

# Hazardous Materials Transportation using Bi-level Linear Programming *Case-study of Liquid Fuel Distribution*

Madalena S. Rodrigues<sup>1</sup>, Marta C. Gomes<sup>1</sup>, Alexandre B. Gonçalves<sup>2</sup> and Sílvia Shruballs<sup>1</sup>

<sup>1</sup>CESUR, Instituto Superior Técnico, Universidade de Lisboa, Av. Rovisco Pais, Lisboa, Portugal

<sup>2</sup>ICIST, Instituto Superior Técnico, Universidade de Lisboa, Av. Rovisco Pais, Lisboa, Portugal

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**Abstract:** Hazardous materials (hazmats) are essential for the competitiveness of contemporary societies, however their transportation is potentially dangerous and expensive. In Portugal, despite the interest of both academia and industry, studies enabling the identification of preferable road routes for hazmats distribution were not identified. Hence, this research aims at contributing to advancing knowledge in identifying these routes in the national current context by balancing two frequently intrinsically conflicting aspects of hazmats transportation: the safety and the economic viability of the available routes. For that, a bi-level linear programming model was implemented in the GAMS modelling system and applied to a real-world case study using petrol and diesel fuels delivery data from a prominent energy group acting in the country. The company shared data of distribution loads over one calendar year to both petrol stations and direct clients in Lisbon. A geographical information system (GIS) was used to map Lisbon road network, which was found to be significantly larger than other networks used in similar studies described in the literature. The model was solved to optimality in a short computation time leading to the clear identification of the preferable road routes for liquid fuel distribution in the Lisbon district of Olivais. The success of the methodology applied in this study, including the generic implementation of the bi-level linear programming model, offers an optimistic prospect for a gradual increase of the geographical coverage, assessed risks and general complexity of the initial model.

## 1 INTRODUCTION

Hazardous materials (hazmats) are substances dangerous to handle because of their flammable, explosive or toxic nature, comprising serious threats to human safety and health, to property or to the environment (HMCPR, 2009). However, the economic success of societies requires the transportation of considerable amounts of hazmats: indeed, in Portugal, 10% of the total materials carried by road are hazmats (ANPC, n/d). Although the transportation of hazmats is associated to relatively few accidents, their consequences can be serious due to the nature of the cargo (PHMSA, 2011). The transportation of hazmats is, therefore, subject to special safety requirements to protect populations and the environment, and it has been recognized as important to find a balance between

these and the economic viability of the operation. The hazmats industry generally places safety at the centre of business and the analysis of safe route definitely requires further studies beyond those presented in the literature.

A political tool commonly used to reduce the risk of hazmats transportation is the interdiction by the regulator of certain road sections identified as more vulnerable; the carrier can then choose the best routes in the available network. Erkut et al. (2007), in a survey on hazardous materials transportation, name this the hazmat transportation network design problem and state that it started receiving the attention of researchers with the work of Kara and Verter (2004). These authors developed a bi-level linear programming model where hazmats are grouped into categories according to the risk impact. The model, which assigns an available road network to each hazmat category, was applied to the region

of Western Ontario, Canada.

Later on, the same authors proposed a linear programming model based on the shortest path formulation (Verter and Kara, 2008). In this formulation, routes considered economically infeasible were left out of the model, assuring that the carriers would not be forced to use routes that were their least preferable choices. Other approaches to this problem are the ones of Erkut and Alp (2007) and Erkut and Gzara (2008). The first authors developed an algorithm in two phases: in the first stage a minimum risk network is found, while in the second stage the network is expanded in an iterative procedure. This enables the regulator to control the density of the hazmats network and the freedom given to the carriers. Erkut and Gzara (2008) used a flow problem formulation in a bi-level network, and compare four networks scenarios related to different decision levels: non-regulated model, over-regulated model, two-step model and bi-level model.

In the present research the model of Kara and Verter (2004) was implemented so as to identify safe routes for the transportation of hazmats using real data provided by a major company of the energy sector in Portugal. Particularly noteworthy is the case study dimension, which significantly surpasses the ones of similar studies described in the reviewed literature and posed a significant challenge to model implementation.

## 2 A BI-LEVEL LINEAR PROGRAMMING MODEL FOR HAZARDOUS MATERIALS TRANSPORTATION

This section defines the problem under study and presents the bi-level linear programming formulation of Kara and Verter (2004), followed by an equivalent mixed-integer linear programming (MILP) model, which was the one implemented and solved with available computational tools.

### 2.1 Problem Definition

A bi-level model consists of two optimization problems that are hierarchically related and belong to two distinct decision makers, in which the optimal decision for one decision maker is constrained by the choices of the other one (Bianco et al., 2009). In the present problem the regulator assumes the leading role, as its decisions are taken at the first level and the choice of routes by the carriers will

depend on them. Thus, the outer-level problem is the regulator concern and determines the links to include in the network to be made available, according to the criteria of total risk minimization, while the inner-level problem, pertaining to the carriers, consists of the route choice in this network. Kara and Verter (2004) present a model that determines the network of minimum total risk and assumes the cost minimization of the carriers, achieving significant risk reductions in the transportation of hazmats in Western Ontario, Canada.

An accident resulting in a release of the hazmat is called an incident (Erkut et al., 2007). The model of Kara and Verter (2004) assumes that the undesired consequences of an incident involving hazmats occur within a given distance from the place where it happened, which varies according to the type of hazmat. Thus, hazmats are grouped into categories according to the impact of incidents associated with each of them.

To assess the risk associated to hazmats transportation, additivity of impacts is assumed. The risk for each link is considered to be known and independent of the direction of each shipment. It is also assumed that every point in the same link has the same incident probability and level of consequences. The sum of risk of the transportation activity in each link then results in the linearity of the objective function (Erkut et al., 2007). In Kara and Verter (2004) model the risk is measured by population exposure and the travelled distance is the criteria for the choice of routes by the carriers, but the methodology can be easily used with other risk and cost measures.

One of the strategies to solve bi-level linear programming problems consists in the application of the Karush-Kuhn-Tucker conditions (KKT) that transform the bi-level model into a single level one. This model, which is equivalent to the initial formulation, can then be solved with a commercial solver (Bianco et al., 2009). Kara and Verter (2004) used the KKT transformation to solve the proposed bi-level model.

### 2.2 Mathematical Formulation of the Model

Based on the problem description, the following sets, parameters and variables are defined:

#### Indices

- $c$  – shipment
- $p$  – population centre
- $i, j, k$  – node
- $m$  – type of hazmat

Sets

$C$  – all shipments across the network,  $c \in C$ . Each shipment is characterized by an origin node, a destination node and a type of hazmat transported  
 $P$  – population centre,  $p \in P$ . Set of population centres affected by the activity of transport of hazmats  
 $N$  – nodes,  $i, j \in N$   
 $A$  – links,  $(i, j) \in A$ , where  $(i, j)$  designates the link connecting nodes  $i$  and  $j$ , with direction  $i \rightarrow j$   
 $M$  – hazmat types,  $m \in M$

Parameters

$\rho_{ij}^{p,m}$  – number of people in  $p$  exposed to a truck carrying hazmat  $m$  through link  $(i, j)$   
 $l_{ij}$  – length of link  $(i, j)$   
 $n^c$  – number of trucks used for shipment  $c$   
 $R$  – a large positive number

Auxiliary variables

These variables appear only in the transformed model with KKT conditions.  
 $\nu_{ij}^c$  and  $\lambda_{ij}^c$  are positive real variables, while  $\omega_i^c$  is a real variable (positive or negative).

Decision variables

The model decision variables are binary:  
 $Y_{ij}^m = 1$  if link  $(i, j)$  is available for transportation of hazmat type  $m$ ,  $Y_{ij}^m = 0$  otherwise.  
 $X_{ij}^c = 1$  if link  $(i, j)$  is used for shipment  $c$ ,  $X_{ij}^c = 0$  otherwise.

**2.2.1 Bi-level Linear Programming Model**

Using the above definitions, the bi-level model is formulated as follows:

Objective function

$$\min \sum_{p \in P} \sum_{(i,j) \in A} \sum_{c \in C} n^c \rho_{ij}^{p,m(c)} X_{ij}^c \tag{1}$$

Subject to:

$$Y_{ij}^m \in \{0,1\} \quad \forall i, j \in A, m \in M \tag{2}$$

Where  $X_{ij}^c$  solves:

$$\min \sum_{c \in C} \sum_{(i,j) \in A} n^c l_{ij} X_{ij}^c \tag{3}$$

Subject to:

$$\sum_{(i,k) \in A} X_{ik}^c - \sum_{(k,i) \in A} X_{ki}^c = \begin{cases} +1 & i = o(c) \\ -1 & i = d(c) \\ 0 & \text{otherwise} \end{cases} \quad \forall i \in N, c \in C \tag{4}$$

Where:

$o(c)$  – origin node of shipment  $c$   
 $d(c)$  – destination node of shipment  $c$

$$X_{ij}^c \leq Y_{ij}^{m(c)} \quad \forall (i, j) \in A, c \in C \tag{5}$$

$m(c)$  – hazmat carried in shipment  $c$

$$X_{ij}^c \in \{0,1\} \quad \forall (i, j) \in A, c \in C \tag{6}$$

The outer-level problem (with objective function (1)) regards the decisions of which links should be made available for hazmats transportation, while the inner-level problem, represented by objective function (3) and constraints (4) - (6), deals with the choice of the transportation routes by the carriers. The binary decision variables ( $Y_{ij}^m$ ) of the outer-level problem are parameters for the inner-level problem, wherefore given the values of  $Y_{ij}^m$  the inner problem consists of determining the minimum cost flow in the network, by minimizing the total distance covered by the trucks (objective function (3)).

Constraints (2) establish the binary nature of decision variables  $Y_{ij}^m$ .

The requirements of flow balance are expressed in constraints (4). For an intermediate node (which is neither the origin nor the destination node of a shipment) equation (4) ensures the hazmat enters and leaves the node (right hand side equal to zero). For an origin node, it ensures the hazmat leaves the node (right hand side equal to +1) while for a destination node it guarantees the hazmat enters the node (right hand side equal to -1).

Constraints (5) assure that only the links that the regulator makes available for a given hazmat ( $Y_{ij}^{m(c)} = 1$ ) can be used by the carriers in a shipment of that hazmat ( $X_{ij}^c = 1$ ).

Constraint (6) sets decision variables  $X_{ij}^c$  to be binary.

**2.2.2 Transformed Model with KKT Conditions (MILP Model)**

Objective function

$$\min \sum_{p \in P} \sum_{(i,j) \in A} \sum_{c \in C} n^c \rho_{ij}^{p,m(c)} X_{ij}^c \tag{1}$$

Subject to:

$$\sum_{(i,k) \in A} X_{ik}^c - \sum_{(k,i) \in A} X_{ki}^c = \begin{cases} +1 & i = o(c) \\ -1 & i = d(c) \\ 0 & \text{otherwise} \end{cases} \quad \forall i \in N, c \in C \tag{2}$$

$$X_{ij}^c \leq Y_{ij}^{m(c)} \quad \forall (i, j) \in A, c \in C \tag{3}$$

$$n^c l_{ij} - \omega_i^c + \omega_j^c - v_{ij}^c + \lambda_{ij}^c = 0 \quad \forall c \in C, (i, j) \in A \quad (4)$$

$$v_{ij}^c \leq R(1 - X_{ij}^c) \quad \forall c \in C, (i, j) \in A \quad (5)$$

$$\lambda_{ij}^c \leq R[1 - (Y_{ij}^{m(c)} - X_{ij}^c)] \quad \forall c \in C, (i, j) \in A \quad (6)$$

$$v_{ij}^c \geq 0, \lambda_{ij}^c \geq 0 \quad \forall c \in C, (i, j) \in A \quad (7)$$

$$\omega_i^c \in \mathbb{R} \quad \forall c \in C, i \in N \quad (8)$$

$$X_{ij}^c \in \{0,1\} \quad \forall (i, j) \in A, c \in C \quad (9)$$

$$Y_{ij}^m \in \{0,1\} \quad \forall (i, j) \in A, m \in M \quad (10)$$

The model comprising expressions (1) to (10) is a mixed-integer linear programming model (MILP), with binary and continuous variables, and consists in the problem of identifying routes for hazmat transportation that minimize the risk of accident without compromising economic viability.

### 3 MODEL APPLICATION AND RESULTS

This section describes the MILP model application to obtain optimal routes for white oils (petrol and diesel) distribution to the company clients in the Olivais district of Lisbon.

#### 3.1 Data Collection and Processing

The Lisbon area road network available for this study (mapped in the ArcGIS software) comprised 35,981 links. This was foreseen as much larger than what could be supported in a successful computational implementation of the MILP model, and hence the network dimension was reduced. To this end, network parts (links and nodes) that neither belonged to Lisbon city nor were one of the main accesses to it were excluded. Route alternatives in zones without petrol stations or direct clients of the company were also eliminated and crossings reconfigured (by doing link junctions). As a result, a 7,276 link network was obtained, depicted in figure 1. Since this number was still very large, to achieve a successful model implementation in the first stage of this on-going research project, only the Olivais district and its main access routes (even if outside of the district) were considered (figure 2). The road network of this case study comprises 682 links and 461 nodes.

To characterise the white oils distribution, data concerning the location of clients in Lisbon municipality and the amount of fuel delivered daily during one calendar year was collected at the company, and then aggregated to obtain annual

values. Clients comprised petrol stations and direct clients supplied by the company in Lisbon.

To implement the objective function (1), where the risk is proportional to the amount of fuel delivered, the number of equivalent trucks was considered. These are the number of trucks needed to deliver the annual amount of fuel to each client and is obtained by the division of the total fuel distributed per year by the capacity of one truck (30 m<sup>3</sup>).

In order to incorporate risk in the network, census 2011 data was used to quantify the population living in each census ward (BGRI in Portuguese, meaning "geographical information referencing basis"). Population density was obtained by dividing the population by the area of each census ward polygon. Population exposure was computed with the following expression:

$$Pop \text{ exp.} = l_{link} \times (50 + 50) \times Density,$$

where *Pop exp.* is the exposed population, *l<sub>link</sub>* is the arc or link length, *Density* is the population density and 50+50 corresponds to a buffer of 50m for each side of the arc called *evacuation distance*.

Several strategies were used to deal with the complexity of integrating the company data with GIS software data. Thereby, the following simplifications were assumed:

##### *Transported materials*

- All white oils belong to the same model category;
- Each truck containing the same type of hazmat imposes the same risk.

##### *Population density*

- The population density around a road segment is constant;
- For links that cross two census wards, the population density considered corresponds to the average of both values;
- Accident probability is constant in each link.

##### *Road network*

- The shipments origin was considered to be the A1 highway entrance in the city of Lisbon (all the company shipments enter the city through this highway);
- Trucks were assumed to circulate at the maximum speed allowed for heavy trucks in each network link, lowered by a degradation coefficient of 5% to take traffic congestion into account;
- Speed in curves was assumed to be equal to speed in straight links;

- Additivity of impacts was assumed.

*Destination points*

- The destination points (petrol stations and direct clients) were represented by the projection of their real location in the nearest node of the road network;
- Only the 6 shipments to fuel stations and direct clients of the company in the Olivais district were considered.

Figure 1 displays the graphic representation of the road network and the liquid fuel destinations. Although dozens of destinations appear in figure 1, only those in the Olivais district (figure 2) were considered when solving the model.



Figure 1: Lisbon road network and location of the company liquid fuel destinations. The area depicted in Figure 2 is highlighted.

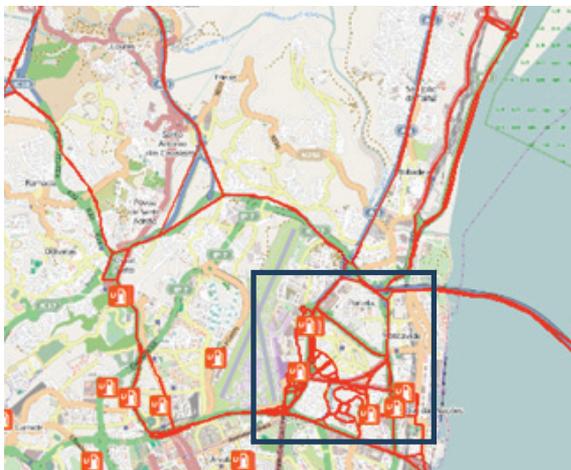


Figure 2: Detail of the Lisbon road network and location of the Olivais district destinations (highlighted area).

Finally, table 1 depicts the number of equivalent trucks per year of white oils delivered to Olivais district clients.

Table 1: Number of equivalent trucks per year in Olivais district destinations/clients.

Destinations	No. of trucks
1	264
2	216
3	68
4	97
5	22
6	186

**3.2 Results and Discussion**

The MILP model (transformed model with KKT conditions) was implemented in GAMS modelling system and solved with CPLEX version 12.4, on an Intel Core computer with an i3-2350M processor (2.3 GHz), 6 GB of RAM and running Windows 7 Home Premium. It should be highlighted the extremely useful xls2gms and.gdx GAMS functionalities for data input and output in spreadsheet format (Excel files).

Table 2 presents the numeric characteristics of the model (number of variables and constraints), the CPU time and number of iterations of the branch-and-bound search, the optimality gap and the objective function value.

Table 2: Summary of numeric characteristics and results of the MILP model.

Characteristic/result	Value
No. of variables	15,763
No. of binary variables	4,788
No. of constraints	33,007
No. of iterations	3,222
CPU time (s)	1.92
Optimality gap (%)	0
Value of objective function	838,335

Table 3: Detailed characteristics of the MILP model solution.

Destination	No. of links	Travel time (min)	Population exposed
1	58	21.1	966
2	38	18.5	706
3	43	19.5	710
4	73	27.9	1,117
5	71	27.1	1,117
6	72	27.8	1,342
<b>Total/year</b>		19,366	838,335
<b>Total/truck/year</b>		22.7	983

Table 3 presents the solution characteristics. For each destination, the number of links of the chosen route, the corresponding travel time and measure of

exposed population are shown. By multiplying these values by the number of trucks, the total travelling time spent and exposed population were obtained (line before the last in Table 3). Note that the latter value (838,335) is equal to the objective function value when solving the model (Table 2). By dividing these values by the total number of trucks, average travel time and exposed population per truck for the Olivais district were finally obtained (last line in Table 3).

As shown in table 2, the model was solved to optimality (0% gap) in less than 2s of CPU time, for a network dimension significantly larger than the ones used in similar studies. In fact, the Olivais district network displays 461 nodes and 682 links, while the network in the Kara and Verter (2004) study features 48 nodes and 57 links. This computational performance underlines the powerful tools that are currently available to solve optimization models.

As a final step in addressing this case study, an analysis was made to parameter R (often termed Big-M in the literature), varying it by powers of ten between 10 and  $10^{12}$ . For R values below  $10^4$  the model is infeasible. Feasible models were solved three times (for each R value) and the average of CPU time computed. This varied between 1.6s and 9.7s, with no observable increasing or decreasing trend regarding the R value variation.

## 4 CONCLUSIONS

It is the authors' understanding that there is a consensual belief, amongst the academia and the industry, that both further knowledge and practical tools to identify safe and economic viable routes for hazmats distribution, including in complex urban systems, requires further developments. This study applied a bi-level linear programming model in a consolidated area of Lisbon using data from one energy group operating in Portugal, aiming to contribute to find a balance between population risk and the economic viability of white oils (petrol and diesel fuels) transportation. The dimension of the road network (number of nodes and links), which was mapped in a GIS software, significantly surpasses those of similar studies described in the literature. The optimal solution (computed in negligible CPU time) displays the road links to be used for hazmats transportation in the Olivais district of Lisbon and identifies the routes for each company shipment in this area. Model results also quantify

population exposure to risk and the routes travel time.

As future developments, the authors intend to address the problem of liquid fuel distribution by this company in the city of Lisbon, which totals 26 destinations. A simplified road network may be then considered, presenting a smaller number of arcs than the aforementioned 7,276. Additionally, risk may be broken down in relation to population vulnerability in different stretches of roads. Indeed, the current study considered the resident population, when greater accuracy would be provided by considering the population effectively present in any area during the day, by identifying generator poles of each zone, such as services and jobs/schools. It is also recommended for future analysis the comparison between the routes currently used by the carriers trucks with those resulting from the optimization model.

This is a pioneer study in Portugal, which benefited from a successful collaboration between the academia and the industry, and is expected to be a first step in a hopefully gradual expansion of applied knowledge.

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