iteration. Experimental results are also provided on graphs with size of billions of vertices and edges. The *MRMM* algorithm is also suitable for inexact pattern matching, moreover, it is capable of detecting even more complex patterns.

In (Tong et al., 2007), Tong et al. suggest an algorithm that returns a specified number of “best effort” matches. The complexity of the Graph X-ray algorithms is linear to the number of nodes in the source graph. In contrast, the *MRMM* algorithm returns all exact matches and can handle not only vertex labels but edge labels as well.

In (Liu et al., 2009), Liu et al. present a MapReduce algorithm for “pattern finding”. Similarly to *MRMM*, their algorithm increases the size of partial mappings. However, the algorithm was used on a graph with 2000 edges with a cluster of 48 nodes. Since they assume the full adjacency matrix is available at each cluster node, the algorithm may have difficulty with larger graphs. The *MRMM* algorithm does not use the whole adjacency matrix at each mapper, this way it is more scalable.

In (Kim et al., 2013), Kim et al. developed a method to efficiently process multiple graph queries based on MapReduce. The method uses a filter-and-verify scheme to reduce the number of subgraph isomorphism test. In contrast to the *MRMM* algorithm, their work focuses on multiple graph queries instead of pattern matching.

The purpose of the solution presented in (Mezei et al., 2009) is to compute one of the possible matches for a pattern. The algorithm is executed in a computing grid consisting of several worker and a master node coordinating their operation. The individual worker nodes are performing the same matching algorithm on the same graph, but the matching is started from different points of the host graph. The selection of the starting point is coordinated by the master node. Once either of the nodes finds a match, it notifies the master node that lets the other worker nodes stop the algorithm. In contrast, our algorithm is capable of computing all the possible matches. Another important drawback of this approach is that it needs the complete host graph to be stored on each worker

nodes, while our approach is more scalable since the workers always see only a small portion of the complete graph.

The pattern matching approach presented in (Dörr, 1995) can be used for matching multiple patterns having isomorphic sub-patterns at the same time. The algorithm runs on single-core environments, but by discovering the identical parts of the different patterns and performing their matching at once, it can achieve noticeable performance gain, since only the different parts of the patterns must be matched separately afterwards.

6 CONCLUSIONS AND FUTURE WORK

In this paper, we have introduced a new pattern specification language. The presented concept makes it possible to define inexact patterns in a concise way. We have also presented the algorithm *MRMM*, a MapReduce-based method for detecting inexact patterns in large graphs. The algorithm finds all subgraphs corresponding to the defined pattern in the host graph.

The MapReduce framework is designed to support processing large data sets. Therefore, it can be suitable for graph related algorithms if the graphs are represented as textual files. In this paper, we have also described the applied data structure.

Because of the lack of space, in this paper, we focused on the description of the new language concept and the new matcher algorithm. We plan to present the detailed evaluation of the performance and the experiences collected during the application of the approach in a separate publication.

Other future work contains the analysis of the presented algorithms. Since the MapReduce framework is not optimized for I/O operations, the sizes of the produced outputs are critical. In order to evaluate the efficiency of the algorithms, we also intend to perform different measurements.

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